A derivation of the laws of physics from pure information

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I suggest and formalize an alternative approach to understand the universe in which a large blob of facts, rather than a small list of axioms, define the theory. The blob of facts is given a physical interpretation when it is structured as a Gibbs ensemble. In the case where the facts are logically verifiable, the Gibbs ensemble describes a thermal universal Turing machine which spawns the whole system. The Lagrange multipliers of the partition function are the Planck units. The background, a thermal spacetime, emerges as a consequence of the limits applicable to the conjugate pairs. The background obeys Special and General relativity, dark energy, the arrow of time, the holographic principle, the Schrödinger equation and the Dirac equation.

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0.1 Notation

We will use the following notations: The double vertical lines |X| means the length of the string X. The suffix $b$, for example in $110_b$, refers to the binary notation.
1 Introduction

It is generally understood that a final and ultimate theory of everything (ToE) in physics should be elegant (in the mathematical sense). Indeed, it is hoped that a ToE could be formulated as a relatively small set of axioms, which as a group explains all of the physics of the universe. The size of the set could (hopefully?) be between 10 to 30 axioms which would make the theory particularly elegant. Finding such a final theory is a major unsolved problem of physics.

Pondering on the difficulty of finding such a theory, I asked myself; what would happen if we were to eliminate the requirement that it should be elegant. Would the problem become simpler, and if so by how much? In the first part of this paper, if you allow me the indulgence, we will formulate a mathematically inelegant theory to describe the universe. This exercise will be justified under the thesis that any theory is better than no theory. Then in the second part, we will see that the exercise was well worth the effort as we will be able to improve the elegance of the theory quite significantly.

Relaxing the requirement on elegance allows us to define the universe using as many axioms as we want (even possibly infinitely many axioms). We can think of this method as a fool-proof way to produce a theory capable of describing all of the universe. The idea is to define the universe by listing, one after the other, each of its facts. Since we allow ourselves arbitrarily many axioms each of them representing a single and independent fact, we never risk running out of axioms before running out of facts. Therefore, our axioms will necessarily cover all facts. For example, using such a methodology, we could encode the state of the universe with a series of yes or no questions. "Is there milk in the fridge?-yes." "Did the chicken cross the road?-no." etc. We might not be able to compress the facts, and therefore the formulation would not be mathematically elegant, but at least we can reasonably convince ourselves that the blob of facts does correspond to the state of the universe. In the end we do have a "ToE" but its complexity is equal to that of the universe itself! Nonetheless, we do take note that it is a correct description of the universe and that whatever properties this blob of facts has, then necessarily so does the universe.

We notice that the specific questions are not so important - it is the fact that the universe can be encoded as such that is. The facts can be discussed abstractly. For example, we can label them as Fact1, Fact2, Fact3, etc without ever defining them explicitly. If any question is without an answer, then it is simply left out of the set of axioms (it would not be a fact). After-all, not all questions need to have an answer.
I noticed that this formulation could be connected to the arrow of time when I realized that any group of \( n \) logically independent axioms contained every axioms in its subgroups but cannot by itself determine the axioms found in its supergroups. For example, the group comprised of Fact1, Fact2, Fact3, Fact4 contains at least the information that is contained in the subgroup of Fact1, Fact2, Fact3. But, in the case where the facts are logically independent axioms, the smaller groups cannot determine what Fact5 is. All that was left to do was to define each instant of time as a specific group of a certain size. As time moves forward, the size of the group would increase to accommodate more information. The result is that the information of the past would be encoded in the group associated with the present, but the future would be undecidable until it occurs - yielding the arrow of time. At that point I was hooked to the idea: Time has an arrow because the number of axioms grows with it.

After some research, I was able to show that encoding the universe as a series of such facts is exactly enough to recover many of the familiar laws of physics. The fundamental equation describing the universe in this interpretation is a Gibbs ensemble of statistical physics that connects a large blob of facts to an entropy. The equation, stated here, will be formally derived and justified in this paper;

\[
Z_\Omega = \sum_{i=1}^{n} 2^{-\beta(D|p_i|+2\pi S_f)} \tag{1.1}
\]

This previously unknown yet relatively simple equation yields a surprising amount of physics. We can explicitly show that a thermal spacetime is emergent from the entropy. We can further show that such a spacetime obeys special relativity, general relativity, dark energy, the Schrödinger equation and the Dirac equation and that it embeds various versions of the holographic principle within its foundation. Finally, the Lagrange multipliers of the partition function can be shown to be the Planck units. The conjecture (and conclusion) is that the laws of physics are simply a consequence of encoding the universe as a large blob of facts. Surprisingly, nothing more appears to be required to derive its laws.

Formulating a system of arbitrary facts as a Gibbs ensemble turned out to be a key insight because, as it is spawned from statistical physics, the Gibbs ensemble ensures, so to speak, the "physicality" of the system. Therefore, it should not be too surprising that from such a formulation a connection between arbitrary facts and physical laws is possible.
1.1 Avoiding the errors of the past

Many philosophers, notably Leibniz, Spinoza and Bertrand Russel, have attempted to construct a yes/no description of all of reality. Specifically, Leibniz tried to create a language that would be able to decide any questions. As expected by the Gödel incompleteness theorem, the project ultimately failed. This is often and incorrectly interpreted as a failure of the yes/no project. To understand why the failure interpretation is incorrect, an important nuance must be understood. It is not the project of encoding a system as yes or no questions per se that as failed. In fact, this form of encoding is always possible when no restrictions are applied to the decidability of the questions. The failure occurs when one tries to formalize the project into a set of axioms of lesser complexity than the facts it encodes so as to improve its elegance. In the most complex systems, such as with the universe, many questions are undecidable and must remain so for the system to be consistent.

When relaxing the requirement such that some questions can be left unanswered, we obtain a set of facts that is recursively enumerable. The construction of the decidable language (now a ”semi-decidable” language) can be revisited in this context. In a recursively enumerable system, every yes-question can be answered in a finite time (they can be enumerated), but the no-questions may require infinite time to prove (they are undecidable in the general case). As an example of an undecidable problem, we can think of the halting problem in computer science.

Having the no-questions undecidable might appear to be quite the limitation to the purest of Platonists, but as we will see it is what actually gives complexity to the universe. Specifically, we will show that using a recursively enumerable set of axioms to encode the universe gives us a way to connect time to a growing set of facts. In this interpretation, both time and entropy exist precisely because there is such a thing as undecidable questions. To paraphrase, improving the elegance of a set of undecidable questions can be done and the consequence of such will be the emergence of an arrowed time.

2 Preliminaries

We must be careful with our use of language from this point on. Referring to the yes/no encoding as questions and answers will not suffice. Indeed, we can show that some problematic questions, known as logical paradoxes, do not have a yes or a no answer. For example, the barber paradox can neither be yes nor no without introducing a contradiction. Another example is the note-on-a-wall statement.
Imagine a note on a wall with a single statement that reads "All statement on this note are false" - is the statement on the note true or false? Let’s see. First, assume it is false. Then there must be one true statement on the note. But, we just assumed that its only statement is false. Hence the assumption must be wrong. Okay, now assume it is true. Then as per the statement, all statements must be false. But, we just assume that it is true. A contradiction is obtained in both cases. This is an undecidable statements which contradicts the idea that all questions have a yes or no answer.

To get rid of these problematic statements, we simply re-align the scope of the yes/no encoding to a more modern formulation. To properly define the encoding, we will introduce the terms; provable, theorem and sentences. The use of the term question will be replaced by the term sentence. For example, we could say; a sentence of a language contains symbols. If a sentence of a language is provable within a logical system, we will say that the sentence is a theorem of the logical system. In the general case, it is undecidable if a sentence is or is not a theorem of a formal system. Reprising the note-on-a-wall example, we would ask: "is the statement on the note a theorem" - the answer is simply no.

As a result, we no longer use the word true or false to describe sentences but will rather use theorem or not-a-theorem and the problem evaporates. For example, we will not refer to a sentence as being ‘true in the universe’ but instead as being a ‘theorem of the universe’. Explicitly, the expression ‘a theorem of the universe’ means a sentence which is a theorem of the theory which explains the universe.

2.1 Formalism

Let us consider a specific language, say binary, and associate to each of its sentences a boolean value to be interpreted as related to the provability of the sentence within a formal logic system. The boolean value will be true if its associated sentence is a theorem and will be false otherwise. We list all sentences of binary in shortlex order (sorted both alphabetically and by length). Then, we associate a boolean value to each sentence. If the sentences are to be interpreted as the questions, we can easily see that all questions are associated with an answer by noting that all sentences of binary are in the list. As an illustrative example, consider the following case;
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Historically, Émile Borel suggested a know-it-all number which would encode the answer to all yes/no questions of a language. Here, we revisit this concept but using the modernized terms. We replace the concept of yes/no questions with theorems/non-theorems.

**Definition 2.8 (Borel number).** A Borel number is a real number between 0 and 1. It starts with 0 followed by a period and followed by an infinite expansion of binary digits. The digits of the expansion are obtained by concatenating all boolean answers back to back. Each digit corresponds to the boolean answer associated with its corresponding sentence. The $i$th digit after the decimal corresponds to the answer to the $i$th sentence. The Borel number of the above example would be $0.1011100..._b$. A Borel number encodes the provability of each sentence of a language as a single real number. A Borel number is an example of a real number that is non-computable in the general case.

The Chaitin construction, also called an Omega number, is a generalization of the above modern formulation of the Borel number. Instead of associating each sentence to a natural number, they are instead associated to a prefix-free code.

**Definition 2.9 (Prefix-free code).** A prefix-free code is a set of sentences with the property that no member of the set is the prefix of another. For example, the sentence $0_b$ is a prefix of $00_b$ hence the set of these two sentences would not be a prefix-free code. But the set of $0_b$, $10_b$, $110_b$, $1110_b$, $11110_b$, ... would.

If each sentence of a language ($s_1, s_2, s_3, ...$) are associated to a prefix-free code ($p_1, p_2, p_3, ...$) then the kraft inequality holds.

---


\[ 1 \geq \sum_{i=1}^{n} 2^{-|p_i|} \geq 0 \quad (2.10) \]

The inequality guarantees that the sum over the exponential decay of the length of the codes will be between 0 and 1 inclusively. The length of a prefix-free code is a natural number. In the case where the sentences are encoded with the unary code (a certain prefix-free code defined as: \(0_b, 10_b, 110_b, 1110_b, \ldots\)), the Borel number is recovered.

The Chaitin construction is defined for all sentences of a language encoded with a prefix-free code \(p\). In this construction, each sentence is considered to be a program which either halts (if the sentence is provable), or doesn’t (if it is non-provable).

\[ \Omega = \sum_{i=1}^{n} 2^{-H(p_i)-|p_i|} \quad \text{where} \quad H(p_i) = \begin{cases} 0 & p_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \quad (2.11) \]

With this construction it is possible to prove that \(\Omega\) is normal, algorithmically random, non-computable and non-compressible. Like a Borel number, \(\Omega\) encodes which sentences of a language are theorems. Unlike the Borel number however, the digits of \(\Omega\) do not necessarily have a one-to-one correspondence to a specific sentence. As a result, the interpretation of \(\Omega\) is different.

The Chaitin construction connects to the halting problem of computer science. Consider a universal Turing machine running a program that is searching for a proof of \(p_1\). If a proof is found, the program halts and \(H(p_1)\) is equal to 0. Consequently, the term of the sum associated to \(p_1\) does not vanish. In the case where \(p_2\) is not a theorem, then the program will search forever and will never halt. In this case \(H(p_2)\) is equal to \(\infty\) and its contribution to the sum vanishes.

Knowledge of \(n\) bits of \(\Omega\) allows an observer to count the total number of programs of length less than or equal to \(n\) that halts. The observer can then use this information to solve the halting problem for programs of length less than or equal to \(n\). Hence, as the halting problem is unsolvable, the infinite expansion of the bits of \(\Omega\) must be non-computable. As it is normalized between 0 and 1, \(\Omega\) is often interpreted as the halting probability of a random program for a prefix-free universal Turing machine.

2.2 Statistical physics

Before continuing to the next section, we will provide a brief recap of statistical physics. In statistical physics, we are interested in the
distribution that maximizes entropy
\[ S = -k_B \sum_{x \in X} p(x) \ln p(x) \] (2.12)

<table>
<thead>
<tr>
<th>Observable</th>
<th>Conjugate variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (E)</td>
<td>Temperature (\beta = 1/(k_B T))</td>
</tr>
<tr>
<td>Volume (V)</td>
<td>Pressure (\gamma = p/(k_B T))</td>
</tr>
<tr>
<td>Number of particles (N)</td>
<td>Chemical potential (\delta = -\mu/(k_B T))</td>
</tr>
</tbody>
</table>

subject to the fixed macroscopic observables. The solution for this
is the Gibbs ensemble. Taking the observables listed in Table 1 as
examples, the partition function becomes
\[ Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \] (2.13)

The probability of occupation of a micro-state is;
\[ p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \] (2.14)

The average values and their variance for the observables are;
\[ \overline{E} = \sum_{x \in X} p(x) E(x) \quad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \quad \langle \Delta E \rangle^2 = \frac{\partial^2 \ln Z}{\partial \beta^2} \] (2.15)
\[ \overline{V} = \sum_{x \in X} p(x) V(x) \quad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \quad \langle \Delta V \rangle^2 = \frac{\partial^2 \ln Z}{\partial \gamma^2} \] (2.16)
\[ \overline{N} = \sum_{x \in X} p(x) N(x) \quad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \quad \langle \Delta N \rangle^2 = \frac{\partial^2 \ln Z}{\partial \delta^2} \] (2.17)

The laws of thermodynamics can be recovered by taking the following derivatives
\[ \frac{\partial S}{\partial E} \bigg|_{V,N} = \frac{1}{T} \quad \frac{\partial S}{\partial V} \bigg|_{E,N} = \frac{p}{T} \quad \frac{\partial S}{\partial N} \bigg|_{E,V} = -\frac{\mu}{T} \] (2.18)

which can be summarized as
\[ dE = TdS - pdV + \mu dN \] (2.19)

This is known as the equation of state of the thermodynamic system.
3 Philosophical justifications

Developing the philosophy behind the methodology will be of the utmost importance as it will allow us to define the scope of the theorems that are encoded by the universe. Without this part, we would not know if the universe encodes all possible theorems, or a fraction of such and if so which fraction specifically. We want to avoid the situation where we over-scoped the set of theorems as that would mean that we would recover properties not applicable to the universe. For example, should the theorems encode the positions of particles? Or should they encode the solutions to mathematical problems? etc. This is what we want to answer in this section. But, before we dive in, let us do a brief recap of select philosophical results that will be of use to us.

