Analytic Demonstrations on the Fourfold Root Topics of Primes

Maximal Gaps between Consecutive Primes, the Number of Primes at a Given Magnitude, the Location of Nth Prime and the General Behavior of Primes

By Shaban A. Omondi Aura

E-mail: nabongoknowledge.musindalomath@yahoo.com

Abstract

This paper is concerned with formulation and demonstration of new versions of equations that can help us resolve problems concerning maximal gaps between consecutive prime numbers, the number of prime numbers at a given magnitude and the location of nth prime number. There is also a mathematical argument on why prime numbers as elementary identities on their own respect behave the way they do. Given that the equations have already been formulated, there are worked out examples on numbers that represent different cohorts. This paper has therefore attempted to formulate an equation that approximates the number of prime numbers at a given magnitude, from $N = 3$ to $N = 10^{25}$. Concerning the location of an nth prime number, the paper has devised a method that can help us locate a given prime number within specified bounds. Nonetheless, the paper has formulated an equation that can help us determine extremely bounded gaps. Lastly, using trans-algebraic number theory method, the paper has shown that unpredictable behaviors of prime numbers are due to their identity nature.

Introduction:

From the days of Pharaonic Africa, through those of Euclid and Eratosthenes and those of Gauss, Euler and Legendre and finally to ours, the encryption of prime numbers remains like unresolved scandal in Analytic Number Theory. Cognitions of professional and hobbyist mathematicians have at least for better times been dedicated to solving the 3000 year question (Guy, 2013). To my view, this question is fourfold and it entails realization of equations or arguments that can determine and explain maximal gaps between prime numbers, the number of prime numbers at a given magnitude, the location of nth prime number in the prime sequence and reasons why primes behave the way they do. There seem to be no predictable occassionalism in prime numbers: a prime number can pop up at any point from 1 to ∞. The effect of this behavior has been realization of gaps between prime numbers whose sizes cannot be fully predicted.

There are two general theoretical themes being articulated in *Analytic Demonstrations*:

1. Natural trends in prime numbers are influenced by continuous and successive interactions of natural exponentials.
2. Prime numbers, being original elementary identities with no unifiers or pre-images for one-to-one correspondence, cannot be aligned entirely with the same relationship on an extended field.

The whole essay is organized in two parts. PART I involves maximal gaps between prime numbers, the number of prime numbers at a given magnitude and the location of nth prime number in the prime sequence. Part II is concerned with trans-algebraic number demonstrations and resolution on identity behaviors of prime numbers.

Epicureans knew very well that the purpose of Philosophy, and hence that of Mathematical Logic, has to be making life happier, more tranquil and self-sufficient— with peace and freedom—and without pain and fear. I am pleased to read in literatures of currency that despite the known applications of prime numbers in cryptography, there are greatly proposed relatedness between trends in prime numbers and those in complex regions of science such as Quantum Mechanics (Cook, 2015). So it is my great hope that the current paper will just be so much applicable in those areas and even others.

I have at large used services of trans-algebraic number method and statistical calculus and probability to derive and demonstrate my arguments. Hume argued in the *Treatise of the Human Nature* that when elementary substances are compounded, their properties in relation to other substance can be inferred (Hume, 2007). It is by this motivation that this analysis has considered the use of trans-algebraic number method to explain and resolve issues concerning general identity behaviors of prime numbers. Proper use of the Theory of Probability, and says Laplace, is essential to reduce ignorance and avoid inchoate assumptions about events that do not seem to be regulated by Laws of Nature. Laplace also noted in the *Philosophical Essay on Probabilities* that indeed laws of calculus are essential in realizing how statistical magnitudes decrease or increase (Laplace, 1902). I am aware of underway researches in the Riemann's hypothesis and other hypotheses motivated by zeta (ζ) functions. *Analytic Demonstrations* has no developed faculties to make clear and developed critic of judgments on them. Therefore, safe Euler's zeta (ζ) function, no allusion will be made on the entire class of zeta (ζ) functions in the ranks of current analysis. *Analytic Demonstrations* will be concerned with proposing equations and rendering discussions on them. Importantly, demonstration will be carried out on different numbers that represent different number cohorts. Without hazards, we will use results of such demonstrations to make opinions concerning the power of every equation to give results it purports to give. Applications of works of Euclid, Gauss, Legendre, Euler and Abel cannot be underestimated, and, therefore, we cannot begin this paper without appreciating their excellent intelligences with greatest esteems.

PART I: INFLUENCE OF EXPONENTIALS ON NATURAL TRENDS OF PRIME NUMBERS

This part is concerned with exploiting the fact that natural trends of prime numbers are influenced by natural trends of exponentials. Primarily, the exponentials talked about involve common logarithm$(\log N)$, natural logarithm $(\ln N)$ and Euler's number, $e = \mathbf{2.718281828459045235360287471352 7}$. The exponentials can have various derivatives depending on their levels of exponentiation. For instance, at the second level we can have $\log\log, \ln\log, \log\ln,$ and $\ln\ln$. At the third and fourth levels we may have $\log\log\ln$ and $\log\ln\ln\ln$ respectively. I have noted, as it will be shown in demonstrations, that natural trends of prime numbers are influenced by natural trends of exponentials. So, one has to use the right and well appropriated exponentials to get better results about natural trends of prime numbers.

A. ON THE MAXIMAL GAPS BETWEEN CONSECUTIVE PRIME NUMBERS, $\boldsymbol{g(P_n)}$

The aim of this section is to formulate equations for maximal gaps and demonstrate how they can be determined and used in other areas of prime number analysis. Essentially, I am interested in finding an extremely bounded gap equation for ascertaining the maximum level at which no gap can exceed however large it is. A gap between consecutive prime numbers can, in simple terms, be defined as the number of composites between two consecutive prime numbers. Conventionally, a gap between consecutive prime numbers is denoted as $g(P_n)$, so that the question of finding the next prime number, P_{n+1}, after the gap is represented by:

$$\boldsymbol{P_{n+1} = P_n + g(P_n) + 1,}$$

Explicitly, $g(P_n)$ is the gap between two prime numbers, $\boldsymbol{P_n}$ and $\boldsymbol{P_{n+1}}$**;**

Therefore:

$$\boldsymbol{g(P_n) = P_{n+1} - P_n}$$

Excepting two prime numbers, 2 and 3, that have $g(P_n) = 0$ between them, it is presumed from the twin prime conjecture that the least gap between two consecutive prime numbers is 1. It is also important to note that a gap between two prime numbers can become arbitrarily large, especially as prime numbers become larger (Tapia, & Støleum, 2016). This can easily be verified in the table that follows.

Table 1: ***Occurrences of Prime Gaps***

Gap	After	Gap	After	Gap	After	Gap	After
0	2	33	1327	117	1349533	247	191912783
1	3	25	9551	131	1357201	249	387096133
3	7	43	15683	147	2010733	281	436273009
5	23	51	19609	153	4652353	287	1294268491
7	89	71	31397	179	17051707	291	1453168141
13	113	85	155921	209	20831323	319	2300942549
17	523	95	360653	219	47326693	335	3842610773
19	887	111	370261	221	122164747	353	4302407359
21	1129	113	492113	233	189695659	381	10726904659

A proof of occurrence of arbitrarily large gaps between consecutive primes can be borrowed from Euclid's work in consideration with a group of positive integers that bear the form:

(n+1)! +2, (n+1)! +3, (n+1)! +4, ………, (n+1)! + n, (n+1)! + (n+1).

With the axiom that the first term is divisible by 2, the second by 3, the third by 4 and the last by n+1, we conclude that the numbers (n+1)! + 2 to (n+1)! + (n+1) are composites. Although there is no clear evidence to show a concrete sequential structure of prime number gaps, it is evident from the table and Euclid's work that larger gaps are more likely to appear in larger prime numbers.

