A formal theory where all laws of physics emerge from information

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In this work, I present a minimalist approach to first-order logic and show how it implies an irrefutable equation which connects the field of algorithmic information theory (AIT) to our major theories of physics. The construction of the irrefutable equation produces a new theory of physics where the universe is to be interpreted as the result of a universal Turing machine maximizing the entropy during the calculation of its halting probability, $\Omega$. The construction is sufficiently specific to derive General Relativity and the Dirac equation from pure reason.

The minimalist approach is, in many ways, similar to the constructivist project in mathematics but taken to the extreme. The approach starts from first-order logic with no axioms and further removes all rules of inference with the exception of the proof by construction. Although this severely cripples first-order logic it nonetheless gives it the following advantage:

From an argument originally made by Plato, I argue that any axioms or rules of inference that are removed increases the "epistemological irrefutability" of the theory. Taken to the extreme, once all axioms and all rules of inference are removed, the theory becomes entirely irrefutable and specifically in the case of this approach, as the first-order logic system is minimal, the epistemological irrefutability of its theorems is maximal.

Using this approach, I construct a universal language defined by a small list of first-order sentences. Each of the sentences claims the existence of an object of language which is provable by construction, the only rule of inference allowed by the minimalist system. As a result of being a theorem of a minimal system, the existence of the constructed universal language is therefore maximally irrefutable.

This minimalist method loosely resembles René Descartes' derivation of the 'cogito ergo sum', an irrefutable statement, obtained by progressively removing any and all uncertain statements and cataloguing what remains. Like Descartes' method, the minimalist method also produces a statement that cannot be denied. But, unlike Descartes, as the proof is written in the language of first order logic, the irrefutable statement obtained is an actual equation. The equation applies to the group of all statements that cannot be refuted by the application of Descartes' universal doubt method.

Part I is the minimalist derivation of the irrefutable equation. The equation obtained is formulated as a Gibbs ensemble and relates the algorithmic notions of provable-sentences to that of entropy.

Part II is the derivation of the physical laws. I recover, from the irrefutable equation, the exact mathematical formulation of the major theories of physics; including statistical mechanics, quantum mechanics (QM), special and general relativity (GR) and the holographic prin-
ciple. These equations are derived entirely from pure reason with no appeal to physical observations. All physical laws obtained are shown to be emergent from the entropy associated with the group of state-
ments proven to be irrefutable by the minimalist method.

Naturally, deriving both the Dirac equation and general relativ-
ity from the same theory is highly suggestive that the irrefutable equation should be promoted to a tentative Theory of Everything (ToE)-candidate. This result motivates the liberal use of the ToE label throughout the paper.

Contents

1 Philosophy ......................................................... 4
  1.1 Primary motivation ....................................... 4
  1.2 Explanatory gap ........................................... 4
  1.3 No uniqueness theorem ................................. 5
  1.4 State of physics ........................................... 5

I A minimalist derivation of the Theory of Everything .... 6

2 Logic ............................................................ 6
  2.1 Formal axioms ............................................. 6
  2.2 Language ..................................................... 7
  2.3 Representation of any axiomatic theories ............. 15
  2.4 Universal reason ......................................... 16
  2.5 The Theory of Everything .............................. 17
  2.6 The universal Turing machine .......................... 18

II Derivation of physical laws .................................. 20

3 A universe made of entropy .................................. 20
  3.1 Introduction ............................................... 20
  3.2 Algorithmic thermodynamics ........................... 22
  3.3 Statistical physics ........................................ 22
  3.4 An "entropic UTM" ........................................ 23
  3.5 Prior and related work ................................... 26
  3.6 Physical interpretation ................................... 28
  3.7 State equation ............................................. 29

III The Theory of Everything ................................... 31

4 Thermodynamics ............................................. 32
  4.1 Energy ....................................................... 32
1 Philosophy

What is the minimalist position?

Plato recognised that most of the disagreements in philosophy are ultimately linked to the choice of assumptions made by the parties involved. He believed that by grinding away at those assumptions, one could recover a kind of universal truth. He believed that this universal truth, comprised of whatever survives the grinding process, could ultimately be used to build a logical framework in a manner that is entirely irrefutable. Furthermore, any argument that would be constructed exclusively from this logical framework will inherit its irrefutable character.

The goal of obtaining such a framework was revisited by René Descartes in 1641. He used a universal doubt method to obtain the 'cogito ergo sum' or 'I think, therefore I am', claiming the existence of the thinking self to be an absolute truth and suggested that it should be used as the foundation of philosophy.