3.1 René Descartes

To understand how the scoping of the theorems will be achieved, we have to recall the philosophy of René Descartes (1596-1650), the famous 17th century french philosopher most well-known for his derivation of *cogito ergo sum* - I think, therefore I am. As we will see, the proper scoping is naturally obtained when we modernize his universal doubt method into a formal logic system such as first order logic. But first, let us recall what the universal doubt method is and how it applies to the derivation of the cogito.

Descartes’ main idea was to come up with a test that every statement must pass before it will be accepted as true. The test will be the strictest test imaginable. Any reason to doubt a statement will be a sufficient reason to reject it. Then, any statement which survives the test will be considered irrefutable.

Using this test, and for a few years, Descartes rejected every statement he considered. The laws and customs of society, as they have no logical justifications, are obviously the first to be rejected. Then, he rejects any information that he collects with his senses; vision, taste, hearing, etc, on the grounds that a "demon" (think hallucinogens) could trick his senses without him knowing. He also rejects the theorems of mathematics on the grounds that axioms are required to derive them, and such axioms could be wrong. For a while, his efforts were fruitless and he doubted if he would ever find an irrefutable statement.

But, eureka! He finally found one which he published in 1641. He doubts of things! The logic goes that if he doubts of everything, then it must be true that he doubts. Furthermore, to doubt he must think and to think, he must exist (at least as a thinking being). Hence,
cogito ergo sum, or I think, therefore I am.

This quite remarkable argument is, almost by itself, responsible for the mind-body dualism of western culture.

3.2 Miniversal logic

We now refocus our interest to Descartes’ universal doubt method itself and not so much in the cogito. To identify the theorems of the universe, we will repeat Descartes’ universal doubt method within the context of a formal logic system. The method will produce a\textit{ minimal set of rules whose theorems are the theorems of the universe} - hence the name \textit{Miniversal logic}.

Miniversal logic is, in many ways, similar to the constructivist project in mathematics but taken to the extreme. We select first-order logic as our starting point. Then, as we do not know which axioms are the true axioms of the universe, we remove all formal axioms from first-order logic on the ground that they carry doubt. Then, we further remove all rules of inference with the exception of the rule of deduction. This method parallels Descartes’ universal doubt method within first order logic. The main argument is that if we remove from first order logic all formal axioms and all rules of inference which could potentially be controversial, then whatever theorem is left will surely be irrefutable. The result is a system of logic which, essentially, does not deceive its user.

Like Descartes with the cogito, we will also obtain statements that cannot be doubted of, but since we have formalized Descartes’ method within first-order logic, the irrefutable statements obtained will be logic statements and are therefore mathematically usable. Specifically, the irrefutable statements obtained are the theorems of Miniversal logic.

To write sentences in a clear and unambiguous manner, Miniversal logic preserves the syntax of first order logic but does not retain its rules of inference (with the exception of the rule of deduction). As only the rule of deduction remains, let us recall its definition.

\textbf{Definition 3.1 (Rule of deduction).} The rule of deduction formalize the idea that proving a theorem using a set of assumptions is valid within these assumptions. It shows that if by assuming \(A\) one can show that \(A \vdash B\), then \(A \rightarrow B\) is a theorem of the logical system. It is often considered one of the most obvious rule of inference of logic, as without it we cannot extend a logical system with new axioms/assumptions. Using the rule of deduction, we can start from seemingly nothing and rebuild any of the familiar logic systems such as Peano’s axioms (PA) or set theory (ZFC) by assuming their axioms.

Why keep the rule of deduction? For the simple reason that us-
ing it does not introduce doubt but removing it would. It is the only standard rule that has this property. For example lets consider another rule, say the rule of excluded middle. Adopting this rule in the foundation of the theory would increase doubt as it is impossible to determine a-priori if this is a valid rule of the universe or not. However, introducing it by first appealing to the rule of deduction would be fine. Indeed, in the latter case we would say "if we assume the rule of excluded middle (via the rule of deduction), then we can prove by contradiction, for example, that $\sqrt{2}$ is irrational". It only affects the branch of the tree under which it is assumed and not the whole system.

In Miniversal logic, no theorem stands on its own. Any theorem must include, within its description, the list of assumptions that are required to prove it. The user of Miniversal logic is always reminded that the theorems that he proves are of the form ‘Assume A, then A proves C’. Hence, by the rule of deduction, $A \rightarrow C$ is a theorem, but C by itself never is. Miniversal logic can be interpreted as the starting point of all logical work - it is the state of mind a logician is in before having morning coffee and selecting a specific system of axioms to work with. As a result and compared to other logic systems, it more accurately represents reality as it reflects the full freedom available to the logician to select any set of axioms prior to working.

3.3 Discussion on metaphysics

Let me apologize for injecting metaphysics into a physics paper, but if you will allow, we will see that it will be useful. In this paper we are only interested in deriving the following:

**Definition 3.2** (Bridge from metaphysics to physics). A method to remove elements from the set of all possible universes until only one element is left. The derivation should rely only on the application of pure reason. It should not rely on empirical observations.

The goal of this section is to construct a bridge from metaphysics to physics. The completion of the exercise will identify all sentences that are theorems of the universe. To iron out the subtleties we will present, in the long standing tradition of philosophy, an hypothetical dialogue about the thesis. The dialogue is based on a number of real conversations which has been edited and combined to remove repetitions, to accelerate the flow and to help illustrate the points being made.

* Alice: I believe in empiricism. To derive the laws of physics, one must make observations. Without these observations, there is no way to know which of many possible worlds is the actual world. For example, is the geometry of the
universe euclidian or hyperbolic? Is the speed of light maximal? Does the microscopic world obey the Schrödinger equation? Etc. Pure reason alone cannot prove these to be actual. Only continual observations followed by refinements or falsifications can improve our degree of confidence in a scientific theory.

I understand your point of view, but I believe I have found a bridge between metaphysics and physics that allows one to obtain irrefutable knowledge about the universe. I will try to explain the bridge from the following angle. First, lets assume that the cogito is true: I think, therefore I exist. Do you agree with the cogito?

Alice: - Yes.

Then, for I to exist, the universe must be restricted in some way. At the very least, it must be such that it does not contradict the existence of thought. We have now essentially reformulated the anthropic principle as an extension to the cogito. I think, therefore I exist - and to exist, I must actually exist in a universe capable of supporting such existence. Would you agree that this argument rules out some universes?

Alice: - Fair enough, yes - it rules out the [...] universes incompatible with the existence of thought. [...]

OK. From that, we already have a slight connection between metaphysics and physics. An argument from pure reason, the cogito extended with the anthropic principle, can be used to place restrictions on what the universe can be. As it contains very little information, the restriction is very loose, but it is nonetheless a restriction.

Alice: - I agree that the anthropic principle rules out universes that are not capable of producing an observer. But, a scientific theory should make precise and falsifiable predictions and the anthropic principle is not sufficiently specific for that.

Now we enter the core of the argument. We will use Miniversal logic to improve the specificity of the anthropic principle. Each theorem of Miniversal logic that we can provide will serve to further restrict what the universe is. For example, using my mind I can prove the sentence “PA implies that two plus two equals four”, and since my mind is in the universe, then the sentence must be a theorem of both my mind and the universe. This is how we find the theorems applicable to the universe. We have now restricted the universe by two statements instead of one. So the previously poorly defined bound is now slightly better defined. Agree, or disagree?

Alice: Well, you want the phrase “theorem of the universe” to be telling us something about the physical world; to put it in your own words, “...this is how we bridge metaphysics to physics.” But how does this work? If “true in the universe” just means provable in PA or ZFC or whatever (as you seem to have just said), how does this provide any link with physical reality at all?
Hold on, it appears that you might not have noticed a subtlety. "Provable in the universe" means provable in the Miniversal logic system I defined earlier. If you use another logic system than Miniversal logic (such as PA) then the argument does not work. If you use PA or ZFC, then the theorems rely on the axioms PA or ZFC. As the universe might have other axioms than PA or ZFC, we cannot prove that PA's or ZFC's theorems are indeed the theorems of the universe. However, Miniversal logic teaches us that the theorems of the universe are not "two plus two equals four", they are "Assume PA, then two plus two equals four". The "Assume PA" prefix is what the subtlety is all about. "Assume PA, then 2+2=4" is a theorem of the universe because, it is true that in the universe, if you assume PA you can prove (within PA) that 2+2=4. You can easily do the exercise in your head to prove that it can be done in the universe.

- Alice: OK, so you want to think of all mathematical proofs as conditional - if certain axioms hold, then certain consequences follow. Fine. How does that provide any connection with physics or the physical world?\(^7\)

Well yes, mathematical proofs that are explicitly conditional on assumptions derived exclusively from the rule of deduction are theorems of the universe. Whereas those that do not meet this condition are theorems of their respective logic system. For example, "2+2=4" is a theorem of Peano's axioms. But, "Assume PA, then 2+2=4" is a theorem of the universe. So all worlds where "Assume PA, then 2+2=4" is not true are ruled out.

- Alice: This is one point where I am a little confused. Pure logic (call it Miniversal logic if you want) guarantees that PA implies 2+2=4. So it's hard to see what worlds it rules out - unless you mean worlds in which there is a mind, but that [this] mind is too [primitive] to realize that PA implies 2+2=4. Is that the kind of world that you take to have been ruled out? If so, I am OK with what you have said.\(^8\)

Yes - that is part of what I am ruling out. Generally speaking, I am ruling out any world which does not embed universal reason. I also rule out worlds for which logic would be incomplete and worlds which would contradict logic by say, letting you prove any theorems regardless of the axioms that you assume.

Since our mind is able, in principle, to explore all branches of Miniversal logic and since the universe must embed our mind, we can precisely identify all the theorems of the universe: The ultimate theory which describes the universe must have, as its theorem, all theorems of Miniversal logic.

Alice: Here I really don't know what you mean, unless you are just saying that there are no 'violations' of [Miniversal] logic in the world. If that's what you mean, I'm happy with that claim.\(^9\)
I am indeed claiming that there are no violations of Miniversal logic in the universe, but I am also claiming something additional. What I am claiming is that we can use Miniversal logic and the anthropic principle to completely restrict the universe to a single solution.

Consider the following; each theorem of Miniversal logic that we identify can be used to restrict the universe further. In principle, we can supply arbitrarily many theorems. PA has "2+2=4" as a theorem, but it also has "2+3=5", etc. Then, ZFC also has infinitely many theorems as well. If we keep supplying theorems, we will eventually supply all theorems for all branches of Miniversal logic. Furthermore as Miniversal logic is universal, all possible theorems for all possible sets of assumptions will eventually be supplied. No patches of theorems will be left out by the process.

As a result, we will have maximally restricted what the universe can be. Indeed, the universe cannot be simpler than Miniversal logic because that would mean leaving a theorem out (but we already said the work will eventually supply all possible theorems so none can be left out). What about complexity - can the universe be more complex then Miniversal logic? The universe cannot be more complex than Miniversal logic either because that would mean the universe has theorems that Miniversal logic hasn’t (but this cannot be the case because Miniversal logic already embeds all possible theorems within its branches).

Therefore, as the universe is restricted both from the perspective of increasing its complexity as well as reducing it, the bound cannot be improved furthermore. The method herein described fully restricts the universe to a single solution.

Alice: I am not sure [I see where you are going with this]. I’m happy to say that the universe must allow for the possibility of a mind that, in principle, can verify all the theorems of [Miniversal] logic. But what follows from that? \(^{11}\)

TB

Usually a theory is first defined by a set of axioms, then the theorems follows from them. In our present situation, we have a list of theorems but we do not have the theory which explains such theorems. The theory is inside-out. The next step is to reverse the formulation so that we can understand the theory from general principles instead of infinitely many theorems.

Alice: I don’t understand this at all. What we have now are all the tautologies of [Miniversal] logic. What connection is there between that and a physical theory? \(^{12}\)

TB

The connection is that, for the reasons stated, the theorems of Miniversal logic are the theorems of the universe. Hence Miniversal...
logic, as its theorems are identical to those of the universe, must fully describe the universe.

Alice: You say "The theorems of [Miniversal] logic are theorems of the universe.". If by this you just mean that the universe obeys the laws of [Miniversal] logic, then yes, I agree. Then you say "Hence Miniversal logic, as its theorems are identical to those of the universe, must fully describe the universe." This seems clearly wrong. It is true in the universe that there [is the law of gravity]. That there [is the law of gravity] is, however, not a theorem of [Miniversal] logic. Thus, the theorems of [Miniversal] logic do not fully describe the universe.\(^\text{13}\)

There is a misunderstanding. I am not claiming that the laws of the universe can be found within Miniversal logic under a certain set of assumptions. What I am claiming is that Miniversal logic is itself isomorphic to the universe. Miniversal logic along with the anthropic principle has allowed us to establish that whatever theory of physics we construct to explain the universe, it must exactly recover the theorems of Miniversal logic - it cannot do more and it cannot do less. As a result, the properties of Miniversal logic, when studied from a "coarse-grained" perspective -say using a meta-language-, will be found to have the same limits and behaviour as the universe. It is from those limits that the laws of physics will be derived.

Alice: Can you spell out the isomorphism you have in mind?\(^\text{14}\)

Yes, we are now ready to return to our interpretation of the universe as a blob of facts.

3.4 Summary

The main argument of this section can be broken down and summarized in a few points.

1. As Miniversal logic is a reproduction of Descartes’ universal doubt method, its theorems are ‘irrefutable’ for the same reasons and to the same degree as the cogito is ‘irrefutable’.

2. The anthropic principle guarantees that each theorem of Miniversal logic is a ‘synthetic a-priori’ statement. Hence, each theorem restricts what the universe can be. The implication is that the ultimate theory which explains the universe must have, as its theorems, all theorems of Miniversal logic.

3. Miniversal logic is universal - it embeds all possible theorems within its branches. Hence the ultimate theory which explains the universe cannot have more theorems than Miniversal logic.
4. As a result, the 'truth content' of the universe is identical to the 'truth content' of Miniversal logic - no more no less. The universe is reduced to a single solution.

5. The conjecture is that by adopting a "coarse-grained" view applicable to the theorems, we will be able to recover the laws of physics from the properties of Miniversal logic.

4 The universe as a blob of facts

By definition, a blob encodes a system as the answers to multiple yes or no questions. If the questions relate to the identification of sentences that are theorems, then the domain of the blob is related to the provability of the sentences within a system of logic. Specifically in the case where the blob encodes the provability of all possible theorems, we will say that the blob is universal. In this case, knowing the precise microstate of a universal blob is equivalent to knowing which theorems are provable and which are not.