In this regard, we can let the common proportion of any prime number relative to base 10 be $\log P$. From the prime number theorem of Gauss and Legendre, $\pi(x) \sim \frac{x}{\ln x}$, we can elementarily let the natural rate at which such a prime number at the same relative proportion translate itself into the next prime number in the number sequence be represented by $\ln P$ (Apostol, 2013).

Given this way, the next prime number $\boldsymbol{P_{n+1}}$**,** can elementarily be represented by the equation:

$\boldsymbol{P_{n+1} \approx P_n + (log\ P_n\ \times ln\ P_n)}$**,** which can be appropriated well as:

$$\boldsymbol{P_{n+1} \approx P_n + (log\ P_n\ \times ln\ P_n - 1) + 1}$$

So, $\boldsymbol{g(P_n) \approx logP_n\ \times ln\ P_n - 1}$

Although $g(P_n) \approx logP_n\ \times ln\ P_n - 1$ will be our leading equation, there is a need to consider local adjustments by developing appropriate local factors.

General Local Factors

General local factors in this regard act as standard errors for common statistical approximations. They are simply $log\ P_n$ and $ln\ P_n$ depending on desired bounds. For very extreme bounds, near the gap ceiling or gap floor, one will have to use $\pm\ ln\ P_n$. Middle range gaps just require the use of $\pm\ log\ P_n$ as the local factor for making adjustment. Since $logP_n\ \times ln\ P_n - 1$ is the leading equation, it is expected that core gaps will conform to it. The leading equation is expected to represent an average or central region for all gaps regardless of their ranges. For this analysis, any gap that is less than the gap floor will not be considered maximal at its local point. Such a gap will be below the expected lowest range for all maximal gaps.

Gaps for Prime Numbers, N$\leq$ 1000

Before demonstrating how we can estimate sizes of gaps using the formulated equations, it is important to put the gaps into three ranges: core gaps, mid-range gaps and extremely bounded gaps.

Core Gaps:

To estimate core gaps, one does not need to do any form of adjustments. This can be demonstrated in the following instances:

a) **The gap after the prime number 19:** one can simply carry out the following operation:
   $\log 19 \times \ln 19 - 1 =\ 2.7652119475 \approx 3$
b) **The gap after the prime number 293:** carry out the operation:
   $\log 293 \times \ln 193 - 1 = 13.012234 \approx 13$
c) **The gap after the prime number 317:** carry out the following operation:
   $\log 317 \times \ln 317 - 1 = 13.403354622 \approx 13$

Mid-range Gaps:

Mid-range gaps are those found between $logP_n\ \times ln\ P_n - 1$ and $(logP_n\ \times ln\ P_n - 1)\ \pm\ log\ P_n$. For instance:

a) **Finding the gap after 13**: It involves the following operation:
$(\log 13 \times \ln 13 - 1) + \log\ 13 = 2.971151638 \approx 3$

b) **Finding the maximal gap after 23:** the following operation can be carried out:
$(\log 23 \times \ln 23 - 1) + \log\ 23 = 4.6314175894 \approx 5$

Extremely-bounded Gaps:

These are gaps found between the mid-range and the ceiling or floor of maximal gaps. The ceiling or the floor of maximal gaps is simply given by the equation:

$$(logP_n \times ln\, P_n - 1) \pm ln\, P_n$$

To use this equation one can practice on the following prime numbers:

(a) **After 7**: as $(log7 \times ln\, 7 - 1) + ln7 = 2.5903950021 \approx 3$
(b) **After 113**: as $(\log 113 \times ln\, 113 - 1) + ln113 = 13.433085843 \approx 13$
(c) **After 863**: as $(\log 863 \times \ln 863 - 1) - \ln\ 863 = 12.088235825 \approx 12$. The real gap is 13. However, as a matter of categorical approximation, the gap is between the mid-range and the floor. So, it is extremely-bounded.

Gaps for Prime Numbers, N$\geq$ 1000

For prime numbers after N≥1000, maximal gaps start appearing arbitrarily larger to the point of not being estimated by the leading equation $logP_n \times ln\, P_n - 1$ with general local factors alone. Appropriations have to be made with regards to underlying changes in log P and ln P. The leading equation has to be multiplied by a new adjustor, γ.

Where $\gamma = \{|logloglogP_n| + |lnlnlnP_n| + |loglnlnP_n|\}$

So the new complete equation is:

$$g(P_n) = \begin{cases} \{(logP_n \times ln\, P_n - 1)\}\gamma & for\ core\ gaps \\ \{(logP_n \times ln\, P_n - 1)\}\gamma \pm logP_n & for\ mid-range\ gaps \\ \{(logP_n \times ln\, P_n - 1)\}\gamma \pm lnP_n & for\ extremely\ bounded\ gaps \end{cases}$$

Just around the Core:

(a) **After the prime number 10726904659:**

(10.030474421*23.096020877-1)1.6415708223

(230.7) 1.6415708223≈ 379. The real gap is 381.

(b) **After the prime number 42842283925351:**

(13.6318726150482*31.388546673004-1) 1.82953956712 + 13.6318726150482

=782.8≈ 783. The real gap is 777.

In the Mid-range:

(a) **After the prime number 492113:**

(5.6920648378*13.106463644-1)1.4774689955+5.6920648378

=114.43798≈114. The real gap is 113. However due to our categorical approximation, the gap is between the core line and the mid-range line.

(b) **After the prime number 277900416100927:**

(14.44388919705* 33.25828395-1) 1.862933372305−14.44388919705

= 878.6071887509415953152370522050 1

≈ 879. The correct answer is 879. This gap lies on the floor.

In the Extremely-bounded Region:

(a) **After the prime number 1327:**

(3.122871*7.1907-1)1.2803123973 +7.1907

34.66051063≈ 35. The real gap is 33, which is between the mid-range and the ceiling.

(b) **After the prime number 218209405436543:**

(14.33887347*33.0164763-1) 1.85875512805 +33.0164763

878.1113802727034514741840960 5+ 33.0164763= 911. The real gap is 906. Note that the prime number 218209405436543 is less than the previously analyzed prime number 277900416100927 which has the gap of 879.

(c) **After the prime number 1129:**

(3.052694*7.0291-1)1.2724478144$-$ 7.0291=19.0023 $\approx$ 19. The correct answer is 21 composites. Thus, it is between the floor and the mid-range.

Section Remark:

From the above chosen prime numbers for demonstrations, it is evidenced that the gap after a prime number P_n cannot exceed $(logP_n \times ln\, P_n - 1) + ln\, P_n$, as the upper bound, and $(logP_n \times ln\, P_n - 1) - ln\, P_n$, as the lower bound when N$\leq$ 1000. If N$\geq$ 1000, It is also expected that the gap after a prime number P_n cannot exceed $\{(logP_n \times ln\, P_n - 1)\}\gamma + lnP_n$ on the upper end and $\{(logP_n \times ln\, P_n - 1)\}\gamma - lnP_n$ on the lower end. Applications of these extremely-bounded results are essential in section C when determining the bounded location of a given prime number.

B. ON THE NUMBER OF PRIMES AT A GIVEN MAGNITUDE $\emptyset \boldsymbol{P}$

This section involves derivation of a prime number counting function, including local factors for making appropriate adjustments as one goes along the path followed by prime numbers.

The normalizer

We begin with stating the Euler Product Formula such that the zeta of s can be presented as the product of all primes p in the form of $\frac{1}{(1-p^{-s})}$.