In this work, the minimalist position refers to the construction of a theory of logic in such a manner as to not contain any axioms assumed to be true. All foundational statements of the theory will be undeniably proven. Hence, as it will be devoid of assumptions, the irrefutable character argued by Plato will apply to it.

1.1 Primary motivation

A theory of everything (ToE) constructed exclusively from the minimalist position will inherit its irrefutable character. This is the primary motivation for this work.

1.2 Explanatory gap

An axiomatic ToE will necessarily have a foundational gap in its ability to explain the universe. Indeed, no axiomatic theory can explain why its axioms are true over other axioms. The proposed ToE, as it will be derived from the minimalist method, will avoid this explanatory gap.

Remark 1.1. The falsifiability argument is a notable non-constructive exception consisting of claiming that these axioms best reproduce the scientific observations to date. But if such an argument is used to justify its axioms, then the theory cannot provably be the ToE and it would forever be an potentially intermediary theory.
1.3 No uniqueness theorem

An axiomatic ToE cannot have a global uniqueness theorem. Any
uniqueness theorem it may contain will ultimately rest upon the
validity of its axioms, and could be false under different axioms.
Only a ToE obtained from the minimalist approach can fully answer
the question; why this theory and not another?

1.4 State of physics

The formal theories of physics are built upon axioms which are not
proven from reason but are instead justified by a series of exper-
iments or observations. For example, why is the speed of light a
constant? Because of the failure of the Michelson–Morley experiment.
And so on. Since our observational capabilities are limited by our
technology, the consequence of building physics upon a series of
observations is that we end up with multiple logically independent
theories which don’t quite fit together.
Part I

A minimalist derivation of the Theory of Everything

2 Logic

2.1 Formal axioms

In this work, we will use the following definition for a formal axiom.

**Definition 2.1 (Formal Axiom).** A formal axiom is an unprovable sentence of a language that can be true or false within a formal logic system, but is considered to be true without proof.

We emphasize the underlined elements of the definition;

1. A formal axiom must be an unprovable sentence. This is required otherwise we could call any theorem an axiom which would negate the distinction.

2. A formal axiom must be a sentence that can be true or false. This prevents tautologies, necessary truths and contradictions from being formal axioms. For example, tautologies are considered to be theorems because they are provably always true.

3. The choice of a formal logic system must be established before we can formulate a correct formal axiom for it. For example, the sentences of first-order logic are not compatible with propositional logic hence writing formal axioms as correct sentences comes after the choice of a logic system.

In this work, we will use the syntax of first order logic to write statements unambiguously. However, to prove these statements we will not use the full facilities of first order logic. Instead, we will only make use of a minimalist logic system.

To prove first order logic sentences, we will only accept a direct demonstration of the existence of the object which is claimed by the sentence. As a trivial example, we could prove the existence of the symbol 1, represented in first order logic by $\exists x[isSymbol(x) \land (x = 1)]$ by writing 1 as the proof. Sentences proven that way are said to be primitive theorems.

**Definition 2.2 (Primitive Theorem).** A primitive theorem is a sentence of first order logic that is provable by direct demonstration of the existence of the object claimed by the sentence.
Remark 2.3 (Primitive theorems are not axioms). An axiom is an unprovable sentence of a language, whereas primitive theorems are provable with a proof by demonstration of existence.

Definition 2.4 (Minimalist theory). A theory is minimalist if it is constructed exclusively from primitive theorems.

The flexibility of this proof method may seem severely crippled, but what it lacks in flexibility, it gains in epistemological irrefutability. Indeed, to reduce assumptions to a minimum, we have gotten rid of most rules of inference such as the law of excluded middle, etc. which could possibly be controversial. We only preserve the proof by demonstration of existence as anything less would be too weak to prove anything.

In a very real sense the pen and paper we used to write formal logic statements becomes a source of undeniable evidence for some statements about the existence of language, symbols and their properties. As we are limited to only constructing sentences or symbols to offer as proof, it is natural to use language as a starting point to build upon.

2.2 Language

It is quite difficult, and most likely impossible, to even imagine a concept that cannot be formulated in language. As such, language is a very general concept - perhaps even the most general of all concepts.

The limits of my language mean the limits of my world [...] Whereof one cannot speak, thereof one must be silent.

–Ludwig Wittgenstein

So how will we define language? The first potential concern is that we are unavoidably using language to define language. This might seem circular, but it is a consequence of how fundamental the concept is. Furthermore, this will allow us to prove the existence of language within the minimalist framework, so for us it will actually be an advantage.