We can construct the blob corresponding to the universe by having it encode the provability of the sentences of Miniversal logic. As Miniversal logic contains all theorems, the blob will be universal. We are looking for a construction of the blob that will connect its microstate to its "coarse-grained" view. A blob can be structured as such by making use of Chaitin’s construction. As per the standard construction, we begin by listing in shortlex all sentences of Miniversal logic. Each sentence is either a theorem or a non-theorem - this defines $H(p_i)$. We then encode the sentences with a prefix-free code. The degree of contribution of each encoded sentence to the sum is exponentially proportional to the negative of the length of the code. This produces:

$$\Omega = \sum_{i=1}^{\infty} 2^{-H(p_i) - |p_i|} \text{ where } H(p_i) = \begin{cases} 0 & p_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \quad (4.1)$$

In this representation we will say that $\Omega$ is the microstate of the blob, and that the sum is the "coarse-grained" view describing the properties of the blob. It sums all theorems of Miniversal logic (the terms of the sum) and compresses them into a single real number ($\Omega$). The yes or no answers describing the blob are the 0 or 1 bits of $\Omega$. To help fix the idea, consider the following example;

$$= 2^{-\infty} 2^{-1} + 2^{-0} 2^{-2} + 2^{-0} 2^{-3} + 2^{-0} 2^{-4} + 2^{-\infty} 2^{-5} + ... \quad (4.2)$$
The presence of the negative infinity in the term of the exponential causes some terms to vanish to zero. Note that the suffix $b$ indicates the binary notation.

\[
\begin{align*}
&= 0_b + 0.01_b + 0.001_b + 0.0001_b + 0_b + ... \\
&= 0.01110..._b
\end{align*}
\]

which recovers $\Omega$ for the example values.

We will now be working with $\Omega$ extensively. Consequently, we will adopt the language of algorithmic information theory (AIT) and we will replace the following expressions by their AIT-equivalents:

\[
\begin{align*}
\text{Proof-theoretic} & \quad \text{AIT} \\
\text{sentence} & \implies \text{program} \\
\text{the sentence is a theorem} & \implies \text{the program halts} \\
\text{the sentence is not a theorem} & \implies \text{the program does not halt}
\end{align*}
\]

This language will be more in line with the subject matter.

4.1 Entropy

As we have previously stated, Miniversal logic restricts the universe to a single solution. However, the formulation of Miniversal logic is itself not unique. Should we express Miniversal logic in first order logic, or second order logic? Should we use English, French, binary, etc.? Since the choice of formulation of Miniversal logic is not itself fixed by Miniversal logic, then to accurately represent reality, we must somehow adjust the Chaitin construction to account for multiple possibilities in the formulation of Miniversal logic.

We have previously presented Chaitin’s construction has a generalization of a (modern) Borel number. We will now present this exercise as a generalization of Chaitin’s construction.

We can represent the available choices in regards to the formulation of Miniversal logic with an entropy. We must also consider that the halting probability does impose that the sentences of Miniversal ultimately be encode as a prefix-free code. Under these circumstances, the $\Omega$ construction is able to accommodate an entropy by adding a multiplication constant applicable to the length of the codes. This multiplication constant “decompresses” the information of $\Omega$ but does not reduce its domain. With the multiplication terms, the equation becomes:
\[
\Omega_D = \sum_{i=1}^{\infty} 2^{-\beta (H(p_i) + D(p_i))}
\] (4.8)

This construction of \( \Omega_D \), known as the Tadaki D-random number, represents a thermal universal blob. It describes the properties of a universal blob that carries an entropy applicable to choice of prefix-free code. The entropy of the ensemble is given by the standard relation:

\[
S = k_B \left( \ln \Omega_D + \beta D[p] \right)
\] (4.9)

and its equation of state is

\[
\frac{1}{\ln 2} TdS = Dd[p]
\] (4.10)

In this case, the \( \Omega_D \) construction is also a Gibbs ensemble of statistical physics and includes the usual Lagrange multiplier term \( \beta D \). Many authors have discussed the similarity between statistical physics and algorithmic information theory. This entropy-bearing formulation \( \Omega_D \) is best understood as a generalization of the halting probability which connects the two domains via the Gibbs ensemble. We note that \( \Omega \) is recovered as a special case when \( \beta D = 1 \).

4.2 Feasibility cutoffs

In the previous section regarding bridging metaphysics to physics, we have argued that an observer can produce, in principle, the proof of any theorem of Miniversal logic. Furthermore, we have argued that as the observer is in the universe, then any such proofs he finds can be used to identify a theorem of the universe (anthropic principle). Furthermore, as Miniversal logic is universal, then the process will eventually enumerate all possible theorems. Hence the methodology can uniquely define the solutions to the theory which explains the universe. We note that for the argument to work we must allow the notion ‘in principle’, which in this context means that an observer can submit proofs of arbitrary size for theorems of arbitrary size.

In practice however an observer will hit a size limit to any proof or theorem he can produce. What if a theorem is true but its proof requires \( \text{TREE}(9999) \) bits? Alternatively, what if a theorem has a short proof but the theorem itself requires a googolplex amount of bits to express? To accurately represent what the observer can feasibly do, we must add a feasibility cutoff to the Gibbs ensemble. With such a cutoff, the blob contains only the size-bounded proofs to size-bounded theorems. It describes all ‘synthetic a-priori’ statement that
an observer can feasibly produce to restrict the universe to a solution. The resulting blob, which is no longer universal, will recover its universality as the feasibility bound grows to infinity.

To adjust the equation such that it describes a feasible blob, we must first introduce the concept of dovetailing. Consider an observer attempting to prove the theorems of Miniversal logic. The observer could pick a sentence at random and work at it until the proof is found. The problem is that if the sentence is non-provable, the observer will hang attempting to prove it. So instead, the observer might try to write one bit of proof for each sentence, then the second bit for each sentence, etc. However, since there are infinitely many sentences, the observer will never return to write the second bit. The solution is to dovetail the work.

**Definition 4.11 (Dovetailing).** Dovetailing is a proof-finding strategy for a system of logic to guarantee that progress will be made on arbitrarily-many theorems even in the presence of non-provable sentences.

**Definition 4.12 (Simple dovetailing).** Consider the case of simple dovetailing. First, an observer write one bit of the proof of the first sentence. Then, the observer write a bit of the proof for both the first and the second sentence. Then, the observer write a bit of the proof for the first, the second and the third sentence. And so on. The observer stops writing bits for sentences whose proof is completed. In the case of a non-provable sentence, the observer would write a bit for it for all future iterations. Using this method, progress will eventually be made on every sentence and no sentence will cause the observer to hang indefinitely.

There are a few additional concerns and pitfalls that we want to avoid when introducing the feasibility cutoff.

- Consider that an observer can attempt to calculate $\Omega$ by working on each theorem in a ‘dovetailing’ manner. As proofs are found, the observer adds their contributions to $\Omega$. After an infinite amount of time, $\Omega$ will indeed be recovered. However, the calculation does not converge towards $\Omega$ as it progresses and discontinuously yields $\Omega$ only at infinity. To see why, consider the case where the first zero-valued bit of $\Omega$ is at position $i$. Since the general non-halting problem is unsolvable, at most the calculation of $\Omega$ differs from the real value of $\Omega$ by $2^{-i}$. The error rate does not decrease during the calculation and only vanishes at infinity when all halting programs are known. As a result, at any time only a handful of the bits of $\Omega$ can be known and the irreducible entropy of the universe would be very small and unable to grow. So simple dovetailing would hit a very small complexity limit for the universe.
• When we define the state of the universe by a proof-size bounded subset of $\Omega$ (let’s use the symbol $Z_\Omega$ to denote it), we want to avoid the situation where the bits are overwritten as the calculation progresses. For example, say at some time $Z_\Omega(t_1) = 0.001100...b$ and at some other time $Z_\Omega(t_2) = 0.101100...b$ (In this example the proof associated with the first sentence is completed and the first bit flips to a 1). The unfortunate result is that the previously calculated microstate of the blob is overwritten during the calculation. As this calculation will be connected to time in a future section, this would basically imply that the future can rewrite the past. To avoid this, we would prefer $Z_\Omega$ to monotonically converge towards $\Omega$ as $t \to \infty$.

• As we have chosen the path of entropy to discuss the macroscopic properties of a non-computable $\Omega$ then the dovetailing algorithm that we introduce must not dissolve the Gibbs ensemble. Therefore, it must be introduced as a thermodynamic conjugate pair.

• Finally, as the concept of feasibility is the last metaphysical argument in our toolbox, introducing it into the Gibbs ensemble must be sufficient to recover the familiar laws of physics such as; the arrow of time, general relativity, dark energy, the Dirac equation, etc.

With all of these requirements in mind, we might expect the feasibility term to be quite complicated. However this is not so at all. Simply adding; the action, represented by $S$, to the frequency, represented by $f$, as a conjugate pair is enough to meet all of these requirements! In the proof-theoretic paradigm, the action can be interpreted as the “effort” required to complete a proof and the frequency can be interpreted as the rate at which steps of the proof must be made to complete the proof within the allocated effort. Adding this conjugate pair replaces $H(p_i)$ and gives this equation:

\[
Z_\Omega = \sum_{i=1}^{\infty} 2^{-\beta(D|p_i|+2\pi Sf_i)}
\]

(4.13)

Here, $Z_\Omega$ encodes for each theorem of Miniversal logic and as a microstate of the Gibbs ensemble, its prefix-free code and, in the form of a frequency, the number of bits its proof requires\(^{16}\). Looking in more detail at this equation, we will see how it produces a new form of dovetailing which is thermal.

**Definition 4.14 (Thermal dovetailing).** **Thermal dovetailing is an algorithm according to which the work done on programs is scheduled so as to**

---

\(^{16}\) The use of the factor $2\pi$ will become clear when we connect $S$ to the reduced Planck constant and $f$ to the angular frequency $\omega$ in section 9. It is added to this Gibbs ensemble so as to recover the definition of the Planck units in terms of $\hbar$. The reader who finds the use of the $2\pi$ factor unpalatable without full justification is advised to skip to section 9 and come back.
maximize the entropy of the system during the computation. Thermal dovetailing is expressed as a Gibbs ensemble which guarantees that the entropy is maximized.

We will now study in more detail the equation. First, let's prove this theorem.

**Theorem 4.15.** $2\pi S f_i$ recovers $H(p_i)$ when $S \to 0^+$. 

**Proof.** An arbitrary sentence $i$ may require a proof of any size $f_i \in [0, \infty]$. If the sentence is a theorem the size of its proof will be $f_i \in [0, \infty]$. If the sentence is not a theorem, its "proof" will be of size $f_i = \infty$. When the effort required to prove a theorem goes to zero (e.i. $S \to 0^+$), all theorems are proven effortlessly. In this case,

$$\lim_{S \to 0^+} 2\pi f_i S = \begin{cases} 
0 & p_i \text{ is a theorem} \\
\infty & \text{otherwise}
\end{cases} \quad (4.16)$$

This is the definition of $H(p_i)$. We recall it here;

$$H(p_i) = \begin{cases} 
0 & p_i \text{ is a theorem} \\
\infty & \text{otherwise}
\end{cases} \quad (4.17)$$

Therefore,

$$\lim_{S \to 0^+} 2\pi f_i S = H(p_i) \quad (4.18)$$

The theorem is proven. Replacing $H(p_i)$ by $2\pi f_i S$ in the Gibbs ensemble still allows us to recover $\Omega$ when the effort required to perform computation goes to $0^+$. 

\[ \square \]

Let us now expand $Z_\Omega$ explicitly with an example. Assume a system comprised of three microstates with program length $|p_1| = 1$, $|p_2| = 2$ and $|p_3| = 3$ and with the following frequencies;

$$f_1 = \frac{5}{2\pi} \quad f_2 = \infty \quad f_3 = \frac{5}{2\pi} \quad (4.19)$$

In this example, $f_1$ and $f_2$ are theorems and $f_3$ is a non-theorem. For the purposes of simplicity we can assume that all other sentences are non-theorems. In this case the system is not universal but let us nonetheless use it as a simplified numerical example. The sum $Z_\Omega$ becomes;
\[ Z_\Omega(S) = 2^{-1-5S} + 2^{-2-\infty S} + 2^{-3-5S} \]  

We will now produce a series of numerical calculations with progressively smaller values of \( S \) and we will look at the evolution of the error rate \( \xi(S) = \Omega - Z_\Omega(S) \). For this system, \( \Omega = 0.101\overline{0} \).

<table>
<thead>
<tr>
<th>( S )</th>
<th>( Z_\Omega(S) )</th>
<th>( \xi(S) )</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \infty )</td>
<td>0</td>
<td>( \Omega )</td>
<td>max</td>
</tr>
<tr>
<td>1</td>
<td>0.000000101... ( _b )</td>
<td>0.10011011 ( _b )</td>
<td>( \approx 2^{-1} )</td>
</tr>
<tr>
<td>0.1</td>
<td>0.011000010... ( _b )</td>
<td>0.00101110... ( _b )</td>
<td>( \approx 2^{-3} )</td>
</tr>
<tr>
<td>0.01</td>
<td>0.100110101... ( _b )</td>
<td>0.00000010... ( _b )</td>
<td>( \approx 2^{-6} )</td>
</tr>
<tr>
<td>0.001</td>
<td>0.01100010... ( _b )</td>
<td>0.00000000... ( _b )</td>
<td>( \approx 2^{-9} )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>0</td>
<td>( \Omega )</td>
<td>0</td>
<td>none</td>
</tr>
</tbody>
</table>

As we can see, reducing the effort to perform computational steps causes the system \( Z_\Omega \) to monotonically converges towards \( \Omega \). The feasible blob (defined up to the error rate) grows as more valid bits of \( \Omega \) are obtained. This yields an asymmetric arrow connected to the non-computability of \( Z_\Omega \). This will be presented in section 10 as the proper solution to the problem of the arrow of time in physics.

We are now in a position to define the following conjecture;

**Definition 4.27** (Thermal UTM conjecture). *It is the conjecture that a thermal universal Turing machine; defined as a prefix-free universal Turing machine which maximizes its entropy during the calculation of its own halting probability \( \Omega \), is isomorphic to the universe. The calculation is performed via thermal dovetailing. Like other forms of dovetailing, it allows work to be done on arbitrarily many sentences even in the presence of non-provable sentences. The calculation is expressed via the Gibbs ensemble*

\[ Z_\Omega = \sum_{i=1}^{\infty} 2^{-\beta(D_p|p_i|+2\pi f_i)} \]

*and occurs when \( S \) goes from \( \infty \) to 0.*

We are all set! From here on, extracting the laws of physics from this Gibbs ensemble will be a near trivial.