In this regard, $\zeta(\mathrm{s}) = \prod_p \frac{1}{(1-p^{-s})}$

The expanded form of this function can be given as:

$$\zeta(\text{s}) = \mathbf{1}+ \frac{\mathbf{1}}{\mathbf{2}^{s}} + \frac{\mathbf{1}}{\mathbf{3}^{s}} + \frac{\mathbf{1}}{\mathbf{4}^{s}} + \frac{\mathbf{1}}{\mathbf{5}^{s}} + \frac{\mathbf{1}}{\mathbf{6}^{s}} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\infty$$

Just as we do with the sieve of Eratosthenes, one can eliminate all prime numbers together with their multiples from the right hand side, as one makes necessary treatments on the left hand of the equation (Abramowitz, & Stegun, 2012). Therefore, as one approaches infinity, to obtain the nth prime numbers, and hence the number of prime numbers $\emptyset P$ at $N,$ one will have to remain with the equation:

$$\left(\mathbf{1}-\frac{\mathbf{1}}{p_n^s}\right).\left(\mathbf{1}-\frac{\mathbf{1}}{p_{n-1}^s}\right).\left(\mathbf{1}-\frac{\mathbf{1}}{p_{n-2}^s}\right)\ldots\ldots\left(\mathbf{1}-\frac{\mathbf{1}}{\mathbf{5}^s}\right).\left(\mathbf{1}-\frac{\mathbf{1}}{\mathbf{3}^s}\right).\left(\mathbf{1}-\frac{\mathbf{1}}{\mathbf{2}^s}\right).\ \zeta(\text{s}) = 1$$

This equation can be simplified to acquire the form:

$$\prod_{p} \frac{\mathbf{1}}{(\mathbf{1}-p^{s})} .\ \zeta(\text{s}) = 1$$

Visibly, there are two spectral distributions that have to be normalized for one to consider before carrying out statistical operations of finding a counting function. Further, it should also be noted that the distributions are inform of a product and one has to consider getting their products to form a general normalizer value of the whole product distribution (Kim, 2015). But first let us consider the Gaussian integral which is given as:

$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ As the normalizer of a single spectral distribution (Major, 2013).

In case of two product spectral distribution, Fubini’s theorem can be applied (Morozov, & Shakirove, 2009; Taillie, Patil, & Baldessari, 2012).). In this regard, a product integral is seen as an integral of an area, so that:

$$\int_{-\infty}^{\infty} e^{\{\zeta(s)\}^2}\, d\zeta(s) \int_{-\infty}^{\infty} e^{\left\{\prod_{p}\frac{\mathbf{1}}{(\mathbf{1}-p^{s})}\right\}^2}\, d\left(\prod_{p} \frac{\mathbf{1}}{(\mathbf{1}-p^{s})}\right\}$$

This yields $(\sqrt{\pi})\times(\sqrt{\pi}) = \pi$ = 3.1415926535897932384626433832795

Probability of a number in the distribution

For a large N, for instance N=1 million, the probability that a given number will be chosen can be given by $1/n$. However, there is a need to integrate over the range that $1/n$ is defined. This yields $\ln n$, and as the distribution goes to infinity, there is need to multiply the probability integral with Euler's number e to result in $e.\ln n$ (Wasserman, 2013).

Error Term

In statistics, it is usually believed that an experimenter is likely to record more erroneous results when the sample of the population being studied is small. Thus, as the sample size increases, average recorded errors are more likely to be reduced (Wasserman, 2013).

So, let $1/\log n$ represent the rate at which such average errors are occurring. Integrating the range over which $1/\log n$ is defined will result in $\ln \log n$.

The Leading Equation for Prime Number Counting

One has to normalize the number that represent the magnitude at which the number of prime numbers is being sought by multiplying it by the normalizer. The resulting product, πn has to be divided by appropriated probability value $e.\ln n$. The quotient can be added to the error term, $lnlogn$, to get the approximate number of primes at such magnitude. The leading equation for prime counting is:

$$\boldsymbol{\emptyset P \approx \frac{\pi . n}{e.\ln n} + \ln \log n}$$

The Number of Prime Numbers at N< 2000

Calculating $\emptyset P$ at N< 2000 does not need any local adjustments on the equation, and therefore it can be used without alterations.

Table 2: ***Selected Examples***

At Magnitude N=	Commentary
8: $$\emptyset P = \frac{\pi . 8}{e.\ln 8} + \ln \log 8$$ $\emptyset P$ = **4.3444 ≈4**	Correct approximation
25:	

$$\emptyset P = \frac{\pi.25}{e.\ln 25} + \ln\log 25$$ **$\emptyset P$ = 8.9711 ≈ 9**	Correct approximation
100: $$\emptyset P = \frac{\pi.100}{e.\ln 100} + \ln\log 100$$ **$\emptyset P$ = 25.78945 ≈ 26**	The correct $\emptyset P$ is 25. The result is approximately 26 since N=100 is closer to the 26th prime number 101 than it is to the 25th prime number 97.
520: $$\emptyset P = \frac{\pi.520}{e.\ln 520} + \ln\log 520$$ **$\emptyset P$ = 97.096806581 ≈ 97**	Correct approximation
1000: $$\emptyset P = \frac{\pi.1000}{e.\ln 1000} + \ln\log 1000$$ **$\emptyset P$ = 168.4073 ≈ 168**	Correct approximation
2000: $$\emptyset P = \frac{\pi.2000}{e.\ln 2000} + \ln\log 2000$$ **$\emptyset P$ = 305.29690023 ≈ 305**	The correct count is 303. Although N=2000 is just close to the 304th prime number, it is evident that the equation is now drifting from the natural prime number trend, and hence it needs adjustments from that point.

The Number of Prime Numbers at N≥ 2000

Continued use of the already discussed leading equation will lead to large errors due to higher result when $N \geq 2000$. There is a significant exponential transition between N=1000 and N=10000. One has to make tables of exponentials and their derivatives to realize the exponential or the derivative that is responsible for the transition. Once the right exponential is identified, it has to be appropriated properly to render good results. So, the leading equation has to be adjusted to become as follows:

$$\emptyset P \approx \left(\frac{\pi . n}{e . \ln n} + \ln \log n\right)\left(\Psi^{\psi}\right)$$

Where $\Psi = 0.9950940298 \ldots.$

$\psi = f(\ln N, logglogln\ N, lnloglog\ N)$

Specifically, $\psi = \alpha \pm \beta$

α Is calculated as:

$$\alpha = \begin{cases} \{1 - (|loglogln\ 3| - |loglogln\ N|)\} \times \ln N & if\ loglogln\ N < 0 \\ \{1 - |loglogln\ N|\} \times \ln N & if\ loglogln\ N > 0 \end{cases}$$

β Is calculated as:

$$\beta = \begin{cases} \frac{(Resd.\ln N)^2 \times lnloglog\ N}{(loglog\ N \times lnlnln\ N)^2} & if\ both\ loglogln\ N\ and\ lnloglog\ N > 0 \\ \frac{(Resd.\ln N - 1) \times |loglog\ N| \times |lnloglog\ N|}{\{1 + |lnloglog\ N|\}} & if\ loglogln\ N > 0\ while\ lnloglog\ N < 0 \\ \frac{(Resd.\ln N - 2) \times |loglog\ N| \times |lnloglog\ N|}{\{2 + |lnloglog\ N|\}} & if\ both\ loglogln\ N\ and\ lnloglogN < 0 \end{cases}$$

$Resd. \ln N$ Is the magnitude of $\ln N$ that remains after taking away α from the total value of $\ln N$.