A real problem however is that of the infinite regression of the definitions. Suppose we define a language by its symbols. Then how do we then define symbols? Do we say that symbols are unique identifiers? If so, then what is an identifier - is it a shape in one’s mind? If so, what is a mind, or a shape? This goes on forever. To break the cycle we introduce a precision cutoff in our definitions and we instead assume that the reader knows intuitively what is talked about.

Formally speaking, these cutoffs are primitive notions.
Definition 2.5 (Primitive notion). A primitive notion is a term that we use but that we do not define. The term should be understood by a mixture of examples, intuition and by the theorems and definitions that result from its usage. Primitive notions are used to define formal axioms. They are not formal axioms themselves.

For example, Euclidean geometry under Hilbert’s axioms has six recognized primitive notions; point, line, plane, congruence, betweenness, and incidence. Set theory has two; set and element of.

To define language, we will introduce two primitive notions.

Primitive Notion 2.6 (Symbol). A symbol is a unique distinguishable identifier. It can be a shape, a sound, a sign, etc. In first order logic, the symbol will be represented with the following predicate

\[
\text{isSymbol}(x) := \begin{cases} 
\text{true} & \text{x is a symbol} \\
\text{false} & \text{otherwise} 
\end{cases}
\]  

(2.7)

Primitive Notion 2.8 (Sentence). A sentence consists of taking multiple symbols and joining them together in a single group or unit. The order of occurrence of the symbols in the sentence matters and repetitions are allowed. In first order logic, the sentence will be represented with the following predicate

\[
\text{isSentence}(p) := \begin{cases} 
\text{true} & \text{p is a sentence} \\
\text{false} & \text{otherwise} 
\end{cases}
\]

Remark 2.9. For completeness, single symbols are also considered to be sentences.

We will now pose definitions relying on these primitive notions.

Definition 2.10 (Alphabetical sentence). An alphabetical sentence \(S_\alpha\) is a sentence of finite length with no repetitions of symbols.

Definition 2.11 (Language). A language \(L\) is defined by a specific alphabetical sentence \(S_\alpha\) such that;

1. The order of occurrence of the symbols in \(S_\alpha\) is the alphabetical order of \(L\).

2. If a sentence contain symbols not present in \(S_\alpha\), then it is not a sentence of \(L\).

As examples, the following languages (on the left) are defined by their \(S_\alpha\) (on the right).
**Remark 2.20 (Notation).** When writing the sentences of a language, we adopt the following notational conventions to eliminate ambiguity. For example, ambiguity occurs if we write 10; is it the decimal number ten, or the binary number two.

- If we list all possible sentences of a language from shortest to longest and from alphabetical first to alphabetical last, we suffix the sentences with 1 for unary, 2 for binary, 3 for ternary, 4 for quaternary, etc. For example, binary would be enumerated as 0₂, 1₂, 00₂, 01₂, 10₂, etc.

- If we list all possible sentences of a language according to positional notation, we suffix the sentences with u for unary, b for binary, t for ternary, q for quaternary, V for quintary, VI for sextary, etc. For example, positional binary would be enumerated as 0₃₀, 1₃₀, 1₀₃₀, etc.

- We note that for unary, both enumeration methods are identical hence the suffix u can be used interchangeably with 1. However, to my eye 1₁ reads a bit more confusing than 1u, therefore we will pick u for unary in this work.

- By convention, we will name positional decenary to be decimal and no suffix is used for its sentences. The sentences of decimal are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc.

- For languages with multiple special symbols, or where ambiguity could otherwise arise, a sentence can be placed in quotation marks. For example, we prefer to write a sentence of arithmetic as “1 + 1” instead of 1 + 1. The quotation marks are not part of the sentence.