5 **Broad overview of the main results**

In this section we will present an overview of the main results in order to build an intuition which will help in the next sections.
5.1 **An algorithmic representation of the universe**

We are now representing the universe as a partition function summing over all sentences that are provable in it up to certain limits on size. These are the facts of the universe. The partition function associates to each fact a program capable of arbitrary computation whose execution finds the proof of the fact (it explains the why the fact is true). Furthermore, as the partition function admits a frequency \( f_i \) which is individually adjustable to the program it is associated to, it can be adjusted to correctly account for when a fact becomes true. It is easy to see how this flexibility allows the construction of the partition function corresponding to a system of arbitrary complexity.

As we will soon see, space and time are to be interpreted as emergent from the blob. Consequently, almost every point of spacetime will be represented by a program capable of arbitrary computation. Although in some cases the full power of arbitrary computation might not be needed, it will be in the general case. For example, a region might have an observer or a computer in it - hence any small region of space holds the potential to execute arbitrary computation.

We stress however that such a flexible representation of a system is only useful if the laws of physics can be extracted from the representation. After we are done with this broad overview, we will not only show that they indeed can, but also that they come out quite simply. Such laws are to be interpreted as describing the limits and the properties applicable to the microstates of the partition function.

5.2 **Thermal spacetime**

Deriving the familiar laws of physics from the equation is now quite straightforward. The state equation for the partition function (4.28) is,

\[
\frac{1}{\ln 2} TdS = Dd|p| + 2\pi S df
\] (5.2)

Solutions to this equation of state yields all universes which can be encoded by a series of theorems (questions) whose provability (answer) can be verified via arbitrary computation up to a bound on proof-sizes and theorem-sizes and such that universality is recovered when the bound is removed. It can also be interpreted as a universal Turing machine operating at thermal equilibrium. We will now rewrite this equation of state, using standard mathematical transformation, to a form which is easier to conceptualize from the physical perspective.

As \( Dd|p| \) is an non-trivial function, we will now take its Taylor expansion. To do so, we first pose \( L(p) := |p| \). Then we obtain,
\[ DL(p) = DL(0) + L'(0)p + \frac{1}{2}L''(0)p^2 + \frac{1}{6}L'''(0)p^3 + ... \quad (5.3) \]

\[ DdL(p) = DL'(0)dp + DL''(0)pdp + D\frac{1}{2}L'''(0)p^2 dp + ... \quad (5.4) \]

switching the notation from \( p \) to \( x \), we get

\[ DdL(x) = DL'(0)dx + DL''(0)x dx + D\frac{1}{2}L'''(0)x^2 dx + ... \quad (5.5) \]

then further posing \( F := DL'(0), k := DL''(0), p := DL'''(0) \) (here \( p \) means the pressure, not a program), we get

\[ DdL(x) = Fdx + kxdx + px^2 dx + ... \quad (5.6) \]

We will now replace \( Dd|p| \) with its Taylor expansion. The state equation is transformed to;

\[ \frac{1}{\ln 2} TdS = (F + kx + px^2 + ...) dx + 2\pi S df \quad (5.7) \]

We can interpret the terms \( (F + kx + px^2)dx \) as three different regimes. \( Fdx \) should be interpreted as a one-dimensional term, \( kxdx \) as a two-dimensional term, and finally \( px^2dx \) as a three-dimensional term. We can rewrite them as \( Fdx, kdA \) and \( pdV \).

To final step is to use the relation \( f = 1/t \) to convert \( S \) to a power \( P \).

\[ 2\pi S df = 2\pi S \left( \frac{1}{t} \right) \quad f = 1/t \quad (5.8) \]

\[ 2\pi S df = 2\pi S (-t^{-2}) dt \quad df = -t^{-2} dt \quad (5.9) \]

\[ 2\pi S df = -P dt \quad \text{where}\ P = 2\pi St^{-2} \quad (5.10) \]

We can then rewrite the equation of state to

**Definition 5.11** (Physical equation of state).

\[ \frac{1}{\ln 2} TdS = Fdx + kdA + pdV + ... - Pdt \quad (5.12) \]

Note that as the Taylor expansion was taken, the physical equation of state is only defined for smooth functions of \( L(p) = |p| \). Therefore, a smoothness of the space of \( L(p) \) is implicitly assumed. Furthermore, the smoothness approximation will apply in a domain where the lengths are large with respect to the program step size. In the case of the universe, where the program step length could be on the order of the Planck length, the smoothness approximation would apply for lengths \( \gg \) Planck length.

The physical equation of state is equivalent (within the few approximations mentioned) to the algorithmic equation of state.
5.3 Regimes and cycles

We will study the physical equation of state as we would study any thermodynamic equation of state. We will pose certain conjugate-derivatives to be zero, while allowing others to vary. The equation contains 6 conjugate-derivatives \((dS, dt, dx, dA \text{ and } dV)\). Each can be set to 0 or allowed to vary. This yields a total of 57 thermodynamics regimes (57 is obtained by taking the total of 64 permutations and subtracting those for which all variables would be 0). In this paper, we will only study the ones that I was able to connect to known laws of physics and will leave the others as open problems for future research. The list below serves as an outline for the future sections of the paper. In these sections, the law (left column) associated to each of the regimes will be formally derived from the regime it is associated to.

Each of the three space regimes \((dx,dA,dV)\) are specified and are studied in isolation by the applicability of these three approximations:

\[(dx = dA = 0) \land (dV \neq 0) \quad \Leftrightarrow \quad F \gg (kx + px^2 ... ) \quad (5.13)\]
\[(dx = dV = 0) \land (dA \neq 0) \quad \Leftrightarrow \quad kx \gg (F + px^2 + ...) \quad (5.14)\]
\[(dA = dV = 0) \land (dx \neq 0) \quad \Leftrightarrow \quad px^2 \gg (F + kx + ...) \quad (5.15)\]

The regimes we will cover will be the following:

\[\begin{array}{c|cccccc}
\text{The Universe} & TdS = & -Pdt & +Fdx & +kdA & +pdV & + \ldots \\
\text{Maximum speed} & TdS = & -Pdt & +Fdx & 0 & 0 & 0 \quad (5.17) \\
\text{Maximum viscosity} & TdS = & -Pdt & 0 & +kdA & 0 & 0 \quad (5.18) \\
\text{Maximum vol. flow rate} & TdS = & -Pdt & 0 & 0 & +pdV & 0 \quad (5.19) \\
\text{Special relativity} & 0 = & -Pdt & +Fdx & 0 & 0 & 0 \quad (5.20) \\
\text{Arrow of time} & TdS = & -Pdt & 0 & 0 & 0 & 0 \quad (5.21) \\
\text{Law of Inertia} & TdS = & 0 & +Fdx & 0 & 0 & 0 \quad (5.22) \\
\text{General relativity} & TdS = & 0 & 0 & +kdA & 0 & 0 \quad (5.23) \\
\text{Dark energy} & TdS = & 0 & 0 & 0 & +pdV & 0 \quad (5.24) \\
\text{Darker energies?} & TdS = & 0 & 0 & 0 & 0 & +c(dx)^4 \quad (5.25) \\
\end{array}\]
For example, the regime associated with the provability of the "maximum speed" occurs when $dA = 0$, $dV = 0$ and $(dx)^2 = 0$.

5.4 Universal constants

The force $F$, the power $P$ and the action $S$ are the Lagrange multipliers of the Gibbs ensemble. For a system at algorithmic equilibrium, these values are constant throughout the system. They are the primary constants defining the system. We will show in section 9 from first principles that these are indeed the Planck force, the Planck power and the Planck action. This will be enough to recover all Planck units.

5.5 Limiting relations

**Theorem 5.26.** The equation of states (5.12) implies a maximum speed.

**Proof.**

\[
\begin{align*}
(ln 2)^{-1} T dS &= F dx - P dt \quad \text{regime 5.17} \quad (5.27) \\
\frac{1}{\ln 2} \frac{T dS}{dt} &= \frac{dx}{dt} - \frac{P}{F} \quad \text{division by } F dt \quad (5.28)
\end{align*}
\]

To see why this implies a maximum speed, first consider that the units of this equation are meters per second. Second, consider the following three cases;

\[
\begin{align*}
\frac{dx}{dt} &= \frac{P}{F} \quad \implies \frac{dS}{dt} = 0 \quad (5.29) \\
\frac{dx}{dt} &< \frac{P}{F} \quad \implies \frac{dS}{dt} < 0 \quad (5.30) \\
\frac{dx}{dt} &> \frac{P}{F} \quad \implies \frac{dS}{dt} > 0 \quad (5.31)
\end{align*}
\]

To prove that the speed $P/F$ is a maximum, it suffices to note that a reversal of the second law of thermodynamics occurs at the $P/F$ barrier. Let us recall that the second law of thermodynamics is a provable consequence of the Gibbs ensemble of statistical physics and can be reviewed and any introductory textbook. Then, it follows that the barrier cannot be overcome. A system evolving faster than $P/F$ will experience a reversal of the second law compared to a system slower than $P/F$ (and vice-versa), but neither will be able to cross $P/F$ and flip its direction.
Remark: When $P$ is the Planck power and $F$ is the Planck force, we do recover $c$ the speed of light;

$$P \left( \frac{1}{F} \right) = \frac{c^5}{G} \left( \frac{G}{c^4} \right) = c$$  \hspace{1cm} (5.32)

Theorem 5.33. The following relations each characterize a maximum quantity.

approx.

\[
\begin{align*}
\text{none} & \quad \frac{1}{\ln 2} \frac{dS}{F dt} = -P & \text{maximum power (J/s)} \quad (5.34) \\
S \propto L & \quad \frac{1}{\ln 2} \frac{dS}{F dt} = \frac{dx}{dt} - \frac{P}{F} & \text{maximum speed (m/s)} \quad (5.35) \\
S \propto A & \quad \frac{1}{\ln 2} \frac{dS}{k dt} = \frac{xdx}{dt} - \frac{P}{k} & \text{maximum viscosity (m}^2/\text{s)} \quad (5.36) \\
S \propto V & \quad \frac{1}{\ln 2} \frac{dS}{p dt} = \frac{x^2 dx}{dt} - \frac{P}{p} & \text{max. vol. flow rate (m}^3/\text{s)} \quad (5.37)
\end{align*}
\]

Proof. Each relation can easily be obtained from (5.12) by posing the other observables to 0. To prove that the quantities are a maximum, it suffices to notice that they each correspond to the point at which the second law of thermodynamics is reversed. \qed

It is well-known that a maximum speed implies special relativity (section 6), but what about to other two? It is less well known, but nonetheless, a maximum viscosity does implies general relativity. In this context, we can interpreted space as being encoded by bits moving very slowly (like molasses) on the surface of a sphere (section 7.3). The maximum volumetric flow rate is associated with dark energy and is responsible for the Hubble horizon - beyond which the flow rate would be exceeded (section 7.4).

5.6 Holographic principle

The equation suggests three entropic growth rates; linear, quadratic and cubic. Each is responsible for a "holographic" principle of a different dimensional level.

Theorem 5.38. The state equation (5.12) implies a holographic principle in the area-dominant regime, where $xdx$ is the dominant contributor to the entropy.
Proof.

\[(\ln 2)^{-1} T dS = k dA \quad \text{regime 5.23} \quad (5.39)\]

\[
\int T dS = (\ln 2) \int k dA \quad (5.40)
\]

\[
TS = (\ln 2)k \frac{1}{2} A + C \quad (5.41)
\]

\[
\Rightarrow S \propto A \quad \text{holographic principle} \quad (5.42)
\]

The laws of physics, which will be derived from the area-dominant approximation will necessarily contain a holographic principle linking the entropy to the area enclosing the volume. However, the holographic principle need not necessarily hold at other entropic growth scales, for example, where the volumetric entropy \(pdV\) is dominant. Indeed, equation of state (5.12) would appear to suggest three different scales, each having a "holographic" principle of a different dimensional size.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Dominant Term</th>
<th>Entropy</th>
<th>Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>(F dx)</td>
<td>(S \propto L)</td>
<td>inertia (5.43)</td>
</tr>
<tr>
<td>2D</td>
<td>(kdA)</td>
<td>(S \propto A)</td>
<td>general relativity (5.44)</td>
</tr>
<tr>
<td>3D</td>
<td>(pdV)</td>
<td>(S \propto V)</td>
<td>dark energy (5.45)</td>
</tr>
</tbody>
</table>

In this scenario, the universe would be dominated by the linear scale at short distances, which would then be overtaken by the area scale and finally by the volume scale.

5.7 Universal Brownian motion

Here we consider regime 5.20 (special relativity) and 5.22 (inertial law). The conjugate pairs applicable to these regimes are \(Fx\) and \(Pt\). The \(F\) and \(P\) are constant throughout the system. However \(x\) and \(t\) are each adjusted for their specific microstate and are not fixed within the system. Therefore, unlike the Planck units, \(x\) and \(t\) are described as a statistical average and undergo thermal fluctuations. The equations describing them are;
Using the original argument made by Einstein in 1905 which lead to the derivation of Brownian motion, we argue here that fluctuations of the $t$ and $x$ variables produce a Brownian motion along the axis. The consequences of such are nothing to be feared. Indeed, we will show in section 8.1 that a random walk over $dx$ will transform the inertial regime ($F = ma$) into the Schrödinger equation. Furthermore, we will show in section 8.48 that a random walk over both $dx$ and $dt$ will transform the special relativity regime into the Dirac equation.

We are suggesting that the quantum version of the classical laws is recovered when the thermal fluctuations are taken into account. As both the quantum laws and the classical laws are spawned from the microstates, one interpretation could be that the microstates are in a sense "sub-quantum" (as in, they are more fundamental).

Note that we are not suggesting a pilot-wave type of interpretation where the particle would exist punctually but would be undergoing Brownian motion until a measurement is made. Rather, we suggest that any positional or time information undergoes a "Dirac equation-like diffusion" so as to make positional or time information perishable over time.

To illustrate, we can imagine placing a mark at a position in space. After a certain time, the random walk will diffuse the position of the marker at any number of possible locations until its actual position is measured again. Instead of being punctual, the marker could be continuous and weighted and the same diffusion-like behaviour will be observed. This Brownian motion would universally apply to the axis themselves.