Thus, $Resd. \ln N$ can simply be determined as:

$$\ln N - \alpha = Resd. \ln N$$

Previously, we have written that $\psi = \alpha \pm \beta$.After the above definitions, we are now allowed to state in definite terms what ψ has to be.

$$\psi = \begin{cases} \alpha + \beta & if\ and\ only\ if, \ln loglog\ N < 0 \\ \alpha - \beta & if\ and\ only\ if, lnloglog\ N > 0 \end{cases}$$

Appropriation for Large Numbers

Since ln log n is usually a small quantity, even in N≤ 1000, it contniously becomes insignificant as numbers being analyzed increase in size. In this regard, the equation

$\emptyset P \approx \frac{\pi . n}{e . \ln n} + \ln \log n$ can become:

$\emptyset P \approx \frac{\pi.n}{e.\ln n} + o(\ln \log n)$ as N goes to infinity. It is in this regard that we will consider the segment $\frac{\pi.n}{e.\ln n}$ only. Thus, the appropriated equation for larger numbers is:

$$\boldsymbol{\emptyset P \approx \left(\frac{\pi.n}{e.\ln n}\right)\left(\Psi^{\psi}\right).}$$

***Selection of the Constant* $\boldsymbol{\Psi = 0.9950940298.........}$**

From the formulation of ψ, it is indeed visible that it will depend on α only when $\ln loglog\ N = 0$. But $\ln loglog\ N = 0$ occurs at N=10^{10}. At N=10^{10}, the calculated $\emptyset P$=501, 925, 988.97, whilst the physically counted $\emptyset P =$ 455,052,511. Since the calculated value is higher than the physically counted one, there is need to reduce it, by $\frac{455{,}052{,}511}{501{,}925{,}988.97} = 0.906612769611335$, to become approximately equivalent to the correct figure. So, Ψ^{ψ} at 10^{10} is 0.906612769611335. We can now get Ψ by simply finding the inverse of $\Psi^{\psi} =$ 0.906612769611335. Thus,

$$\Psi^{1/\psi} = 0.906612769611335^{1/\psi}$$

However, since $\psi = \alpha$ we have to find α.

$$loglogln\ N > 0 \quad \text{So, } \alpha = \{1 - |loglogln\ 10^{10}|\} \times \ln 10^{10}$$

$$= 19.93472535$$

The original $\boldsymbol{\Psi}$ can then be retrieved as: $0.906612769611335^{1/19.93472535} =$ $\mathbf{0.99509402982660239227775946583523}$.

Table 3: ***Selected Examples***

At Magnitude N=	Workings	Commentary
3500: $\boldsymbol{\emptyset P} \approx (\frac{\pi.3{,}500}{e.\ln\ 3{,}500} +$ $lnlog\ 3500)(\Psi^{\psi})$. (497. 95018171) *(	For convenience one has to begin with finding components of ψ. $loglogln\ N < 0$, so $\alpha = \{1 - (\lvert loglogln\ 3\rvert - \lvert loglogln\ 3500\rvert)\} \times \ln 3500$ = 2.8457956991. $both\ loglogln\ N\ and\ lnloglogN < 0,$	Correct approximation . Consider that N=3,500 is still not really a large

$0.9950940298^{3.6240811608}$) $\emptyset P$ =**488.48 ≈ 489**	thus, $\beta = \frac{(Resd.\ln\ 3500-2)\times\lvert loglog\ 3500\rvert\times\lvert lnloglog\ 3500\rvert}{\{2+\lvert lnloglog\ 3500\rvert\}}$ $= 0.7782854617$ $\ln loglog\ N < 0$, hence $\psi = 2.8457956991 + 0.7782854617$ $= 3.6240811608$ One can then calculate $\frac{\pi.3{,}500}{e.\ln\ 3{,}500}$= 495.68490648	number and hence $lnlog\ N$ is still significant.
$\mathbf{10^4}$ $\emptyset \boldsymbol{P} \approx (\frac{\pi.10^4}{e.\ln 10^4})(\Psi^{\psi})$. (1254.8150265)* $0.9950940298^{3.895913872}$ $\emptyset P$ = **1231.001292** **≈ 1231**	Finding components of ψ. $logglogln\ N < 0$, so $\alpha = \{1-(\lvert logglogln\ 3\rvert - \lvert logglogln\ 10{,}000\rvert)\}\times \ln$ 10,000 = 3.4360844988. $both\ logglogln\ 10^4\ and\ lnloglog\ 10^4 < 0,$ thus, $\beta = \frac{(Resd.\ln 10^4-2)\times\lvert loglog 10^4\rvert\times\lvert lnloglog\rvert}{\{2+\lvert lnloglog 10^4\ \rvert\}}$ $= 0.7782854617$ $\ln loglog\ N < 0$, hence $\psi = 2.8457956991 + 0.4598293732$ $= 3.895913872$ One can then calculate $\frac{\pi.10{,}000}{e.\ln\ 10{,}000}$= 1254.8150265	The correct $\emptyset P$ is 1229. The result is 1231 because N=10000 is nearer to twin primes 10007 and 10009, than it is to the 1229th prime, 9973
$\mathbf{10^6}$**:** $\emptyset \boldsymbol{P} \approx (\frac{\pi.10^6}{e.\ln 10^6})(\Psi^{\psi})$. (83654.3351)* $0.9950940298^{13.06047599}$ $\emptyset P$ = **78449.982419** **≈78, 450**	Finding components of ψ. $logglogln 10^6\ > 0$, so $\alpha = \{1-\lvert logglogln 10^6\ \rvert\}\times \ln 10^6$ = 13.02741003 $logglogln 10^6\ > 0\ while\ lnloglog\ 10^6 < 0$ thus, $\beta = \frac{(1-Resd.\ln 10^6)\times\lvert loglog 10^6\rvert\times\lvert lnloglog 10^6\rvert}{\{1+\lvert lnloglog 10^6\ \rvert\}}$ $= 0.0330659599$ $\ln loglog\ N < 0$, hence $\psi = 13.02741003 + 0.0330659599$ $= 13.06047599$ One can then calculate $\frac{\pi.10^6}{e.\ln 10^6}$= 83654.3351	Results due to average approximation The correct $\emptyset P$ is 78,498.
$\mathbf{10^{10}}$**:** $\emptyset \boldsymbol{P} \approx (\frac{\pi.10^{10}}{e.\ln 10^{10}})(\Psi^{\psi})$.	Finding components of ψ. $logglogln 10^{10}\ > 0$, so $\alpha = \{1-\lvert logglogln 10^{10}\ \rvert\}\times \ln 10^{10}$	Correct approximation . The correct $\emptyset P$ is

(501, 925, 988.97)* $0.9950940298^{19.93472535}$ $\emptyset P$ =455052510.76 $\approx \mathbf{455,052,511}$	$= 19.93472535$ $logglogln10^{10} > 0 \; while \; lnloglog \; 10^{10} = 0$ thus, $\beta = \dfrac{(1 - Resd.\ln 10^{10}) \times \lvert loglog10^{10} \rvert \times \lvert lnloglog10^{10} \rvert}{\{1 + \lvert lnloglog10^{10} \rvert\}}$ $= 0.0$ $\ln loglog\ N < 0$, hence $\psi = 19.93472535 + 0 = 19.93472535$ One can then calculate $\frac{\pi.10^{10}}{e.\ln 10^{10}}$= 501, 925, 988.97	455,052,511
$\mathbf{10^{11}}$**:** $\emptyset \boldsymbol{P} \approx (\frac{\pi.10^{11}}{e.\ln 10^{11}})(\Psi^{\psi})$. 4562963536.1* $0.9950940298^{21.22091345}$ $\emptyset P$ ≈**4110,755, 884.4**	Finding components of $\boldsymbol{\psi}$. $logglogln10^{11} > 0$, so $\alpha = \{1 - \lvert logglogln10^{11} \rvert\} \times \ln 10^{11}$ $= 21.598926373$ $Both \; logglogln \; N \; and \; lnloglog \; N > 0$ therefore, $\beta = \dfrac{(Resd.\ln 10^{11})^2 \times lnloglog10^{11}}{(loglog10^{11} \times lnlnln \; 10^{11})^2}$ $= 0.3780129153$ $\ln loglog\ N > 0$, now $\psi = 21.598926373 - 0.3780129153 = 21.220913458$ One can then calculate $\frac{\pi.10^{11}}{e.\ln 10^{11}}$ = 4562963536.1	Barely predictive. The physical count is 4,118,054,813
$\mathbf{10^{19}}$**:** $\emptyset \boldsymbol{P} \approx (\frac{\pi.10^{19}}{e.\ln 10^{19}})(\Psi^{\psi})$. $\emptyset P$ ≈**231,546,466,577, 122,376.**	Finding components of $\boldsymbol{\psi}$. $logglogln10^{19} > 0$, so $\alpha = \{1 - \lvert logglogln10^{19} \rvert\} \times \ln 10^{19}$ $= 34.338662$ $Both \; logglogln \; N \; and \; lnloglog \; N > 0$ thus, $\beta = \dfrac{(Resd.\ln 10^{19})^2 \times lnloglog10^{19}}{(loglog10^{19} \times lnlnln \; 10^{19})^2} = 7.5356766899$ $\ln loglog\ N > 0$, hence, $\psi = 34.338662 - 7.5356766899 = 26.802985310$ One can then calculate $\frac{\pi.10^{19}}{e.\ln 10^{19}}$ = 264171584525719288.	The correct figure is: 234,057,667,276,344,607. We can say this is just a bare approximation
$\mathbf{10^{25}}$**:** $\emptyset \boldsymbol{P} \approx (\frac{\pi.10^{25}}{e.\ln 10^{25}})(\Psi^{\psi})$.	Finding components of $\boldsymbol{\psi}$. $logglogln10^{25} > 0$, so $\alpha = \{1 - \lvert logglogln10^{25} \rvert\} \times \ln 10^{25}$ =	The correct figure is: 176,846,309,399,143,769,411,680.

