On Gormaund Numbers and Gormaund's Theorem

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Dedicated to my loving children

1 Gormaund Numbers

A Gormaund number is any $g \in \mathbb{N}$ with exactly three factors. In this, I have been inspired by the prime numbers, which have exactly two factors. The first few Gormaund numbers are:

g	factors		
4	1	2	4
9	1	3	9
25	1	5	25
49	1	7	49
121	1	11	121

You may see a pattern beginning to emerge, but is it provable?

2 Gormaund's Theorem

I say that for $g \in \mathbb{N}$, g is a Gormaund number iff it is the square of a prime.
To prove this, we consider all possible cases
For $g = 1$, g has only one factor and thus is not a Gormaund Number.
Otherwise, the Fundamental Theorem of Arithmetic tells us that g is a product of primes.
If g is prime, it has only two factors, and thus is not a Gormaund number.
If $g = p^2$ for some prime p, then g has factors $1, p$ and g, and therefore is a Gormaund number. If g is some higher power of p, then g has at least factors of $1, p, p^2$ and g, and is therefore not a Gormaund number.
If g is a product of several distict primes, let two of them be p and q. g has at least factors of $1, p, q, g$ and so is not a Gormaund number.

Therefore, by examination of all the cases, only the square of a prime can be a Gormaund number, and all squares of primes are Gormaund numbers.
Q.E.D.

3 Gormaund's Corollary

It thus follows that there are infinitely many Gormaund numbers. For let g_n denote the nth Gormaund number, and let $g_1, g_2, \ldots g_n$ be the finite sequence of all Gormaund numbers. Then, by Gormaund's Theorem, $\sqrt{g_1}, \sqrt{g_2}, \ldots \sqrt{g_n}$ creates a finite sequence of all primes. But by Proposition 20, Book IX of Euclid's Elements, this is impossible. Therefore, there are infinite Gormaund numbers.

4 Gormaund's Second Corollary

It is therefore possible to relate the Gormaund numbers to the Riemann zeta function, like so.

$\zeta(s) = \prod_{i=1}^{\infty} \frac{1}{1-g_i^{-\frac{1}{2}s}}$

Thus showing a deep link between the Gormaund numbers and the Riemann Hypothesis, a great unsolved problem of mathematics.