**Remark 2.21.** Positional notation skips some sentences from its enumeration, notably those with leading zeros.

If we were to construct a conventional axiomatic theory of knowledge, we would pose our first formal axioms at this stage. A favorite

<table>
<thead>
<tr>
<th>Language</th>
<th>Alphabetical Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>nullary</td>
<td>( \epsilon )</td>
</tr>
<tr>
<td>unary</td>
<td>1</td>
</tr>
<tr>
<td>binary</td>
<td>01</td>
</tr>
<tr>
<td>ternary</td>
<td>012</td>
</tr>
<tr>
<td>decenary</td>
<td>0123456789</td>
</tr>
<tr>
<td>roman alphabet</td>
<td>abcdefghijklmnopqrstuvwxyz</td>
</tr>
<tr>
<td>roman numbers</td>
<td>IVXG</td>
</tr>
<tr>
<td>arithmetic</td>
<td>0123456789(⋅ = + − × /)</td>
</tr>
</tbody>
</table>
is usually the axiom of existence taking the paraphrased form of: 
"We pose that a simple language exists, such as the binary language".
Then using this axiom and possibly other formal axioms we would 
derive theorems, such as the existence of other languages, etc. For ex-
ample, in a certain formulation of set theory, the axiom of the empty 
set takes the following form:

**Remark 2.22** (Axiom of the empty set in set theory). In some formulation of set theory, an axiom is introduced to obtain a first set.

\[ \exists x \forall y (\neg (y \in x)) \]

In other formulations of set theory, the axiom of subsets, or the axiom of an infinite set takes the place of the generator of the first set.

Here however, we are not planning to describe sets or numbers but 
language. Since language is required to describe any formal system of 
logic, its existence is guaranteed (within logic). In the theory of knowledge that we will introduce, all initial statements of the theory will be provable as primitive theorems. Hence no formal axioms will be used in the making of this theory.

This is a unique property of using language to describe language. It cannot be used with any other abstract notions such as with sets. The sets of set theory although describable via language are nonetheless too abstract for language to provide a proof of their existence by construction thereof. Hence, set theory must be derived from axioms taken as true.

**Definition 2.23** (Universal language). Universal language \( U_L \) adopts the syntax of first order logic but not its rules of inference.

Production rules

\[
\begin{align*}
\forall x [ (\text{isSymbol}(x) \implies \text{isSentence}(x)) ] & \quad \text{First sentence (2.24)} \\
\forall x \forall p [ (\text{isSymbol}(x) \land \text{isSentence}(p)) \implies \text{isSentence}(px) ] & \quad \text{Concatenation (2.25)} \\
\forall p \forall q [ (\text{isSentence}(p) \land \text{isSentence}(q)) \implies p \rightarrow q ] & \quad \text{Unrestricted replacement (2.26)}
\end{align*}
\]

Rules of inference

\[
\begin{align*}
"x" \vdash \exists x [ \text{isSymbol}(x) ] & \quad \text{Demonstration of existence (symbol) (2.27)} \\
"p" \vdash \exists p [ \text{isSentence}(p) ] & \quad \text{Demonstration of existence (sentence) (2.28)} \\
(U_L \cup \{ k \}) \vdash t \implies (k \vdash t) & \quad \text{Deduction rule (2.29)}
\end{align*}
\]

Formal axioms

\[ \text{none} \] (2.30)
Remark 2.31. The formulation of universal language adopts the syntax of first order logic. We use it to write its laws in a clear and unambiguous manner but we do not use the full facilities of first-order logic to prove theorems.

Remark 2.32. The only proof method that we accept is the simplest proof method possible: demonstration of the object the existence of which we want to prove. We call this a proof by demonstration of existence.

Remark 2.33. All other theorems are derived from the deduction rule. Once a specific set of axioms $k$ is posed, the sentences that are provable as a result of these axioms are only so within the context of these axioms. Hence, if $k$ proves $t$, then $k \vdash t$ is a theorem of $U_L$, but $t$ by itself is not.

In what follows, we will prove the laws of universal language as primitive theorems.

Theorem 2.34 (Existence of a symbol).
\[ \exists x [\text{isSymbol}(x)] \]

Proof. We demonstrate the existence of a symbol by writing a symbol.

\[ \square \]

Theorem 2.35 (Existence of a sentence).
\[ \exists p [\text{isSentence}(p)] \]

Proof. We demonstrate the existence of a sentence by writing a sentence.

\[ \square \]

Theorem 2.36 (Reflexivity).
\[ \forall x [\text{isSymbol}(x) \implies (x = x)] \]  \hspace{1cm} (2.37)
\[ \forall p [\text{isSentence}(p) \implies (p = p)] \]  \hspace{1cm} (2.38)

Proof. Equality is definable from second order logic by the following principles of Leibniz;

\[ \forall x \forall y [x = y \implies \forall P (P x \leftrightarrow P y)] \]  \hspace{1cm} (Indiscernibility of identicals)
\[ \forall x \forall y [\forall P (P x \leftrightarrow P y) \implies x = y] \]  \hspace{1cm} (Identity of indiscernibles)