### 5.8 Universal averages - length and time

We have just explained how fluctuations implies a random walk along the $x$ and $t$ axis. But why are there axis to begin with? Consider these equations (repeated for convenience from the previous section):

<table>
<thead>
<tr>
<th>Variable</th>
<th>Average</th>
<th>Fluctuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ (time)</td>
<td>$\bar{t} = -\frac{\partial \ln \Omega}{\partial P}$</td>
<td>$(\Delta t)^2 = \frac{\partial^2 \ln \Omega}{\partial P^2}$</td>
</tr>
<tr>
<td>$x$ (space-linear)</td>
<td>$\bar{x} = -\frac{\partial \ln \Omega}{\partial F}$</td>
<td>$(\Delta x)^2 = \frac{\partial^2 \ln \Omega}{\partial F^2}$</td>
</tr>
<tr>
<td>$A$ (space-area)</td>
<td>$\bar{A} = -\frac{\partial \ln \Omega}{\partial k}$</td>
<td>$(\Delta A)^2 = \frac{\partial^2 \ln \Omega}{\partial k^2}$</td>
</tr>
<tr>
<td>$V$ (space-volume)</td>
<td>$\bar{V} = -\frac{\partial \ln \Omega}{\partial F}$</td>
<td>$(\Delta V)^2 = \frac{\partial^2 \ln \Omega}{\partial p^2}$</td>
</tr>
</tbody>
</table>
average fluctuation

\[ \begin{align*}
\bar{t} &= -\frac{\partial \ln Z_{\Omega}}{\partial P} \quad \langle (\Delta t)^2 \rangle = \frac{\partial^2 \ln Z_{\Omega}}{\partial P^2} \\
\bar{x} &= -\frac{\partial \ln Z_{\Omega}}{\partial F} \quad \langle (\Delta x)^2 \rangle = \frac{\partial^2 \ln Z_{\Omega}}{\partial F^2} \\
\bar{A} &= -\frac{\partial \ln Z_{\Omega}}{\partial k} \quad \langle (\Delta A)^2 \rangle = \frac{\partial^2 \ln Z_{\Omega}}{\partial k^2} \\
\bar{V} &= -\frac{\partial \ln Z_{\Omega}}{\partial F} \quad \langle (\Delta V)^2 \rangle = \frac{\partial^2 \ln Z_{\Omega}}{\partial F^2}
\end{align*} \]

(5.50) (5.51) (5.52) (5.53)

From these equations, it is possible to see how the information of the universe can structure itself in a manner so as to permit the emergence of a length and a time as an averaging effect over the information. This is what the average relation for \( \bar{t} \) and \( \bar{x} \) imply. An observer who is unable to probe the microstates of the Gibbs ensemble will see a macroscopic length and a macroscopic time as an averaging effect on the microstates.

In the next section we will see explicitly how this averaging effects has the properties and laws that we associate with length and time; such as special relativity, general relativity, etc. - hence they do indeed correspond to our idea of what a physical length or time is. The implication is that the time and space axis should not be understood as fundamental concepts. They should be understood as a thermal spacetime as they represent thermodynamic averages over the microstates.

5.9 Physical language

We will now introduce the physical language as an equivalent to the proof-theoretic language. In this equivalence, the proof-size is renamed to time with units seconds, and the theorem-size is triple-renamed (Taylor expansion) to length with units meters, to area with units meters-squared and to volume with units meters-cubed. Summarizing these equivalence, we obtain Table 2 as the thermal mapping between algorithmic thermodynamics and physical thermodynamics. In the next sections we will liberally alternate between the equivalent languages (physical, proof-theoretic and algorithmic) as appropriate.

5.10 Summary

Since the last summary,

1. We have seen that the information of the blob can be representation by a real number which is the halting probability of a prefix-free universal Turing machine.
2. We have seen that the properties of this number, when adjusted to allow only feasible proofs (time cutoff) to feasible questions (size cutoff), yields a thermodynamic equation.

3. As the equation encodes and calculates a halting probability, it describes a universal system. It can be interpreted as a thermal universal Turing machine.

4. We have claimed that the the equation of state suggests multiple regime of physics, each associated with a specific law of physics. Important notions are; A) The Planck units $F$, $P$ and $S$ are the thermodynamic constants of the system and correspond to the Lagrange multipliers (proof of such in section 9). B) The limiting relations corresponds to the limits of special relativity, general relativity and dark energy. C) At least three holographic principles of different dimensionality applies to the universe and D) a universal Brownian motion applies to all axis which encodes space or time information. E) The axis $t$ and $x$ are to be interpreted as thermal axis resulting from averaging effects over the information of the universe.

We will now derive the laws of physics themselves.

6 Special relativity

6.1 Light cones as thermodynamic cycles

In this section, we look at the thermodynamic cycle of the system transiting through time and space starting at $O$ to $A$ to $B$ and back to $O$, as illustrated on Figure 1. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles, such as those involving pressure and volume but also of relativistic light cones.

We select regime 5.20 (special relativity) for our cycle.

$$\frac{1}{\ln 2} TdS = -Pdt + Fdx$$ (6.1)
O to A: As O is translated forward in time to A while keeping the distance constant \((dx = 0)\), the entropy must decrease over time to compensate.

\[
\left( \frac{1}{\ln 2} \right) TdS = -Pdt + Fdx \bigg|_{dx=0} \quad \text{(6.2)}
\]

\[
\Rightarrow \frac{dS}{dt} = -(\ln 2) \frac{P}{T} \quad \text{(6.3)}
\]

A forward translation in time causes the system to know more bits of \(\Omega\). As the unknown bits of \(\Omega\) carry an entropy, knowing more reduces the entropy. Conversely, a backward translation in time causes the system to erase bits from its pool of information which increases its entropy. A backward translation in time is equivalent to erasing halting information about the system’s present.

A to B: As A is translated forward in space to B while keeping the time constant \((dt = 0)\), the entropy must increase over space to compensate.

\[
\left( \frac{1}{\ln 2} \right) TdS = -Pdt + Fdx \bigg|_{dt=0} \quad \text{(6.4)}
\]

\[
\Rightarrow \frac{dS}{dx} = (\ln 2) \frac{F}{T} \quad \text{(6.5)}
\]

We conclude that the further away from A a region is, the higher its entropy will be. Since \(dt = 0\), no change in time is experienced.

O to B: As O is translated forward both in time and in space to B while keeping the entropy constant \((dS = 0)\), the system has a speed \(c\).

\[
\left( \frac{1}{\ln 2} \right) TdS = Fdx - Pdt \bigg|_{dS=0} \quad \text{(6.6)}
\]

\[
\Rightarrow \frac{dx}{dt} = \frac{P}{F} = c \quad \text{(6.7)}
\]

We conclude that an object traveling at speed \(c\) is neither encouraged nor discouraged by entropy. However, the type of entropy changes. The rate \(P/F\) is the rate of conversion of time entropy to space entropy. At O, the system is comprised exclusively of time entropy as its future is not yet determined. As the system evolves towards B, its time entropy is decreased over time as the system replaces its future entropy with a singular past. Its space entropy

Figure 1: A thermodynamic cycle through space, time and entropy as observables.
(which encodes the singular past), however is increased to offset the reduction.

As a backward translation in time erases the most recently calculated bits of $\Omega$, we conclude that the system "forgets its future" during the backward translation.

6.2 Lorentz’s transformation

To recover the Lorentz’s factor $\gamma$, let us consider figure 2. Two observers start at the origin $S$ and travel in space-time respectively to $O$ and $O'$. We regard $O'$ as traveling at speed $|v|$ in the reference frame of $O$. From standard trigonometry, we derive the following values for the length of the segments;

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>SO</td>
</tr>
<tr>
<td>$</td>
<td>SO'</td>
</tr>
<tr>
<td>$</td>
<td>OO'</td>
</tr>
</tbody>
</table>

We start with the Pythagorean theorem and solve for $\cos \theta$.

\[
|SO|^2 = |SO'|^2 + |OO'|^2
L^2 = (L \cos \theta)^2 + (L \sin \theta)^2
1 = (\cos \theta)^2 + (\sin \theta)^2
\]
\[
\sqrt{1 - (\sin \theta)^2} = \cos \theta
\] (6.14)

We consider that the distance between two observers moving at constant speed is simply $vt$. Hence, $|OO'| = vt$. Solving for $\sin \theta$, we obtain

\[
|OO'| = vt = L \sin \theta
\]
\[
\Rightarrow \sin \theta = \frac{vt}{L}
\] (6.15)

From equation (6.14) and (6.16), we get the reciprocal of the Lorentz factor,

\[
\sqrt{1 - \frac{v^2 t^2}{L^2}} = \cos \theta = \gamma^{-1}
\]
\[
\Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2 t^2}{L^2}}}
\] (6.18)
Finally, we consider that $L$ is the distance travelled in time by $O$ in its own reference frame. This is given via the relation $dx = cdt$. Hence $L = ct$. We obtain,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$  \hspace{1cm} (6.19)$$

which is the well-known Lorentz factor and is the multiplication constant connecting $\mid SO\mid$ to $\mid SO'\mid$.

7 Holographic principles

7.1 1D "Holographic" principle (Law of inertia)

First, let us derive a relation between $dS$ and $dN$. Here $N$ represents the number of bits.

**Theorem 7.1.** $dS = (\ln 2)k_BdN$

**Proof.**

$$S = Nk_B \ln(2) \hspace{1cm} \text{binary entropy} \hspace{1cm} (7.2)$$

$$\implies dS = (\ln 2)k_BdN \hspace{1cm} (7.3)$$

\[\square\]

Second, let us look at the implications of the first term, $Fdx$ in the $S \propto L$ regime.

**Theorem 7.4.** The $S \propto L$ scale implies the law of inertia, $F = ma$.

**Proof.** First, consider the equation for an entropic force $F = T\Delta S/\Delta x$ such as the case of a polymer or of osmosis. In the case of a binary entropy, the entropic force takes the form;

$$Fdx = (\ln 2)^{-1}TdS \hspace{1cm} \text{regime 5.21} \hspace{1cm} (7.5)$$

$$F = (\ln 2)^{-1}T \frac{dS}{dx} \hspace{1cm} \text{divide } dx \hspace{1cm} (7.6)$$

$$F = (\ln 2)^{-1}T \frac{\ln 2)k_BdN}{dx} \hspace{1cm} \text{binary entropy} \hspace{1cm} (7.7)$$

$$F = T k_B \frac{dN}{dx} \hspace{1cm} \text{entropic force} \hspace{1cm} (7.8)$$

This equation connects to $F = ma$ provided that we interpret it as the Compton wavelength multiplied by the Unruh temperature \(^7\) which is experienced by a body undergoing constant acceleration.

---

well-defined temperature implies that the regime occurs at thermodynamic equilibrium, hence the thermal derivation holds.

\[
F = \left( \frac{\hbar}{2\pi c k_B} \right) k_B \frac{dN}{dx} \quad \text{Unruh temperature} \quad (7.9)
\]

\[
F = \left( \frac{1}{2\pi c} \right) \frac{dN}{dx} \quad \text{clean up} \quad (7.10)
\]

Finally, the equation \( F = ma \) can be recovered provided that the ratio \( dx/dN \) is the reduced Compton wavelength multiplied by \( 2\pi \).

\[
\Rightarrow 2\pi \frac{dx}{dN} = \frac{\hbar}{mc} \quad (7.11)
\]

What does this mean from the algorithmic perspective and why is \((2\pi)dx/dN\) the reduced Compton wavelength? \( dx/dN \) is the ratio between the position of an object and the number of bits required to express such a position. It implies that each increment of an object’s position by its reduced Compton wavelength must use one additional bit of entropy. The algorithm to keep track of positions used by the thermal UTM is of the form \( x = n\lambda \), where \( n \) indicates the number of times its reduced Compton wavelength is repeated to reach its position. The entropy usage is optimized as the position of an object does not need to be specified more accurately than its reduced Compton wavelength.

7.2 Note on the Schwarzschild radius

As we have seen, the first term of the Taylor expansion is associated with the inertial mass as it implies \( F = ma \). We have also seen that the first term of the Taylor expansion implies a one dimensional version of the holographic principle. As a result, we would expect that the mass in the universe is bounded linearly. Is that the case?

Consider the Schwarzschild radius,

\[
R = \frac{2GM}{c^2} \quad (7.12)
\]

As we can see, the radius grows linearly with the mass \( M \). Hence, the one dimensional holographic principle associated with the inertial mass holds.

7.3 Holographic principle -2D- (General relativity)

In this section, we will show how the regime 5.23 suggests that general relativity is an emergent entropic phenomenon attributable to the second term, \( kxdx \), of the Taylor expansion of \( d|p| \).
Theorem 7.13. The area-dominant regime implies general relativity.

Proof. Our goal in this proof is to derive the Einstein field equation of general relativity starting from the holographic principle. First consider that the units of $TdS$ are Joules, hence we can pose $dE = TdS$ where $dE$ is an energy.

\[
\frac{1}{\ln 2} TdS = k x dx \tag{7.14}
\]

\[\Rightarrow S \propto A \tag{7.15}\]

\[\Rightarrow dE = \gamma dA \tag{7.16}\]

Deriving general relativity from $dE = \gamma dA$ has indeed been done before in the literature, notably by Ted Jacobson, then later (and differently) by Erik Verlinde\textsuperscript{18}. Furthermore, Christoph Schiller argues that a maximum power (5.34) implies the Field equation\textsuperscript{19}. Here, we will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

Jacobson, starting from $dE = TdS$, first connects $dE$ to an arbitrary coordinate system and energy flow rates,

\[dE = \int T_{ab} k^a d\Sigma^b \tag{7.17}\]

Here $T_{ab}$ is a energy-momentum tensor, $k$ is a killing vector field and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that, assuming that the holographic principle holds (and in here it does according to 5.23), the right part of (7.16) can be rewritten to

\[dA = \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b \tag{7.18}\]

where $R_{ab}$ is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

\[\int T_{ab} k^a d\Sigma^b = \gamma \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b \tag{7.19}\]

which can only be satisfied if

\[T_{ab} = \gamma \frac{c^2}{a} \left( R_{ab} - \left( \frac{R}{2} + \Lambda \right) g_{ab} \right) \tag{7.20}\]

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant). \hfill \qed
7.4 3D "Holographic" principle (Dark energy)

Associating dark energy to a volumetric entropy has been suggested and discussed by other authors before. Here, we suggest that dark energy provides the physical interpretation for the third term of the Taylor expansion.

\[ TdS = (\ln 2)pdV \]

regime 5.24 (7.21)

To determine the value of the pressure \( p \) associated with volumetric entropy, we consider the case of an entropic force. In this case, the pressure relates to the force as

\[ F = -pA \]

\[ \implies p = -\frac{F}{A} = -\frac{F}{4\pi x^2} \]

(7.23)

The sign of the force is negative because the force points in the direction of increased entropy, which is oriented outward the enclosing area.