$\emptyset P$ $\approx$176,723,008,631, 601,940,568,114.	$Both\ logslogln\ N\ and\ lnloglog\ N > 0$ therefore, $\beta = \frac{(Resd.\ln 10^{25})^2 \times lnloglog 10^{25}}{(loglog 10^{25}\ \times lnlnln\ 10^{25})^2} = 17.48868748564596$ $\ln loglog\ N > 0$, now $\psi = 43.429571 - 17.48868748564596 = 25.9408831$ One can then calculate $\frac{\pi.10^{25}}{e.\ln 10^{25}}$ = 200770404239027481258881.	So, this is also a bare approximation , but at least it gives a blue-print.

Section Remark:

We have finished our demonstrative survey on the number of primes at a given magnitude along the formulated general path of prime numbers. We have seen that the leading equation is only reliable when $N < 2000$. The number of primes continue to increase in diminishing rates every step one makes along the path described by the equation. One has to find right exponentials in appropriate forms to keep one on the same course with natural trends of prime numbers. From the selected examples, it is evident that the equation’s power continue to reduce. Reasons for this scenario can be the fact that we are using rational numbers that reduce clarities of calculations, as many and larger figures are plugged into the equation. Perhaps, secondly we have not used right and well appropriated exponentials to guarantee clear answers. Given that this was a statistical venture, we could only be sure of approximate figures.

C. ON THE LOCATION OF NTH PRIME NUMBER $\mathcal{L}_{pth}$

This section involves formulation of a prime locating equation and its local adjustors. The leading equation for prime location has to involve appropriation of some elements of the prime counting equation, since each of the two equations can be inverse of the other at some elementary levels.

Therefore, invariable components of the prime counting function can be reversed. For simplicity, I will represent $\emptyset P$ with t. Moreover, from gap analysis, we saw that two exponentials, log t and ln t, can explain existence of maximal gaps between consecutive prime numbers. However, although prime numbers increase exponentially with regards to the two exponentials in their sequence, this increase is based on additive exponentiation ($\log t + \ln t$). The first equation can now be presented as:

$$\boldsymbol{\mathcal{L}} \sim \frac{\boldsymbol{t \times e \times (\ln t + \log t)}}{\boldsymbol{\pi}}$$

The next step has to involve appropriation of error term. We understand that all prime numbers, except 2 and 3 can be put in modules to become simply as 6 ± 1. This allows us to say that a prime number after 2 and 3 is just $[6 + \text{error term}]$. But as from the prime counting function, we saw that the error term in prime number is $lnlog\ N$; it can be $lnlog\ t$ for the current case. Normalization of the variable part will make prime location error presentation to appear like: $6 + \frac{lnlog\ t}{\pi}$. The results for this small equation are also variable and hence have to be normalized. This leads to the new equation:

$$\frac{6 + \frac{lnlog\ t}{\pi}}{\pi}$$

As already seen, it is easier to predict results in small numbers than it is for larger numbers. This indicates that the magnitude of the error term increases as prime numbers become larger. Increment in prime numbers is relatively modest and we can just say that they are brought by common logs of t. The common logs can be accelerated at other modest rates, so that resulting rates are equal to $loglog\ t$. But $loglog\ t$ is variable and hence it needs to be normalized. Normalization will lead to:

$$\frac{\log \log t}{\pi}$$

To determine the likely error term at every nth prime number, we simply find the product v of the two equations:

$$v = \left(\frac{\frac{lnlog\ t}{\pi} + 6}{\pi^2}\right)(loglog\ t)$$

Therefore, the whole leading equation can be presented as:

$$\boldsymbol{\mathcal{L} \approx \frac{t \times e \times (\ln t + \log t)}{(\pi + v)}}$$

The Process of Locating a Prime Number

At this point, it is important to note that due to unpredictable nature of prime numbers, they cannot be easily "handpicked". What we can do in the current analysis is to find the bounds in which a given prime number can be found.

There are two options here. For relatively small primes, one can make use of maximal gaps. In this regard, we use ceilings of gaps to determine the location of P_{th} prime number. For relatively large prime numbers, the constant $\Psi = 0.9950940298$ can be used. The constant Ψ is used to adjust drifting results to conform to the desired trend. For operational purpose, let large numbers begin after $\boldsymbol{P_{th}}$**=1,500.**

For relatively small primes, a prime is located between $\mathcal{L}$ and $(\mathcal{L} - \boldsymbol{g}(\boldsymbol{P}_{\boldsymbol{\mathcal{L}\ Ceiling}})$

Where $\boldsymbol{g}(\boldsymbol{P}_{\boldsymbol{\mathcal{L}\ Ceiling}})$**,** is the ceiling gap after the calculated $\mathcal{L}$, since it reveals a clue on the expected gap at the local point of $\mathcal{L}$.

The location of a large prime number is between $\mathcal{L}$ and $\Psi\mathcal{L}$.

Locating a Prime Number, $\boldsymbol{P_{th} \geq 367}$

Locating small prime numbers to 167th prime number does not need any alterations on the leading equation, and it can be used the way it is. Therefore the equation $\mathcal{L} \approx \frac{t \times e \times (\ln t + \log t)}{(\pi + v)}$ will be used.

Table 4: ***Selected Examples***

Location of	**Workings**	**Commentary**
9th prime: $\mathcal{L} \approx \frac{9 \times e \times (\ln 9 + \log 9)}{(\pi + v)}$ $\approx \frac{9 \times e \times (\ln 9 + \log 9)}{(3.1313183886)}$ $\approx \frac{77.099181433}{3.1313183886}$	For convenience begin with finding v. $v = \left(\frac{\frac{lnlog\ 9}{\pi} + 6}{\pi^2}\right)(loglog\ 9)$ = −0.010274265 Calculate $9 \times e \times (\ln 9 + \log 9)$ = 77.099181433	The 9th prime is 23. The bounds are true.