Replacing $x$ by $x$ in any sentence of a language $L$ has no bearing on the value of the predicate applicable to the system. Hence reflexivity is proved. \[ \square \]
Theorem 2.39 (Existence of another symbol).
\[ \exists x \exists y [\text{isSymbol}(x) \land \text{isSymbol}(y) \land \neg(x = y)] \tag{2.40} \]

Proof. First we offer the proof of the existence of a candidate symbol by demonstration of existence.

\[ 0 \]

To prove that this symbol is different than 1, recall the principles of Leibniz

\[
\forall x \forall y [x = y \Rightarrow \forall P (Px \leftrightarrow Py)] \quad \text{(Indiscernibility of identicals)}  \\
\forall x \forall y [\forall P (Px \leftrightarrow Py) \Rightarrow x = y] \quad \text{(Identity of indiscernibles)}
\]

If we are to replace the symbol 1 by the symbol 0 in say the sentence given by 10, some predicates of the replaced state would be different than those of the prior state. Hence the two symbols are not equal. If they are not equal, then it implies that 0 is a different symbol than 1.

\[ \Box \]

Theorem 2.41. There exist the unary language

Proof. As a proof by demonstration of existence, take the language generated by the following alphabetical sentence

\[ 1 \]

Definition 2.42 (Unary language). The unary language is defined by the alphabetical sentence of one symbol: 1. Some examples of the sentences of unary are: 1_u, 11_u, 111_u, 1111_u, \ldots. The subscript u (for unary) is optional but it is added to avoid confusion with other usages of the symbol 1.

Other languages can be defined (and proved) in a similar manner. For example, binary.

Theorem 2.43. There exists the binary language

Proof. As a proof by demonstration of existence, take the language generated by the following alphabetical sentence

\[ 01 \]

Definition 2.44 (Binary language). The binary language is defined by the alphabetical sentence of two symbols: 01. Some examples of the sentences of binary are: 02, 12, 002, 012, 102, 112, 0002, \ldots. The subscript 2 (for binary) is optional but it is added to avoid confusion with other usages of the symbols 0 and 1.
**Primitive Theorem 2.45.** For all languages L, there exists an alphabetical enumeration of its sentences.

*Proof.* As a proof by demonstration of existence, consider this enumeration of sentences from the unary language.

\[ 1_u \quad (2.46) \\
11_u \quad (2.47) \\
111_u \quad (2.48) \\
\vdots \\
\]

It is easy to construct the enumeration for other languages. This is shown in table 1.

<table>
<thead>
<tr>
<th>decimal</th>
<th>nullary</th>
<th>unary := 1</th>
<th>binary := 01</th>
<th>ternary := 012</th>
<th>quaternary := 0123</th>
<th>decenary := 0123456789</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(e)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>(e)</td>
<td>11</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(e)</td>
<td>111</td>
<td>00</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>(e)</td>
<td>1111</td>
<td>01</td>
<td>00</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>(e)</td>
<td>11111</td>
<td>10</td>
<td>01</td>
<td>00</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>(e)</td>
<td>111111</td>
<td>11</td>
<td>02</td>
<td>01</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>(e)</td>
<td>1111111</td>
<td>000</td>
<td>10</td>
<td>02</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>(e)</td>
<td>11111111</td>
<td>001</td>
<td>11</td>
<td>03</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>(e)</td>
<td>111111111</td>
<td>010</td>
<td>12</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>(e)</td>
<td>1111111111</td>
<td>011</td>
<td>20</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>(e)</td>
<td>11111111111</td>
<td>100</td>
<td>21</td>
<td>12</td>
<td>00</td>
</tr>
<tr>
<td>11</td>
<td>(e)</td>
<td>111111111111</td>
<td>101</td>
<td>22</td>
<td>13</td>
<td>01</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
</tbody>
</table>

*Table 1:* A natural number is associated to each sentence of select languages.

**Primitive Theorem 2.49.** There exists a model of the naturals within language.

*Proof.* The naturals are defined by recursion against the starting element, such that every natural has a successor and the successor of any natural is also a natural. To recover a model using a language \(L\), we

1. associate the alphabetical first sentence of \(L\) with the starting element, and

2. associate the next sentence as the successor of the previous sentence.
Then it follows that any successor is also a sentence. Recall that we have proven, in the previous theorem, the existence of the alphabetical enumeration by constructing it. Here, we have proven that these alphabetical enumeration follows the successor axiom of the naturals. As a result, the successor axiom of the naturals is a theorem applicable to the alphabetical enumeration of $L$. Hence, there exists a model of the natural within language in the form of the alphabetical enumeration.