To determine \( x \), it suffices to notice that \( Fdx + kxdx + px^2dx \) encodes the informational content of the universe up to a boundary given by \( x \), which is common to all terms of the Taylor expansion. Physically, it makes sense to connect this bound to the Hubble horizon as it defines an event horizon applicable to the "instantaneous" system. As it is an event horizon, its temperature is given by De Sitter's temperature and is constant at the horizon. Therefore, an entropic force is expected. To obtain the magnitude of the force, it suffices to calculate the entropic force as per the Bekenstein-Hawking entropy and the De Sitter temperature, both applicable to event horizons.

\[ dS = 2\pi \frac{k_B c^3}{Gh} xdx \]

Bekenstein-Hawking entropy (7.24)

\[ T = \frac{hH}{k_B 2\pi} \]

De Sitter temperature (7.25)

\[ F = \frac{tdS}{dx} \]

entropic force (7.26)

\[ \implies F = \left( \frac{hH}{k_B 2\pi} \right) \left( 2\pi \frac{k_B c^3}{Gh} x \right) \]

(7.27)

\[ = \frac{c^3}{G} Hx \]

clean up (7.28)

As \( x \) is the radius of the Hubble horizon \( x = c/H \), we obtain the final value of the force \( F = c^4/G \), the Planck force. Finally, the pressure is given by;
\[ F = \frac{c^4}{G} \quad \text{Planck force (7.29)} \]
\[ \Rightarrow p = \frac{F}{A} = -\left(\frac{c^4}{G}\right)\left(\frac{1}{4\pi(c/H)^2}\right) \quad (7.30) \]
\[ p = -\frac{c^2H^2}{4\pi G} \quad \text{(negative pressure)} \]

This is close to the current measured value for the negative pressure associated with dark energy. As we can see, the suggested entropic derivation of dark energy applies to the third term of the Taylor expansion.

8 Universal Brownian motion

As we have seen in section 5.7, thermal spacetime experiences fluctuations along the \(x\) and \(t\) axis. We recall the fluctuation relations:

\[
\begin{align*}
\text{average} & \quad \text{fluctuation} \\
\bar{t} \quad \text{(time)} & = -\frac{\partial \ln Z_\Omega}{\partial p} & (\Delta t)^2 & = \frac{\partial^2 \ln Z_\Omega}{\partial p^2} \quad (8.1) \\
\bar{x} \quad \text{(space)} & = -\frac{\partial \ln Z_\Omega}{\partial F} & (\Delta x)^2 & = \frac{\partial^2 \ln Z_\Omega}{\partial F^2} \quad (8.2)
\end{align*}
\]

8.1 Schrödinger equation

In section 7.1, we have used the program-size to entropy relation
\[ TdS = Fdx \] to recover \( F = ma \). In this section we use the same relation but we extend it with the fluctuations effects of the thermal UTM. Doing so will allow us to recover the Schrödinger equation.

We recall that a thermal UTM encodes position via the \(dx\) conjugate associated with program lengths. As a result, the UTM can only express a position if the program with the corresponding size is part of its partition function (i.e., it halts). In this section, we will argue that the missing non-halting programs are responsible for a universal Brownian motion in space applicable to the \(dx\) variable. This will be enough to recover the Schrödinger’s equation.

**Theorem 8.3.** A position described with missing program-sizes will evolve in time according to Schrödinger’s equation.

\[
i\hbar \frac{\partial}{\partial t} \psi(x, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x, t) \right] \psi(x, t)
\]

The proof is slightly more involved than the preceding theorems. First, here is a sketch of the proof.
1. We will show that non-halting programs leave holes in space such that a position cannot be expressed.

2. We will show that these holes are causing a Brownian motion of the encoded position.

3. We will derive its diffusion coefficient to be $\frac{\hbar}{2m}$.

4. We will consider that the presence of any external field is experienced as acceleration via $F = ma$.

5. Using the well known Brownian motion equations of Langevin, we show that the above reproduces Schrödinger’s equation exactly.

**Lemma 8.4.** A spacial encoding based on programs will leave holes in space corresponding to non-halting programs.

**Proof.** We use regime 5.22 applicable to the inertial law. We have also seen that the conjugate $x$ denotes program lengths. However, not all programs halt hence some lengths are missing from the sum. These missing programs are holes in space the position of which cannot be expressed by the UTM’s positional algorithm. Since $\Omega$ is a normal number, we can expect the position of these holes to be algorithmically random.

**Lemma 8.5.** A particle in space will experience Brownian motion due to the holes.

**Proof.** We will calculate the average displacement $\Delta x$ of a particle subjected to entropic positioning and space holes. Since $Z$ is a normal number, we conclude that half of the program’s lengths are available to describe position and half are not. Therefore, to describe a particle at position $x$, there is a 50% chance there is a halting program available to express it. And in the case where there is no program at exactly $x$, then there is a 50% chance that there will be one at position $x + 1$, and so on. In other words, a particle at $x$ has 50% chance of being at $x$, 25% chance of being at $x + 1$, 12.5% chance of being at $x + 2$, etc. Expressed as a sum, we obtain

$$
\bar{\Delta x} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots
$$

(8.6)

$$
= \sum_{i=0}^{\infty} \frac{i}{2^{i+1}}
$$

(8.7)

$$
= 1
$$

(8.8)

On average, as it moves through space, a position will shift by $\Delta x = 1$ at each iteration of the Brownian motion.
**Lemma 8.9.** The diffusion coefficient of the described Brownian motion is

\[ D = \frac{\hbar}{2m} \]

**Proof.** From Einstein paper the diffusion coefficient of Brownian motion is given by

\[ D = \frac{l^2}{2\tau} \tag{8.10} \]

where \( l \) is the length of the random step and \( \tau \) is the frequency of the occurrence of the steps. As we have previously connected the reduced Compton wavelength to \( F = ma \) taking the role of the system’s characteristic length associated with positional encoding for a mass of bits, it makes sense to use it here as well. We get a scaling factor of

\[ \lambda = \frac{\hbar}{mc} \tag{8.11} \]

Since entropic positioning can only express position as multiples of \( \lambda \), we take it as the Brownian step of length \( l \). The diffusion coefficient becomes

\[ D = \left( \frac{\hbar}{mc} \right)^2 \frac{1}{2\tau} \tag{8.12} \]

This leaves us with the need to define \( \tau \). For \( \tau \), we take the characteristic frequency of the wave \( E = \hbar \omega \). This is related to proof-step frequency. Solving for \( \tau = 1/\omega \), we obtain

\[ \omega = \frac{E}{\hbar} \tag{8.13} \]

\[ \omega^{-1} = \frac{\hbar}{E} = \tau \tag{8.14} \]

Replacing \( \tau \) in the equation for \( D \), we obtain

\[ D = \frac{\hbar^2}{m^2c^2} \left( \frac{E}{2\hbar} \right) \tag{8.15} \]

Using \( E = mc^2 \), and reducing the constants, we obtain our final expression of \( D \),

\[ D = \frac{\hbar^2}{m^2c^2} \left( \frac{mc^2}{2\hbar} \right) \tag{8.16} \]

\[ = \frac{\hbar}{2m} \tag{8.17} \]

\( \square \)
Lemma 8.18. The Langevin equations for Brownian motion with a diffusion coefficient of $\hbar/(2m)$ and an external inertial field experienced as $F = ma$ reproduces Schrödinger’s equation.

Proof. We recall the Langevin equation,

$$d[x(t)] = v(t)dt \quad (8.19)$$

$$d[v(t)] = -\frac{\gamma}{m}v(t)dt + \frac{1}{m}W(t)dt \quad (8.20)$$

where $W(t)$ is a random force and a stochastic variable giving the effect of a background noise to the motion of the particle.

From $F = ma$ and replacing the acceleration $d[v(t)]/dt$ with $F/m$, Edward Nelson $^{22}$ is able to show that the Langevin equation becomes,

$$\nabla \left( \frac{1}{2} \vec{u}^2 + D \nabla \cdot \vec{u} \right) = \frac{1}{m} \nabla V \quad (8.21)$$

where $D$ is the diffusion coefficient of $\hbar/(2m)$ obtained in lemma 8.9, where $\vec{F} = -\nabla V$, where $\vec{u} = v \nabla \ln \rho$ and $\rho$ is the probability density of $x(t)$. For brevity, the proof of 8.21 is omitted here but can be reviewed in Nelson’s paper. Eliminating the gradients on each side and simplifying the constants, we obtain

$$\frac{m}{2} \vec{u}^2 + \frac{\hbar}{2} \nabla \cdot \vec{u} = V - E \quad (8.22)$$

where $E$ is the arbitrary integration constant. This equation in nonlinear because of the term $\vec{u}^2$ but it can be made linear by a change of dependant variable. To make it linear, let us pose

$$\vec{u} = \frac{\hbar}{m \psi} \nabla \psi \quad (8.23)$$

and replace it into equation 8.22, we obtain

$$V - E = \frac{m}{2} \left( \frac{\hbar}{m \psi} \nabla \psi \right)^2 + \frac{\hbar}{2} \nabla \cdot \left( \frac{\hbar}{m \psi} \nabla \psi \right) \quad (8.24)$$

$$= \frac{\hbar^2}{2m \psi^2} (\nabla \psi \cdot \nabla \psi) + \frac{\hbar^2}{2m} \left[ \nabla \cdot \left( \frac{1}{\psi} \nabla \psi \right) \right] \quad (8.25)$$

$$= \frac{\hbar^2}{2m \psi^2} (\nabla \psi \cdot \nabla \psi) + \frac{\hbar^2}{2m} \left[ \frac{\psi \nabla \cdot \nabla \psi - \nabla \psi \cdot \nabla \psi}{\psi^2} \right] \quad (Identity)$$

$$= \frac{\hbar^2}{2m \psi^2} (\nabla \psi \cdot \nabla \psi) + \frac{\hbar^2}{2m} \left[ \frac{1}{\psi} \nabla \cdot \nabla \psi - \frac{1}{\psi^2} (\nabla \psi \cdot \nabla \psi) \right] \quad (8.26)$$

The first and the last terms cancel each other.

$$\frac{\hbar^2}{2m \psi} \nabla^2 \psi = V - E \quad (8.27)$$
Finally, it simplifies to

\[
\left[ -\frac{\hbar^2}{2m} \nabla^2 + V - E \right] \psi = 0 \quad (8.28)
\]

which is the time independent Schrödinger’s equation.

We are now ready to derive the time dependent Schrödinger equation and prove theorem 8.3.

Proof. We use the same proof used by Edward Nelson in the same paper. Starting from the time dependent Schrödinger equation, we show that a replacement of \( \psi = e^{R+iS} \) leads to the Langevin equation of Brownian motion.

\[
\frac{\partial \psi}{\partial t} = i \frac{\hbar}{2m} \nabla^2 \psi - i \frac{1}{\hbar} V \psi \quad (8.29)
\]

Replacing \( \psi \) with \( e^{R+iS} \), we obtain

\[
\frac{\partial}{\partial t} \left( e^{R+iS} \right) = i \frac{\hbar}{2m} \nabla^2 \left( e^{R+iS} \right) - i \frac{1}{\hbar} V \left( e^{R+iS} \right) \quad (8.30)
\]

Taking the derivatives and the gradients, we obtain

\[
\left[ \frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} \right] \left( e^{R+iS} \right) = \frac{i\hbar}{2m} \left[ \nabla^2 R + i \nabla^2 S + (\nabla (R + iS))^2 \right] \left( e^{R+iS} \right) - i \frac{1}{\hbar} V \left( e^{R+iS} \right) \quad (8.31)
\]

Eliminating \( e^{R+iS} \) from each side and simplifying, we obtain

\[
\frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} = \frac{i\hbar}{2m} \left[ \nabla^2 R + i \nabla^2 S + (\nabla R)^2 - (\nabla S)^2 \right] - i \frac{1}{\hbar} V \quad (\text{eliminating } e^{R+iS})
\]

\[
\frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} = \frac{i\hbar}{2m} \left[ \nabla^2 R + (\nabla R)^2 - (\nabla S)^2 \right] - i \frac{1}{\hbar} V \quad (\text{taking the product})
\]

\[
\frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} = \frac{\hbar}{2m} \left[ i \nabla^2 R - \nabla S - i(\nabla R)^2 - 2 \nabla R \cdot \nabla S - i(\nabla S)^2 \right] - i \frac{1}{\hbar} V \quad (\text{distributing the } i)
\]

We obtain two equations by separating the real and the imaginary parts

\[
\frac{\partial R}{\partial t} = \frac{\hbar}{2m} \left[ -\nabla^2 S - 2 \nabla R \cdot \nabla S \right] \quad (8.32)
\]

\[
\frac{\partial S}{\partial t} = \frac{\hbar}{2m} \left[ \nabla^2 R + (\nabla R)^2 - (\nabla S)^2 \right] - \frac{1}{\hbar} V \quad (8.33)
\]

This is equivalent to the Langevin equations with some replacements

\[
\frac{\partial \vec{u}}{\partial t} = -\frac{\hbar}{2m} \nabla (\nabla \cdot \vec{v}) - \nabla (\vec{v} \cdot \vec{u}) \quad (8.34)
\]

\[
\frac{\partial \vec{v}}{\partial t} = \frac{\hbar}{2m} \nabla (\nabla \cdot \vec{u}) + \frac{1}{2} \nabla (\vec{u}^2) - \frac{1}{2} \nabla (\vec{v}^2) - \frac{1}{m} \nabla V \quad (8.35)
\]
Lemma 8.36. Equation 8.32 with the replacements $\nabla R = (m/\hbar)\vec{u}$ and $\nabla S = (m/\hbar)\vec{v}$ produces 8.34.

Proof.

\[
\frac{\partial R}{\partial t} = \frac{\hbar}{2m} \left[ -\nabla^2 S - 2\nabla R \cdot \nabla S \right] \quad \text{(equation 8.32)}
\]

\[
\nabla \frac{\partial R}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\nabla^2 S - 2\nabla R \cdot \nabla S \right] \right) \quad \text{(taking the gradient)}
\]

\[
\frac{\partial \nabla R}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\nabla \cdot \nabla S - 2\nabla R \cdot \nabla S \right] \right) \tag{8.37}
\]

\[
m \frac{\partial \vec{u}}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\vec{v} \cdot \left( \frac{m}{\hbar} \vec{u} \right) - 2 \left( \frac{m}{\hbar} \vec{u} \right) \cdot \left( \frac{m}{\hbar} \vec{v} \right) \right] \right) \quad \text{(replacing $\nabla R$ and $\nabla S$)}
\]

\[
\frac{\partial \vec{v}}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\vec{u} \cdot \left( \frac{m}{\hbar} \vec{v} \right) - 2 \left( \frac{m}{\hbar} \vec{v} \right) \cdot \left( \frac{m}{\hbar} \vec{u} \right) \right] \right) \quad \text{(eliminating $m/\hbar$)}
\]

\[
\frac{\partial \vec{v}}{\partial t} = -\frac{\hbar}{2m} \nabla (\vec{u} \cdot \vec{v}) - \nabla (\vec{u} \cdot \vec{v}) \quad \text{(equation 8.34)}
\]

Lemma 8.38. Equation 8.33 with the replacements $\nabla R = (m/\hbar)\vec{u}$ and $\nabla S = (m/\hbar)\vec{v}$ produces 8.35.