≈ 24.622 But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}$ *and* $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling}\}$ So 9th prime is between 24.622 and 17.95	Calculate $g(P_{\mathcal{L}\,Ceiling}$ $(\log 24.622 \times \ln 24.622 - 1) + \ln 24.622$=6.6762 Get $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ =17.95	
25th Prime: $\mathcal{L} \approx \frac{25 \times e \times (\ln 25 + \log 25)}{(\pi + v)}$ $\approx \frac{25 \times e \times (\ln 25 + \log 25)}{(3.2316109719)}$ $\approx \frac{313.74516465}{3.2316109719}$ ≈ 97.08630382 But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}$ *and* $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ So 9th prime is between 97.1 and 84.43	Find v. $v = \left(\frac{\frac{lnlog\ 25}{\pi} + 6}{\pi^2}\right)(loglog\ 25)$ = 0.0900183183 Calculate $25 \times e \times (\ln 25 + \log 25)$ =313.74516465 Calculate $g(P_{\mathcal{L}\,Ceiling}$ $(\log 97.1 \times \ln 97.1 - 1) + \ln 97.1$= 12.67 Get $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ =84.43	The 25th prime is 97. The bounds are true.
168th Prime: $\mathcal{L} \approx \frac{168 \times e \times (\ln 168 + \log 168)}{(\pi + v)}$ $\approx \frac{168 \times e \times (\ln 168 + \log 168)}{(3.3617426121)}$ $\approx \frac{3356.2025209}{3.3617426121}$ ≈ 998.352 But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}$ *and* $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ So 168th prime is between 998.25 and 971.78	Find v. $v = \left(\frac{\frac{lnlog\ 168}{\pi} + 6}{\pi^2}\right)(loglog\ 168)$ = 0.2201499585 Calculate $168 \times e \times (\ln 168 + \log 168)$ =3356.2025209 Calculate $g(P_{\mathcal{L}\,Ceiling}$ $(\log 998.352 \times \ln 998.352 - 1) + \ln 998.352$= 26.62 Get $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ =971.780	The 168th prime is 997. The bounds are true.
367th Prime: $\mathcal{L} = \frac{367 \times e \times (\ln 367 + \log 367)}{(\pi + v)}$ $\approx \frac{367 \times e \times (\ln 367 + \log 367)}{(3.4026780556)}$	Find v. $v = \left(\frac{\frac{lnlog\ 367}{\pi} + 6}{\pi^2}\right)(loglog\ 367)$ = 0.2610854020 Calculate $367 \times e \times (\ln 367 + \log 367)$ =8449.7797268	The 367th prime is 2477. The bounds are true.

$\approx \dfrac{8449.7797268}{3.4026780556}$ ≈ 2483.2733479 But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}$ and $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ So 367th prime is between 2483.27 and 2449.94	Calculate $g(P_{\mathcal{L}\,Ceiling}$ $(\log 2483.2733479 \times \ln 2483.2733479 - 1) + \ln 2483.2733479$= 33.4 Get $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ =2449.94	
368th Prime: $\mathcal{L} = \dfrac{368 \times e \times (\ln 368 + \log 368)}{(\pi + v)}$ $\approx \dfrac{368 \times e \times (\ln 368 + \log 368)}{(3.4028118406)}$ $\approx \dfrac{8476.7077731}{3.4028118406}$ ≈ 2491.0891845 But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}$ and $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ So 367th prime is between 2491.1 and 2457.72	Find v. $v = \left(\dfrac{\dfrac{lnlog\, 368}{\pi} + 6}{\pi^2} \right) (loglog\ 368)$ = 0.261219187 Calculate $368 \times e \times (\ln 368 + \log 368)$ =8476.7077731 Calculate $g(P_{\mathcal{L}\,Ceiling}$ $(\log 2491.0891845 \times \ln 2491.0891845 - 1) + \ln 2491.0891845$= 33.4 Get $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ =2449.94	The 368th prime is 2503. The bounds are not true.

Locating a Prime Number, $P_{th} > 367$

From previous workings, we can see that the leading equation starts to fail predicting the location of nth prime at $\boldsymbol{P_{th} = 368}$**.** The leading equation thus has to be adjusted to conform to new exponential changes between $\boldsymbol{P_{th} = 367}$ and $\boldsymbol{P_{th} = 368}$**.** Careful look at results for $\boldsymbol{P_{th} = 368}$ indicates that the variable part $\boldsymbol{v}$ could be becoming higher than expected to result in $\boldsymbol{\mathcal{L}}$ that is smaller than the expected one. It is therefore necessary that some adjustments are made on the variable part $\boldsymbol{v}$**.**

Getting the optimum variable part $\boldsymbol{v}$

It is notable that the leading equation drifts below the expected trend between $\boldsymbol{P_{th} = 367}$ and $\boldsymbol{P_{th} = 368}$**.** So, we can presumably say that an optimum $\boldsymbol{v}$ is found between their $\boldsymbol{v}$s. This allows us to get their average as an optimum $\boldsymbol{v}$ where the drift just occurs.

$$\frac{0.2610854020+0.261219187}{2} = 0.2611522945.$$

So the optimum $\boldsymbol{v}^* = 0.2611522945$.

Therefore, for $\boldsymbol{P_{th}} > 367$, appropriations will have to start by subtracting the optimum $\boldsymbol{v}^*$ from the $\boldsymbol{P_{th}\, v}$ to form $\boldsymbol{Q}$. $\boldsymbol{P_{th}\, v}$ is the likely error term $\boldsymbol{v}$ at every nth prime number > 367.

The equation we are looking for is:

$$\boldsymbol{V} = \begin{cases} \frac{\boldsymbol{Q^2}}{\boldsymbol{L}} + \boldsymbol{v}^* & if\ lnlnlnln\ t\ or\ loglnlnln\ t < \boldsymbol{0} \\ \boldsymbol{Q^2.L} +\ \boldsymbol{v}^* & if\ lnlnlnln\ t\ or\ loglnlnln\ t > \boldsymbol{0} \end{cases}$$

$$\boldsymbol{Q} = \boldsymbol{P_{th}\, v} - \boldsymbol{v}^*$$

L is calculated as:

$$\boldsymbol{L} = \{\boldsymbol{1} - (\boldsymbol{lnlnlnln\ t} - \boldsymbol{loglnlnln\ t})^{\boldsymbol{2}}\}^{\boldsymbol{m}}$$

And **m** is calculated as:

$$\boldsymbol{m} = \begin{cases} \boldsymbol{log\ t} \times \boldsymbol{ln\ t} - \boldsymbol{ln\ t} & if\ \ lnlnlnln\ t < 0 \\ \boldsymbol{ln\ t} - \{(\boldsymbol{lnlnlnln\ t} - \boldsymbol{loglnlnln\ t})^{\boldsymbol{2}} \times (\boldsymbol{log\ t}\ \times \boldsymbol{ln\ t})\} & if\ \ lnlnlnln\ t > 0 \end{cases}$$

After those definitions, we expect the final equation to be of the form:

$$\boldsymbol{\mathcal{L}} \approx \frac{\boldsymbol{t \times e \times (\ln t + \log t)}}{\boldsymbol{(\pi + V)}}$$

This is the equation needed for resolving the location of a prime number $\boldsymbol{P_{th}} > 367$.