**Definition 2.50** (Alphabetical position of a sentence). To define a numerical position for the sentences of a language $L$, we apply theorem 2.49 to its alphabetical enumeration. Specifically, we associate a natural to each sentence of $L$ in the following way:

1. The first natural is the alphabetical first sentence.
2. The next natural is associated with the next alphabetical sentence.
3. And so on.

Each natural is associated to a corresponding sentence from shortest to longest and from alphabetical first to alphabetical last. The natural represents the position of the sentence of the language $L$ within its alphabetical enumeration. For example, we can say that the fifth sentence of unary is $11111_u$.

**Theorem 2.51.** There is no proof of inconsistency in universal language

**Proof.** Universal language is defined in minimalist first-order logic. The system has no formal axioms and only two rules of inference. The two allowed rules of inference are;

1. The proof by demonstration of existence.
2. The proof by deduction.

**Lemma 2.52.** The proof by demonstration of existence cannot prove an inconsistency.

To prove an inconsistency, one would have to first construct an object, then show that such object cannot be constructed. However, the proof by demonstration of existence can only prove existence and cannot, by itself, prove non-existence. To prove non-existence, the deduction rule must be used and some assumptions on logic must be posed. Hence, sentences invoking non-existence cannot be theorems of universal language. They can, at best, only be theorems of universal language plus some assumptions.

**Lemma 2.53.** The proof by deduction cannot produce a contradiction as a theorem of universal language.
The deduction rule can show that $\Delta \vdash A \land \neg A$, where $\Delta$ is a set of assumptions implying a contradiction. Then, $\Delta \vdash A \land \neg A$ would be a theorem of universal language, but $A \land \neg A$ would not be. Hence no contradiction will be a theorem of $U_L$.

The implication is that all truths, with the exception of those proven by demonstration of existence, are truths conditional on some assumptions. The minimal approach produces a system that, in a sense, does not lie to its user. The system admits that the user can write any sentence down if he so wishes. Furthermore, the user can replace any sentence by any other sentence. To work within a certain framework of truth however, the user must voluntarily restrict its freedom of use over the language to a set of truth preserving rules, which are assumed. The user is always reminded that the theorems that he proves are of the form $k \vdash t$, and not just $t$ by itself, with the exception of the existence of the language itself. As a result, the logic system is held to be a better representation of the reality of the logician, than would a system having more axioms or more rules of inference.

2.3 Representation of any axiomatic theories

In this section, we will show a method to represent any axiomatic theories $k$ within universal language.

For axiomatic theories $k$ that admit a truth predicate $isTrue(p)$ over the sentences $p$ of a language $L$, we can define a binary $\Omega$ sentences encoding the result of the predicate. For a binary language and if, as an example, $\Omega = 1101...$, then $\Omega$ can be associated with the result of the predicate as follows.

<table>
<thead>
<tr>
<th>truth predicate value $\Omega$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$isTrue(0)$ true 1</td>
<td>1</td>
</tr>
<tr>
<td>$isTrue(1)$ true 1</td>
<td>1</td>
</tr>
<tr>
<td>$isTrue(00)$ false 0</td>
<td>0</td>
</tr>
<tr>
<td>$isTrue(01)$ true 1</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

For more complex theories that are incapable of defining a truth predicate, we can alternatively interpret the symbol 0 of $\Omega$ as the undefined symbol $\exists$. In this case $isTrue$ is no longer a predicate but a function $f$. It can be interpreted as proof verification function over
L under a set of rules $k$. The allowable rules are precisely those of recursively enumerable theories.

<table>
<thead>
<tr>
<th>proof function</th>
<th>value</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(0)$</td>
<td>true</td>
<td>1 (2.58)</td>
</tr>
<tr>
<td>$f(1)$</td>
<td>true</td>
<td>1 (2.59)</td>
</tr>
<tr>
<td>$f(00)$</td>
<td>$\not\exists$</td>
<td>0 (2.60)</td>
</tr>
<tr>
<td>$f(01)$</td>
<td>true</td>
<td>1 (2.61)</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The function or predicate, if defined via an effective method can be computed within universal language using unrestricted replacements. To do so it suffices to pose that the rules of inference of the function are bearers of truth, then with the deduction rule, all theorems $t$ of all recursively enumerable theories $k$ are theorems of universal language in the form $k \vdash t$.