Proof.

\[
\frac{\partial S}{\partial t} = \frac{\hbar}{2m} \left[ \nabla^2 R + (\nabla R)^2 - (\nabla S)^2 \right] - \frac{1}{\hbar} \nabla V \quad \text{(equation 8.33)}
\]

\[
\nabla \frac{\partial S}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ \nabla \cdot \nabla R + (\nabla R)^2 - (\nabla S)^2 \right] \right) - \frac{1}{\hbar} \nabla V \quad \text{(taking the gradient)}
\]

\[
m \frac{\partial \vec{v}}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ \nabla \cdot \left( \frac{m}{\hbar} \vec{u} \right) + \left( \frac{m}{\hbar} \vec{u} \right)^2 - \left( \frac{m}{\hbar} \vec{v} \right)^2 \right] \right) - \frac{1}{\hbar} \nabla V \quad \text{(replacing $\nabla R$ and $\nabla S$)}
\]

\[
\frac{\partial \vec{v}}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ \nabla \vec{u} + \frac{m}{\hbar} \vec{u}^2 - \frac{m}{\hbar} \vec{v}^2 \right] \right) - \frac{1}{m} \nabla V \quad \text{(eliminating $m/\hbar$)}
\]

\[
\frac{\partial \vec{v}}{\partial t} = \frac{\hbar}{2m} \nabla (\vec{u} \cdot \vec{v}) + \frac{1}{2} \nabla (\vec{u}^2) - \frac{1}{2} \nabla (\vec{v}^2) - \frac{1}{m} \nabla V \quad \text{(equation 8.35)}
\]

This completes the proof of theorem 8.3. □

8.2 Dirac equation

In a previous section, we have used $TdS = Fdx$ to recover $F = ma$. In section 6, we have used $TdS = Pdt + Fdx$ to recover special relativity. We have then used a random walk on $dx$ to recover the Schrödinger equation which is the quantum analogue to $F = ma$. Of course, the natural question to ask is, will using $TdS = Pdt + Fdx$ and applying a random walk to both $dt$ and $dx$ be enough to recover the Dirac equation, the quantum analogue to special relativity? The answer is yes!
In this section, we will see that applying a random walk to both the \( dt \) and the \( dx \) variables is enough to recover the Dirac equation for relativistic quantum mechanics. Let us begin by answering why would there be a random walk on \( dt \).

First we consider that, as is the case with program length, program runtime varies from one UTM to the next. Programs that are difficult to solve on one UTM are likely to be difficult to solve on other UTMs. For example the travelling salesman problem is hard to solve on every UTM. The runtime of these programs will be randomly distributed and centred around a mean runtime.

Second, we consider an analogous argument to the one used to justify a random walk on \( dx \), but applied to \( dt \). On some UTM a program of size \( x \) might have halted and on others it might not have. Therefore a particle can be defined to be at a time \( t \) only if a program halting at time \( t \) is in the partition function. If there is no such available halting program at time \( t \), then the particle will be a time \( t \pm \Delta t \), the runtime of the next available halting program. Since the halting problem is algorithmically random and non-computable, we consider this behaviour as a random walk in time.

A connection between a random walk in time and space and the telegraphic equation has been linked to the Dirac equation before\(^{23}\). D. G. C. McKeon and G. N. Ord proposes a random walk model in space and in time. Starting from the equation for a random walk in space, we have

\[
P_\pm(x, t + \Delta t) = (1 - a\Delta t)P_\pm(x \mp \Delta x, t) + a\Delta tP_\mp(x \pm \Delta x, t) \quad (8.39)
\]

then, D. G. C. McKeon and G. N. Ord extend this equation with a random walk in time. They obtain

\[
F_\pm(x, t) = (1 - a_L\Delta t - a_R\Delta t)F_\pm(x \mp \Delta x, t - \Delta t) + a_L a_R\Delta t B_\pm(x \mp \Delta x, t + \Delta t) + a_R a_L\Delta t F_\mp(x \pm \Delta x, t - \Delta t) \quad (8.40)
\]

where \( F_\pm(x, t) \) is the probability distribution to go forward in time and \( B_\pm(x, t) \), backward in time. They then introduce a causality condition such that \( F_\pm(x, t) \) and \( B_\pm(x, t) \) only depends on probabilities from the past.

\[
F_\pm(x, t) = B_\mp(x \pm \Delta x, t + \Delta t) \quad (8.41)
\]

From equation 8.2 and 8.41, they get
\[ B_{\pm}(x, t) = (1 - a_L \Delta t - a_R \Delta t)B_{\pm}(x \mp \Delta x, t + \Delta t) + a_{L,R} \Delta t B_{\mp}(x \mp \Delta x, t + \Delta t) + a_{R,L} \Delta t F_{\pm}(x \mp \Delta x, t - \Delta t) \]  

(8.42)

In the limit \( \Delta x, \Delta t \to 0 \) and with \( \Delta x = v \Delta t \), they get,

\[
\pm v \frac{\partial F_{\pm}}{\partial x} + \frac{\partial F_{\pm}}{\partial t} = a_{L,R}(-F_{\pm} + B_{\pm}) + a_{R,L}(-F_{\mp} + F_{\pm}) \]

(8.43)

\[
\pm v \frac{\partial B_{\pm}}{\partial x} + \frac{\partial B_{\pm}}{\partial t} = a_{L,R}(-B_{\mp} + F_{\pm}) + a_{R,L}(-B_{\mp} + B_{\pm}) \]

(8.44)

Posing these changes of variables,

\[
A_{\pm} = (F_{\pm} - B_{\mp}) \exp[(a_L + a_R)t] \]

(8.45)

\[
\lambda := -a_L + a_R \]

(8.46)

then (8.44) becomes

\[
v \frac{\partial A_{\pm}}{\partial x} \pm \frac{\partial A_{\pm}}{\partial t} = \lambda A_{\mp} \]

(8.47)

Finally, they pose \( v = c, \lambda = mc^2/\hbar \) and \( \psi = F(A_+, A_-) \), they get

\[
\imath \hbar \frac{\partial \psi}{\partial t} = mc^2 \sigma_y \psi - \imath c \hbar \sigma_z \frac{\partial \psi}{\partial x} \]

(8.48)

which is the Dirac equation in \( 1+1 \) spacetime.

9 Characteristic units

Our goal in this section is to show how the definition of the Planck units naturally follows from the state equation (5.12). To do so, we must first obtain definitions for \( G, c \) and \( \hbar \) by deriving from it known laws of physics that contain them. We start by obtaining the gravitational constant \( G \) from Newton’s law of gravitation.

**Theorem 9.1.** The gravitational constant \( G \) is defined as \( c^3 L^2 / \hbar \).

**Proof.** A derivation of Newton’s law of gravitation from the entropic perspective has been done before by Erik Verlinde\(^\text{24} \). Here to obtain the law of gravitation, we work in regime 5.23. This regime contains the 2D-holographic principle and, as a result, the entropy of the system grows via \( x^2 \), an area law. We further consider that the entropy of this area law is given by bits exclusively occupying a small area \( L^2 \) on the surface. In this case, the total number of bits on the surface is given by

The equipartition theorem applies to energy terms of the partition function, which are quadratic. The term $kdx$ is $\frac{1}{2}kx^2$ in the partition function. As a result its average energy is $E = \frac{1}{2}Nk_BT$ as per the equipartition theorem.

$$E = \frac{1}{2} \left( \frac{4\pi x^2}{L^2} \right) k_B T$$

$$\Rightarrow T = \frac{L^2}{2\pi k_B} \frac{E}{x^2}$$

We obtain a constant temperature throughout the system indicating that it is at thermodynamic equilibrium. As our goal is to recover the gravitational constant, we inject this temperature in the entropic force relation.

$$F = Tk_B \frac{dN}{dx}$$  \text{entropic force (7.8)}

$$F = \left( \frac{L^2}{2\pi k_B} \frac{E}{x^2} \right) k_B \frac{dN}{dx}$$  \text{derived temperature (9.6)}

We then replace the ratio $dx/dN$ by the reduced Compton wavelength.

$$F = \left( \frac{L^2}{2\pi k_B} \frac{E}{x^2} \right) k_B \left( 2\pi \frac{mc}{\hbar} \right)$$

$$F = \left( \frac{L^2 c}{\hbar} \right) \frac{Em}{x^2}$$  \text{clean up (9.8)}

We then convert $E$ to its rest mass via $E = mc^2$.

$$F = \left( \frac{L^2 c^3}{\hbar} \right) \frac{Mm}{x^2}$$

We obtain the Newton’s law of gravitation along with a definition for $G$.

$$F = G \frac{Mm}{x^2}$$

$$\Rightarrow G = \frac{L^2 c^3}{\hbar}$$

which further implies that

$$L = \sqrt{\frac{\hbar G}{c^3}}$$  \text{(Planck’s length)}
Theorem 9.12. The speed of light $c$ is defined by $P/F$.

Proof. We refer to the proof for theorem 5.26 where $P/F$ is a characteristic speed associated with an inversion in the direction of the second law of thermodynamics. Then, under the principle that the second law is irreversible, the speed $P/F$ is a boundary and defines $c$. \hfill $\square$

Theorem 9.13. The action $S$ is defined by $\hbar$.

Proof.

$$
\frac{1}{\ln 2} TdS = 2\pi Sdf, \quad \text{regime 5.21} \quad (9.14)
$$

$$
dE = \frac{1}{\ln 2} TdS = 2\pi Sdf, \quad \text{units of energy} \quad (9.15)
$$

$$
dE = 2\pi Sdf, \quad \text{posing } dS = 0 \quad (9.16)
$$

Switching to the angular frequency,

$$
dE = Sd\omega, \quad df = d\omega/(2\pi) \quad (9.17)
$$

$$
\int dE = \int Sd\omega \quad (9.18)
$$

$$
E = S\omega + C \quad (9.19)
$$

Posing $C = 0$, this is the photon angular-frequency to energy relation $E = h\omega \implies S = \hbar$. \hfill $\square$

We have now obtained a definition for three of the fundamental constants.

$$
\hbar = S \quad c = \frac{P}{F} \quad G = \frac{L^2c^3}{\hbar} \quad (9.20)
$$

We can now definite characteristic units applicable to the thermal UTM,

$$
G = \frac{L^2c^3}{\hbar} \implies L = \sqrt{\frac{\hbar G}{c^5}} \quad \text{(Planck’s length)}
$$

$$
t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}} \quad \text{(Planck’s time)}
$$

$$
E = S/t \implies E = \sqrt{\frac{\hbar c^5}{G}} \quad \text{(Planck’s energy)}
$$

$$
P = t^{-2}S = \frac{c^5}{G} \quad \text{(Planck’s power)}
$$

$$
\frac{P}{F} = c \implies F = \frac{c^4}{G} \quad \text{(Planck’s force)}
$$

which agrees with the physical Planck units.
10 Arrow of time

10.1 Time

**Theorem 10.1.** The state equation (5.12) implies a halting entropy decreasing with time.

Proof.

\[
\frac{1}{\ln 2} T dS = -P dt
\]

\[
\Rightarrow \frac{dS}{dt} = -(\ln 2) \frac{P}{T} \quad \text{decreasing entropy (10.3)}
\]

\[\square\]

**Definition 10.4 (Halting entropy).** The halting entropy is the entropy exclusively associated with the calculation of \( \Omega \) over time. It is the entropy obtained in regime 5.21.

As time increases the entropy from the calculation of \( \Omega \) decreases according to the term \(-(\ln 2)P/T\). Why does it decrease over time? Consider that at the beginning of the calculation none of the bits of \( \Omega(t) \) are known, hence the error rate is at its maximum. Each bit with an unknown value contributes \( k_B \ln 2 \) to the entropy. As the calculation progresses and the error rate is diminished, then each additional and correct bit that has been calculated becomes fixed and their entropy contributions are reduced to 0.

As a result, an arrow of time connected to the non-computability of \( \Omega \) can be attributed to the system as follows. A forward translation in time is associated with an increase in halting information. Furthermore, since each bit of \( \Omega \) is algorithmically random, then the future, which can only be described with more bits of \( \Omega \), is guaranteed to be non-computable. While the past, which holds less bits than the present, is guaranteed to be computable from the present. This corresponds more closely to our human experience, as we can remember and even deduce the past based on present evidence, but cannot precisely know the future until it happens.

Furthermore, as the entropy of the valid bits of \( \Omega \) is exactly 0, then it means that the past of the system is fixed and cannot be changed. Again, this more closely matches our human experience as we cannot change our past, so why would its halting entropy be anything other than 0?

10.2 Exfoliation

As an entropy decreasing with time would violate the second law of thermodynamics, we suggest that an entropic exfoliation to space occurs so as to make the second law hold. In this scenario, the entropy
reduction from the calculation of $\Omega$ is compensated by an increase in entropy associated with the exfoliation observables. Consider the following theorem.

**Theorem 10.5.** The state equation (5.12), the second law of thermodynamics and theorem (10.1) implies an entropic exfoliation to space.

**Proof.**

\[
\frac{T \, dS}{\ln 2} = -P \, dt + F \, dx + kA + pdV
\]

regime 5.16 (10.6)

\[
\frac{dS}{dt} = (\ln 2) \frac{1}{T} \left[ \frac{F \, dx}{dt} + \frac{kA}{dt} + \frac{pdV}{dt} - P \right]
\]

exfoliation (10.7)

\[\square\]

**Definition 10.8 (Exfoliation entropy).** The exfoliation entropy is the contribution by the following term to the entropy over time.

\[
(\ln 2) \frac{1}{T} \left[ \frac{F \, dx}{dt} + \frac{kA}{dt} + \frac{pdV}{dt} - P \right]
\]

To investigate this result, let us look at three cases;

\[
\frac{F \, dx}{dt} + \frac{kA}{dt} + \frac{pdV}{dt} < P \quad \Rightarrow \quad \frac{dS}{dt} < 0 \quad \text{decreasing entropy (10.10)}
\]

\[
\frac{F \, dx}{dt} + \frac{kA}{dt} + \frac{pdV}{dt} = P \quad \Rightarrow \quad \frac{dS}{dt} = 0 \quad \text{constant entropy (10.11)}
\]

\[
\frac{F \, dx}{dt} + \frac{kA}{dt} + \frac{pdV}{dt} > P \quad \Rightarrow \quad \frac{dS}{dt} > 0 \quad \text{increasing entropy (10.12)}
\]

At (10.11), a shift occurs in the direction of the production of entropy over time. It is the point at which the exfoliation entropy overtakes and exceeds the reduction in halting entropy. The second law of thermodynamics, which states that $dS/dt \geq 0$ will hold for (10.11) and (10.12), but will be violated for (10.10). In any case, if $(\ln 2) \frac{1}{T} \left( F \, dx + kA + pdV \right) > 0$ then the second law of thermodynamics applicable to the exfoliation observables will be observed.