Table 5: ***Selected Examples***

Location of	**Workings**	**Commentary**
1229th prime: $\mathcal{L}$ $\approx \frac{1229 \times e \times (\ln 1229 + \log 1229)}{(\pi + V)}$ $\approx \frac{1229 \times e \times (\ln 1229 + \log 1229)}{(3.4090929331)}$ $\approx \frac{34087.556734}{3.4090929331}$ $\approx \mathbf{9999.7}$ But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}$ and $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling}\}$ So 1229th prime is between 9954.65 and 9999.7	For convenience begin with finding $P_{th}\ v$. $P_{th}\ v = \left(\frac{\frac{lnlog\ 1229}{\pi} + 6}{\pi^2}\right)(loglog\ 1229)$ =0.3156434249 Calculate Q $0.3156434249 - 0.2611522945$ $= 0.0544911304$ $Q^2 = 0.0029692832$ Next calculate m. $lnlnlnln\ 1229 < 0$, so. $m = \log 1229 \times \ln 1229 - \ln 1229$ $m = 14.864980383$ Determine L. $L = \{1 - (lnlnlnln\ 1229 - loglnlnln\ 1229)^2\}^{14.87}$ = 0.4677523369 Get V: $V = \frac{0.0029692832}{0.4677523369} + 0.2611522945$ V =0.2675002795 Calculate $1229 \times e \times (\ln 1229 + \log 1229)$ =34087.556734 Calculate $g(P_{\mathcal{L}\,Ceiling}$ $(\log \mathbf{9999.7} \times \ln \mathbf{9999.7} - 1) + \ln \mathbf{9999.7}$= 45.051432 Get $\{\mathcal{L} - g(P_{\mathcal{L}\,Ceiling})\}$ =9954.65	The 1229th prime is 9973. The bounds are true.
1 millionth prime: $\mathcal{L} \approx \frac{10^6 \times e \times (log10^6 + ln10^6)}{(\pi + V)}$ $\approx \frac{10^6 \times e \times (ln10^6 + log10^6)}{(3.4705651446)}$ $\approx \frac{53864142.271}{3.4705651446}$	Finding $P_{th}\ v$. $P_{th}\ v = \left(\frac{\frac{lnlog 10^6}{\pi} + 6}{\pi^2}\right)(loglog\ 10^6)$ =0.5180262546 Calculate Q $0.5180262546 - 0.2611522945$ =0.2568739601	The 1,000,000th prime is 15,485,863. The bounds are true.

$\approx \mathbf{15,520,279.847}$ But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}\ and\ \mathcal{L}_{\Psi}$.So 1,000,000th prime is between 15,520,279.847 and 15,444,137.82.	Q^2 =0.0659842313 Next calculate m. $lnlnlnln\ 10^6 < 0$, so. $m = \log 10^6 \times \ln 10^6 - \ln 10^6$ m =69.07755279 Determine L. $L = \{1 - (lnlnlnln\ 10^6 - loglnlnln 10^6\ )^2\}^{69.077755279}$ = 0.9729289309 Get V: $V = \frac{0.0659842313}{0.9729289309} + 0.2611522945$ V =0.328972491 Calculate $10^6 \times e \times (\ln 10^6 + \log 10^6)$ = 53864142.271 Since this is a large number, we can just find $\mathcal{L}_{\Psi}$ as $\Psi\mathcal{L}$= 15,520,279.847 × 0.9950940298 = 15,444,137.82	
37,607,912,018th prime: **Let t=37,607,912,018** $\frac{t \times e \times (\ln t + \log t)}{(\pi + V)}$ $\approx \frac{3570422111001.1}{3.56717572573913917973519949368}$ $\approx \mathbf{1000910071583.67}$ But $\boldsymbol{\mathcal{L}_{pth}}$ is between $\mathcal{L}\ and\ \mathcal{L}_{\Psi}$.So **37,607,912,018th** prime is between 1000910071583.67 and 995999636599.603	Finding $P_{th}\ v$. $P_{th}\ v = \left(\frac{\frac{lnlogt}{\pi} + 6}{\pi^2}\right)(loglog\ t)$ =0.7006084123 Calculate Q $0.5180262546 - 0.2611522945$ =0.4394561178 Q^2 =0.1931216794 Next calculate m… **Beware, not like in the previous examples,** $\boldsymbol{lnlnlnln\ t > 0}$**, so,** $\boldsymbol{m = ln\ t - \{(lnlnlnln\ t - loglnlnln\ t)^2 \times (log\ t\ \times ln\ t)\}}$ **m=22.51777317** Determine L. $L = \{1 - (lnlnlnln\ t - loglnlnln\ t\ )^2\}^{22.51777317}$ =0.8514361420230376335084631125116 7 Get V…….. **Beware again, not like in the previous examples,**	I don’ have an idea about the exact 37,607,912,018th prime. The bounds are seemingly true. The prime number is just slightly below N=10^{12}. I chose this number for illustration.

	$\boldsymbol{lnlnlnln\, t > 0}$**, so,** $$\boldsymbol{V = Q^2 . L + v^*}$$ $V = 0.1931216794 \times 0.85143614202304 + 0.2611522945$ V =0.425583072149345941272556 1104 Calculate $t \times e \times (\ln t + \log t)$ = 3570422111001.066591575250048097 2 Since this is a large number, we can find $\mathcal{L}_{\Psi}$ as $\Psi\mathcal{L}$= 1000910071583.67 × 0.9950940298 = 995999636599.603	

Section Remark:

We have just finished a demonstrative survey of locating an nth prime number. Due to the unpredictable nature of prime numbers, it has not been easy to handpick a particular prime number. Unavailability of data has made me to survey on the 37,607,912,018th prime number just for illustration purpose. I really do not have an idea about the exact 37,607,912,018th prime to tell if it fits in the bounds calculated. However, the journey of finding nth prime location has seemed to be less bumpy compared to those of the two previous sections.

PART II: WHY PRIME NUMBERS BEHAVE AS THEY DO

D. ARGUMENT FOR THE BEHAVIOUR OF PRIME NUMBERS

This section presents an argument for reasons why prime numbers behave the way they do. In so doing, we will be employing trans-algebraic number method to illustrate why prime numbers cannot be identified or generated by a single elementary identity, since each prime number is an identity in its own respect before it is compounded. To illustrate how an elementary identity of a Prime Group, $\boldsymbol{G_p}$ can be determined, we can say that there exist an identity element $e \in G,$ such that $g * e = e * g = g$,$\forall\ g \in G$. where the element g is the group's generator. Therefore, a finite cyclic group G of order m with $< g >$ as the generator would consist of the following elements (Gallian, 2016):

$$e, g^0, g^1, g^2, g^3, \dots \dots \dots \dots \dots .., g^{m-1}, \text{where } g^{m-1} \neq e.$$

For infinite cyclic group, the group G can be defined with $\boldsymbol{G} =< \boldsymbol{g} >= \{\boldsymbol{g^n}: \boldsymbol{n} \in \mathbb{Z}\}$ and $|G| = \infty$. In the above cases, we have assumed that the identity element is an exact real number represented as $g \in \mathbb{R}$. However, there can be cases whereby g is a rational number represented as ϱ ∈ $\mathbb{Q}$.

A finite extended field can therefore be defined by extensions of the field L such that (Gallian, 2016):

$$G = E_0 \subset E_1 \subset Eg_0 \subset E_2 = E_1 g_1 \subset \cdots \subset L = E_n$$

We can subsequently take the translation (mapping) of one element onto another to be isomorphism. Taking $E_0 = G_0, E_1 = G_1 \; and \; E_2 = G_2$ for instance, then $G_1 = gG_0 \; and \; G_2 = gG_1$. In this regard, we can note that there is a one-to-one correspondence among elements of the group generated by g. This definite generator ensures normality in the group, so that there can be a clear sequence. We can now say that every element in the group can just be a direct summand of the preceding one such that (Reis, & Rankin, 2016):

$$G = G_1 \oplus G_2 \oplus G_3 \oplus G_4 \dots \dots \dots \dots \dots \dots \dots .. \oplus G_n \quad with\ |G_i| \geq |G_j|\ if\ i < j.$$

Due to the nature of prime numbers (Euclid already gave a proof of their infinitude), we can let the group of prime numbers to be $\boldsymbol{G} =< ϱ >= \{ϱ^{\boldsymbol{n}}: \boldsymbol{n} \in \mathbb{Z}\}$ and $|G| = \infty$.