We can show the existence all recursively enumerable theories by writing the all possible $\Omega$ sentences. As the $\Omega$ sentences are simply binary sentences, the enumeration can certainly be done for all finite sentences. For infinite $\Omega$ sentences, it must be defined as the result of a recursively enumerable function. As a result, universal language can prove the existence of all recursively enumerable $\Omega$-sentences.

2.4 Universal reason

**Definition 2.62** (Universal reason). A theory $T$ embeds universal reason if this sentence holds for it,

$$\forall k \forall t [(k \vdash t) \implies T \vdash (k \vdash t)]$$

This sentence means that for all axiomatic theories $k$ and all theorems $t$, if $k \vdash t$ then it must be the case that $T$ can prove that $k \vdash t$, or $T$ would contain gaps in reason. Having gaps in reason would imply a partial embedding of reason as opposed to a universal one.

For example, the english language embeds universal reason as we can talk about any mathematical principle using the english language. If the english language did not embed universal reason, then someone speaking Greek could potentially understand theorems that would fundamentally escape any english speaker. Such Greek theorems would not survive translation to english and could only be understood by learning Greek. Clearly, most (and likely all) human languages embed universal reason.
Remark 2.63. Universal reason also contains the minimalist approach. For example, this occurs when \( k \) is comprised of the primitive theorems of universal language.

**Primitive Theorem 2.64.** Universal language embeds universal reason

**Proof.** This is a trivial consequence of the existence of arbitrary \( \Omega \)-sentences, the deduction rule and of unrestricted replacements.

**Theorem 2.65.** First order arithmetic embeds universal reason

**Proof.** Consistent with the Gödel numbering method and for each well formed formula of arithmetic, we associate to it a natural number. Then, for each recursively enumerable theory of knowledge \( k \) we pose a series of equations to be used on the natural numbers. The equations are posed such that they transform the natural number to another natural number in a manner consistent with the rules of inference of \( k \).

This can be repeated for all possible recursively enumerable axiomatic theories of knowledge. As a result, arithmetic embeds universal reason.

2.5 *The Theory of Everything*

We are now ready to derive the main result of this paper. Recall that universal language is constructed with primitive theorems. As a result the existence of universal language is maximally irrefutable in the sense argued by Plato. Furthermore, the existence of arbitrary \( \Omega \)-sentences proves that universal language embeds universal reason. Hence, universal reason, as a primitive theorem of universal language, exists irrefutably.

A theory of everything, by definition, must explain what is occurring in the universe. As a result, it must, at minimum, explain that which is proven to exist irrefutably in it. Hence, the ToE must embed universal reason.

All that is left to be done is show that the laws of physics are a necessary consequence of embedding universal reason within a theory.

The main result of this paper is that I have found that any theory which embeds universal reason will necessarily contain a theory of physics implied by the embedding. The theory of physics is unique and does not depend on the specifics of the embedding. All laws of physics are conjectured to be emergent from this embedding. This will be shown explicitly for special & general relativity, for the Schrödinger equation, for the Dirac equation and for the holographic principle.
The implication is that the laws of physics are a necessary consequence of the existence of universal reason. The conjecture is that no physical observations or confirmation are required to understand the universe (other than to provide helpful insights and to confirm mathematical correctness).

2.6 The universal Turing machine

To make the laws of physics come out of the embedding, we will reformulate universal reason within the framework of a universal Turing machine (UTM). Doing this will unlock the formalism of algorithmic information theory and other UTM-related theorems to help us out.

Recall that first-order arithmetic, like universal language, also embeds universal reason. As a result, augmenting universal language with the axioms of first-order arithmetic will not void the derivation of the laws of physics. First-order arithmetic is selected for two reasons: 1) it is very uncontroversial and 2) almost everyone is familiar with it. With this augmentation, we can now define sums and use other arithmetic tools to simplify the notation and the proofs.