This derivation more closely matches human experience. Indeed,

1. at the beginning of time the future of the system is un-actualized, hence the possibilities are endless. To reflect this, the halting entropy is at its maximum at $t = 0$, and the exfoliation entropy is equal to 0. This matches our current belief that the exfoliation entropy at the Big Bang is very low.

2. during the evolution the future becomes past which is "set in stone". As the past is "set in stone", the halting entropy of the bits defining it are equal to 0. This is because we "remember" or "observe" only one past. This reduction in halting entropy is offset by
a growth in exfoliation entropy, which is related to the size and complexity of the space encoded by the exfoliation observables. This growth in space entropy obeys the second law of thermodynamics.

3. at the end of time there is no future. The value of $\Omega$ has been calculated, and the full history of the system is now "set in stone". The halting entropy is 0 and the exfoliation entropy is at its maximum. This matches the hypothesis of the heat death.

Note that contrary to the halting entropy, the exfoliation entropy of an observer’s past does not need to be equal to 0 as multiple exfoliated micro-states could be compatible with an observer’s present. Indeed, as per the second law of thermodynamics, the observer sees a monotonically increasing exfoliation entropy.

How then do we understand this result from the perspective of algorithmic information theory? The exfoliation variable represents the entropy in the choice of available prefix-free encodings for the programs of the UTM. When no bits of $\Omega$ are known, it doesn’t make sense to speak of the ways to encode this information as there is nothing to encode. Hence, the entropy should be 0. As more bits of $\Omega$ are known then more ways to encode this information exist and the entropy associated with the possible encodings increases.

11 Discussion

A convincing scientific theory is one that survives falsification. Meaning, the theory should make predictions that can be either verified or falsified via physical observations. The concept of falsifiability, in principle, serves as an ideal. In practice however, there is an additional informal criteria whose mention is often neglected but one that is nonetheless also important - the prediction must be remarkable. And indeed, looking into the history of science we find that the more remarkable the prediction is, the more convinced we are of the validity of the theory predicting it. Being remarkable is an aesthetic; it is connected to the uniqueness of the explanation as well as to the impact of the prediction on the current state of the art.

For example, a theory which predicts a slight correction of less than one thousand of a percentage point on some measured quantity (while everything else remains equal) will not be considered a remarkable prediction. The prediction might be absolutely correct, but it would very unlikely come to replace the existing textbook theory within a reasonable timeframe. First, the cognitive burden of learning a new approach hardly justifies the near-negligible improvement. Second, many alternative theories would be presumed to be able to
account for such a small variation and its uniqueness will be questioned. And indeed, in the literature, we find this is quite often the case.

However, the situation is different when the prediction is remarkable. For example, before Einstein’s theory of relativity, time was assumed to be absolute and constant. Hence, the prediction that it was not was remarkable. Once experimental evidence was found to confirm this unexpected prediction, then the adoption of the new theory was favoured.

11.1 Reductio ad 400

Our goal here will be to produce a prediction that is both remarkable and falsifiable. One that, ideally, no other theory has predicted before. One that is of significant impact not only to physics but also to philosophy and one that if confirmed, must alter our deepest conception of reality. This prediction, if it is observed to be true, will lend tremendous credibility to this theory.

Our prediction will be a numerical estimation of the minimum number of bits required to encode the full complexity of the universe. In other words, its compressibility ratio. Evidence of this prediction should be abundant in the universe as it affects all things. Expressed in more technical terms, we consider that the Gibbs ensemble connects the many facts of the universe to an irreducible blob of axioms. The blob of axioms is itself defined by the first $n$ bits of $\Omega$ and up to an error rate. We will calculate the numerical value of $n$ as predicted by this theory for the current size and age of the universe.

To calculate it, we will consider the case where the holographic principle holds as an area law. In this case, the linear and volumetric entropy terms of the Taylor expansion will get their entropy from an exfoliation of the area entropy. As such, the area entropy experiences a reduction to offset the entropic gain in the other dimensions.

The alternative assumption is that the holographic principle does not hold. In this case, the entropy of the universe grows with respect to the sum of its volume, area and length entropy. In this case, the calculation is essentially the same but the final value would be higher.

Let us recall the Gibbs ensemble describing the thermal UTM.

\[
Z_\Omega = \sum_i 2^{-\beta(D|p_i|+2\pi S_i)}
\]  

(11.1)

Each term of the sum describes a fact and together they describe a large ensemble of facts. The facts are summed to produce a numerical value which compresses them. When $\mathcal{S} \to 0^+$, the degree
of compression is so high that the numerical value is provably incompressible via any possible algorithm. Hence, we say that it is algorithmically random. As \( Z_\Omega \) converges towards \( \Omega \) its first \( n \) valid bits are enough to discriminate the halting status of \( 2^n \) programs. As a result, we can calculate \( n \) as follows;

Under the holographic hypothesis, the entropy of the universe is restricted by the number of bits occupying the Planck area that can fit on the surface of a sphere enclosing the universe with radius equal to the cosmic event horizon. This is approximately \( 10^{122} \) bits of entropy. In the case of a volumetric entropy, the number of bits is \( 10^{183} \). Hence, as the first \( n \) bits of \( \Omega \) can decide the first \( 2^n \) facts, we calculate \( n \) as follows:

\[
\begin{align*}
n &= \log_2 \left( 10^{122} \right) \approx 400 \text{ bits} \quad \text{holographic entropy} \\
n &= \log_2 \left( 10^{183} \right) \approx 612 \text{ bits} \quad \text{volumetric entropy}
\end{align*}
\]

The volumetric entropy is presented for comparison purposes - in what follows, we will exclusively discuss the prediction of 400 bits applicable to the holographic principle. \textit{Four hundred} is the number of leading bits of \( \Omega \) required to encode \( 10^{122} \) facts. This calculation suggests that the entire informational description of the universe can be compressed to a mere 400 bits of data, enough to fit into the memory of a pocket calculator. These 400 bits are the leading part of \( \Omega \) which itself is algorithmically random and cannot be compressed any further by any possible algorithm. The value of the bits of \( \Omega \) as they are not given by an algorithm, cannot be deduced from pure reason. Consequently, we will argue that these bits are better interpreted as the axioms of the theory of everything of the universe. Hence, the theory of everything which describes the universe at its current size and age must have approximately 400 algorithmically random bits of axioms.

An observer knowing these 400 bits could calculate the entire informational description of the universe from first principles. To the knowledge of the author, no other theory has suggested such a strong compressibility applicable to the facts of the universe.

How credible is 400 bits? Well, we will grant that it is mind bogglingly low. But, for purposes of falsification this is a good thing. If we had obtained a compressibility of say \( 10^{100} \) bits instead of 400 bits then it would have been a much less remarkable prediction. The fact that it is so low is precisely why it is so remarkable. As far as to its credibility, consider the axioms of vanilla non-relativistic quantum mechanics. Copy pasting the text of the axioms in notepad taken from wikipedia and applying a compression algorithm, I obtain 1235 byte of data as a zip file. The file is very small, yet it can explain...
a large percentage of the universe. The Dirac equation takes only a handful of compressed bytes to express yet it explains an even larger part of the universe. The point is that axioms contain tremendous amount of information in a small amount of bits. Nonetheless, the compressibility of the whole universe to 400 bits of data should still surprise us. Evidence of such a low bound on complexity should be plentiful in the universe.

It is worth mentioning that a similar number was obtained by Paul Davies in the context of the maximal number of qubits usable by a general quantum computer. Here, we suggest that the bound of \( \approx 400 \) as described by Davies in the context of quantum computers and qubits is essentially the same bound described here but in terms of \( \Omega \) bits. The bound should serve as the primary falsifiable prediction of an informational theory of the universe. It predicts an ultimate compressibility of the universe to 400 bits of data. We can consider that the data is so compressed that its decompression algorithm operates over billions of years - the amount of time it takes to produce approximately \( 10^{122} \) uncompressed facts from 400 \( \Omega \) bits using thermal dovetailing.

11.2 An axiom-free theory

If the bits are the theory of everything, and we have not explicitly specified any of these bits, why is it that we were able to obtain physical laws? Are the physical laws not supposed to be encoded within the bits (Therefore, if the bits are unknown then the laws should also be unknown)? And if not, then what exactly do the bits represent?

Before we answer the question, let us imagine a computer program which constructs a virtual world out of a seed. The seed is a short sequence of random numbers. As a result, the seed can be shared between two users and the world building program, as it is deterministic, will always rebuild the same world from the same seed. Hence, the world building program is the same for all worlds. Changing the seed changes the world, but it does not change the rules used to build the world.

We essentially suggest a similar interpretation. The theory of everything is the seed and the laws of physics is the program that builds the world from the seed. This is why the axiom-free methodology of removing all formal axioms and rules of inference from Miniversal logic was so critical to deriving the laws of physics. The theory of everything only contains the seed. As surprising as this might sound, the laws of physics are not part of the theory of everything - they are independently deducible from pure reason by any and all observers. This turns out to be an absolute necessity. When

\[ \text{Paul CW Davies. The implications of a cosmological information bound for complexity, quantum information and the nature of physical law. Fluctuation and Noise Letters, 7(04):C37-C50, 2007} \]
we reproduced the universal doubt method of Descartes within formal logic by removing rules of inference and formal axioms, we set up the only logical system capable of proving the laws of physics.

11.3 The unreasonable effectiveness of mathematics in the natural sciences

As Wigner\(^{27}\) once wrote, mathematics is unreasonably effective in the natural sciences. In the present theory, the universe contains 400 bits of mathematically unexplainable information, and \(10^{122}\) of mathematically explainable information. The entropy of \(10^{122}\) is produced by a deterministic algorithm. This explains why mathematic is so effective. The world building program is deterministic and follows mathematical and repeatable patterns for all of its facts.

11.4 Boltzmann brains

The prediction that the universe is describable by only 400 bits lends weight to the Boltzmann brain hypothesis. The hypothesis states that a brain is most likely to be in the simplest possible universe that is capable of producing a brain. At 400 bits of complexity, this might be as simple as it gets.

The argument is actually stronger because the complexity of the universe is grown bit by bit. At the beginning of the universe, 1 bit of \(\Omega\) was sufficient to produce all of its facts. As time advances, the number of bits required to describe it must also grow. At some point it would have grown to 100 bits, then to 200 bits, and so. Eventually it would become sufficiently large and complex to encode sentient life. The development of sentient life would occur more or less when \(n\) is sufficiently large to allow it. Hence, the first sentient life in the universe are invariably Boltzmann brains.

11.5 Why do subjective observers share the same objective reality?

All subjective observers who can produce complex thought such as the cogito will be able to deduce the same laws of physics independently of any observations. The occurrence of complex thought defined as being able to in principle verify the proof of any theorem of any assumption, guarantees the laws of physics as we know them.

11.6 Mathematics cannot decide the future

The world building program of the universe unpacks the \(\Omega\) seed starting from leftmost bit and moving to the right. This allows it to

extract facts and as they are calculated store them in the large entropy of the universe. Facts stored as such are immediately accessible as part of the entropy of the universe at little algorithmic time cost. The future as it contains more fact, will use more bits of \( \Omega \) to be produced from. Hence the value of \( n \), currently \( \approx 400 \) will grow with time and was smaller in the past. The future as it is connected to a larger \( n \) than the present cannot mathematically be decided from the present as the bits of \( \Omega \) are non-computable.

12 Conclusion

We note an affinity between a thermal universal Turing machine and the laws of physics. The affinity occurs when we consider a prefix-free UTM calculating its \( \Omega \) number in a manner so as to maximize the entropy throughout the calculation. When the entropy is maximized, the halting probability becomes a Gibbs ensemble.

Understanding physics from the perspective of an thermal UTM holds several conceptual advantages. First, the system is at thermal-equilibrium hence it doesn’t impose a ‘special’ case or a ‘fine-tuning’. Second, it is a universal Turing machine hence it defines a universal system capable of arbitrary computation which can match the universe’s complexity. More specifically, the representation can define a non-computable future with a computable singular past whose halting entropy is 0. This provides us with an arrow of time closely matching human experience. The entropy of the complete system (which includes future possibilities as well as an encoding scheme for the past) does stay constant over time as the change of entropy of one is offset by the other. The second law of thermodynamics, understood as an increase in entropy over time, is perceived in the exfoliation variables while the larger system, made to include future possibilities, has a constant entropy over time. In this system future possibilities are consumed to produce encoding possibilities. The second law of thermodynamics is therefore corrected to a law of conservation of entropy for the larger system comprised both thermal time and thermal space.

The decomposition of the program encoding scheme used by the thermal UTM via a Taylor expansion produces terms which can be linked to a scale where a specific entropic forces are dominant. For the first Taylor expansion term, we recover special relativity (speed of light \( (5.26) \), light-cones (figure 1) and the Lorentz’s factor (figure 2)) and the law of inertia \( (7.4) \). For the second term, we recover general relativity \( (7.13) \) and the holographic principle \( (7.3) \). Finally, the third term is related to an entropic origin of dark energy \( (7.21) \). Quantum mechanics is recovered as a result of the random walk produced on
$dx$ and $dt$ and associated with fluctuation thermodynamic variables. The Lagrange multipliers of the partition function are the Planck units.

The derivation of the representation can be achieved from pure reason. It does not require an appeal to experimental evidence. It contains a metaphysical proof that the solution is unique hence it provides an explanation for why the universe is the way it is, and not an alternative. Finally, in the last part of the paper, we have calculated to compressibility of the universe under the holographic assumption to be approximately 400 bits. Those bits are the theory of everything for the current size and age of the universe and can be loosely interpreted as a random seed. This can serve as a remarkable prediction which opens the theory to the possibility of falsification.

As a reference, I presented many of these ideas in an earlier publication\textsuperscript{28}.

\textbf{References}


a derivation of the laws of physics from pure information