We can now specifically write the group of prime numbers as:

$G_p = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29$ … … … … … …. Or

$$G_p = Z_{p_2}, Z_{p_3}, Z_{p_7}, Z_{p_{11}} \dots\dots\dots\dots\dots.. Z_{p_{n-1}}, G_{p_n}$$

Note that we have already discussed that for the group's elements to be represented on an extended field with the same relationship, there have to be a one-to-one correspondence initiated by the same identity e and generated by the same generator g. The contrary will indicate that at least one of the element in the group is non-trivial; that is, it has its own identity and it cannot be defined and generated within the group by the group's identity and generator respectively. Suppose we do an analysis on primes, we may take 2, the first prime number, as an identity. The next step will be analyzing how 2 is translated, by isomorphism, onto other numbers in the prime sequence. This will indeed help us establish if all numbers are being produced by the identity 2 and the same generator. This will in fact be indicated by gaps between the numbers; having the same identity and being generated by similar generator will determine regularity of gaps. From the maximal gap analysis, we saw that the next prime number is a function of preceding prime and the gap that is between them. If we let for instance sizes of gaps to be represented by ϱ, then there have to be regularity in the gaps after each prime number for us to say that elements in prime group are of the same identity e and are being generated by the same generator ϱ, so that $G_1 = \varrho G_0$ *and* $G_2 = \varrho G_1$ or $G_1 = \varrho \pm G_0$ and $G_2 = \varrho \pm G_1$(Reis, & Rankin, 2016).

In case such a group does not include elements with predictable pattern, as indicated by regularity in nth element appearance and gap sizes, then at least one of them will be non-trivial. All of them can be non-trivial if no predictable pattern exist at all to show how every element translate itself onto another. In a group with a single identity and single generator, each element can be represented as $\mathcal{R}\left(\frac{m}{n}\right) = \mathbb{Z}$. In this relationship, $\mathcal{R}$ is the ratio of homomorphism. The m can be the actual size of $\mathbb{Z}$ in sequence. The denominator n indicates the relative position of the element in the sequence, or extension. Thus, if all elements of the group are properly generated by the same generator throughout, we are supposed to have the relationship:

$$\mathcal{R}\left(\frac{m_2}{n_2}\right) = \mathcal{R}\left(\frac{m_3}{n_3}\right) = \mathcal{R}\left(\frac{m_7}{n_7}\right) = \mathcal{R}\left(\frac{m_{11}}{n_{11}}\right) = \mathcal{R}\left(\frac{m_{13}}{n_{13}}\right) = \cdots = \mathcal{R}\left(\frac{m_{n-1}}{n_{n-1}}\right) = \mathcal{R}\left(\frac{m_n}{n_n}\right) = e.$$

e is a common identity (Reis, & Rankin, 2016). In case the contrary exist so that

$$\mathcal{R}\left(\frac{m_2}{n_2}\right) \neq \mathcal{R}\left(\frac{m_3}{n_3}\right) \neq \mathcal{R}\left(\frac{m_7}{n_7}\right) \neq \mathcal{R}\left(\frac{m_{11}}{n_{11}}\right) \neq \mathcal{R}\left(\frac{m_{13}}{n_{13}}\right) \neq \cdots \neq \mathcal{R}\left(\frac{m_{n-1}}{n_{n-1}}\right) \neq \mathcal{R}\left(\frac{m_n}{n_n}\right) \neq$$

e, then each of the element in the group will be non-trivial. Each of them will be an identity in its own respect and we cannot present each of them with a single identity and generate each of them with a single generator. This is the case with prime numbers, as each of them is non-trivial. The group of prime numbers should therefore be presented as:

$$G_p =< g_{p_2},\ g_{p_3},\ g_{p_5},\ g_{p_7}, g_{p_{11}} \dots \dots \dots \dots \dots \dots \dots \dots g_{p_{n-2}}, g_{p_{n-1}}, g_{p_n} >$$

But not elementarily as:

$G =< g >$, since

$$G_p \neq Z_{p_2} \oplus Z_{p_3} \oplus Z_{p_7} \oplus Z_{p_{11}} \dots \dots \dots \dots \dots .. \oplus Z_{p_{n-1}} \oplus G_{p_n}$$

Section Remark:

In this way, we can now establish that it is non-triviality or identity nature of every prime number that makes it hard to generalize prime numbers. Only statistical approximation and other kinds of approximation can be used to attain approximate generalization of prime numbers due to their unpredictable nature.

Concluding Remark

We have finished analyses of all sections. From the last section, we have demonstrated that prime numbers, before being compounded, cannot be generalized with one identity or relationship since each of them is an identity in its own respect. This explains why there are irregular gaps between consecutive primes, and their appearances cannot be predicted. We have therefore opined that only approximations of statistical operations, among other approximation methods, can help us in this regard. We began with formulating models that can assist us in determining maximal gaps between consecutive prime numbers. We subsequently determined extremely bounded gaps, so that one can determine their lowest and highest bounds to resolve questions on locations of prime numbers. We have also formulated an equation that shows a general path of prime numbers, and thus a fair blue-print of estimating the number of prime numbers at a given magnitude, from $N = 3$ to $N = 10^{25}$. I have also noticed that just after $N = 10^{10}$, our results are fairly below those of physical counts; I have already said that despite choosing right and well appropriated exponents, injections of many and large rational numbers in the equation can reduce exactitude of results. The prime locating function has been less bumpy, although it has not been tried on very large prime numbers. All workings were based on data that the researcher had at disposal, and therefore, these equations may be limited to scopes of data used. However, the demonstrations can be perceived as easier to understand and cheaper to implement. With beliefs that the equations can be understood easily and implemented cheaply, I hope to see many innovations on them very soon. We need to improve their precisions as soon as possible. I would like to thank Chris K. Caldwell, the owner of the website *Prime Pages*, for his efforts of preparing prime number tables for novice researchers like me; the tables inspired my essay and made it doable to this extent. And finally, Anaxagoras described the world as a mixture of primary ingredients that cannot perish. Those primary ingredients that cannot perish without rationalism, are, just to say, prime numbers in Mathematical Logic.

References

Abramowitz, M. & Stegun, I. A. (2012). *Handbook of Mathematical Functions*. New York: Dover Publications.

Apostol, T. M. (2013). *Introduction to Analytic Number Theory*. New York: Springer Science & Business Media.

Cook, G. (2015).The singular mind of Terry Tao. *New York Times*. Retrieved on 18 May 2017 from https://www.nytimes.com/2015/07/26/magazine/the-singular-mind-of-terry-tao.html?_r=0

Gallian, J. (2016). *Contemporary Abstract Algebra*. New York: Cengage Learning.

Guy, R. (2013). *Unsolved Problems in Number Theory* (Vol. 1). New York: Springer Science & Business Media.

Hume, D. (2007). *Treatise of the Human Nature*. (Jonathan Bennet, Ed.). New York: Wiley & Sons.

Kim, T. (2015). Degenerate Euler zeta function. *Russian Journal of Mathematical Physics*, *22*(4), 469-472.

Laplace, P.S. (1902). *A philosophical Essay on Probabilities*. (Frederick W. Truscott and Frederick L. Emory, Trans.). London: Chapman & Hall Ltd.

Major, P. (2013).*On the Estimation of Multiple Random Integrals and U-Statistics.* New York: Springer.

Morozov, A., & Shakirove, S. (2009). Introduction to integral discriminants. *Journal of High Energy Physics*, 12(002). doi:10.1088/1126-6708/2009/12/002

Taillie, C., Ganapati P. Patil, & Bruno A. Baldessari. (2012).Statistical distributions in scientific work: Volume 4 — *Models, Structures, and Characterizations*, Proceedings of the NATO Advanced Study Institute held at the Università degli Studi di Trieste, Trieste, Italy, July 10 – August 1, 1980.

Reis, C., & Rankin, S. A. (2016). *Abstract Algebra: Introduction to Groups, Rings and Fields with Applications*. New York: World Scientific Publishing Co Inc.

Tapia, A. O., & Støleum, H. H. (2016). Studies of entropy measures concerning the gaps of prime numbers. *arXiv Preprint arXiv:*1606.08293.

Wasserman, L. (2013). *All of Statistics: A Concise Course in Statistical Inference.* New York: Springer Science & Business Media