Recall the definition of a theory embedding universal reason,

\[ \forall k \forall t [(k \vdash t) \implies \text{ToE} \vdash (k \vdash t)] \]  

As there are countably many, each sentence \( k \vdash t \) can be mapped to a natural number. For example, this can be done according to Table 2. Using this mapping, we can now define the following sum

\[ \Omega = \sum_{n=1}^{\infty} 2^{-E(n) - n} \]  

where \( E(n) = \begin{cases} 0 & [k \vdash t]_n \\ \infty & \text{otherwise} \end{cases} \)  

To see into more detail what happens in the sum, let us unpack it with an example. We get

\[ \Omega = \sum_{n=1}^{\infty} 2^{-E(n) - n} \]  

We recall that if the \( n^{th} \) sentence is provable, then \( E(n) = 0 \), otherwise \( E(n) = \infty \). Using example values for \( E(n) \), we get

\[ = 2^{-\infty}2^{-1} + 2^{-0}2^{-2} + 2^{-0}2^{-3} + 2^{-0}2^{-4} + 2^{-\infty}2^{-5} + ... \]  

<table>
<thead>
<tr>
<th>( n )</th>
<th>sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( k_1 \vdash t_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( k_1 \vdash t_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( k_2 \vdash t_1 )</td>
</tr>
<tr>
<td>4</td>
<td>( k_2 \vdash t_2 )</td>
</tr>
<tr>
<td>5</td>
<td>( k_1 \vdash t_3 )</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>( [k \vdash t]_n )</td>
</tr>
</tbody>
</table>

Table 2: Since there are countably many sentences, they can be associated to a natural number in a one to one correspondence.
The presence of the negative infinity in the term of the exponential causes some terms to vanish to zero. Note that the suffix 2 indicates the binary notation.

$$= 0_2 + 0.01_2 + 0.001_2 + 0.0001_2 + 0_2 + ...$$ (2.70)

$$= 0.01110..._2$$ (2.71)

This sum encodes the truth value of each sentence $k \vdash t$ of universal reason as a binary real number. This representation connects universal reason to the halting probability of a prefix-free universal Turing machine $^2$.

Mapping the sentences of universal reason to the natural numbers is conceptually simple but it is by no means the only way to do it. Indeed, the halting probability is defined for any prefix-free coding of the sentences. As a result, we rewrite the sum as

$$\Omega = \sum_p 2^{-E(p) - |p|} \quad \text{where} \quad E(p) = \begin{cases} 
0 & \text{p halts} \\
\infty & \text{otherwise} 
\end{cases}$$ (2.72)

Here, $p$ represents a prefix-free encoded sentence of universal reason and $|p|$ represents its length. With this representation of universal reason, we have recovered the definition of the halting probability of a prefix-free universal Turing machine. This is what we will use as our starting point to derive the laws of physics.

---

Part II

Derivation of physical laws

3 A universe made of entropy

It has been said that thermodynamics is the most general of all the disciplines of physics. Hence it is expected to be the first derived from a ToE.

A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts.

–Albert Einstein

We have so far obtained a representation of the universe which is information-based and defined by $\Omega$, the knowledge contained within it. It is equivalent to representing the universe as the answers to a series of yes/no questions. The yes/no questions, are of the form; “is the sentence [...] a theorem of the universe, yes or no?”. The universe, like any system can be fully represented given sufficiently many answers to yes/no questions of this form. The quantity of answers, which is astronomical for a system the size of the universe, represent the total entropy of the system. We will see that this entropy connects directly to the physical entropy of thermodynamics provided that the answers to the yes/no questions are maximally compressed. This is the case for $\Omega$, the halting probability. Furthermore, the laws of relativity and quantum mechanics naturally and uniquely come out of this connection.

3.1 Introduction

A connection between algorithmic information theory and statistical physics was established by Tadaki and extended by John C. Baez and Mike Stay. Indeed, Tadaki found that augmenting the halting probability of a prefix-free universal Turing machine (UTM) with a multiplication constant, applicable to the program length, is enough to recover the Gibbs ensemble of statistical physics. John C. Baez and Mike Stay took it a step further by introducing into the halting probability additional terms interpreted as thermodynamic observable equivalents. This was enough for them to construct a thermodynamic cycle applicable to algorithmic information theory (AIT). The


authors’ goal was to import notions of thermodynamic into AIT in order to help understand it better. Here, our goal is reversed as we want to use this connection to step into physics from AIT.

I have found that introducing two new thermodynamic conjugate pairs, each having a dual interpretation both as a physical system and as an algorithmic system, is enough to connect Ω, the halting probability of a prefix-free UTM, to many unsolved problems of theoretical physics. These two conjugate pairs are the minimum required to make the sum converges towards Ω. The first observable, let’s call it entropic time, is the action $S$ conjugated with the frequency $f$ and accounts for the change of entropy in time. It can be interpreted as an internal clock used by the UTM to synchronize the calculation. The second observable, let’s call it entropic space, is a “decompression” factor $D$ and is conjugated with the program length $|p|$. It can be interpreted as a function that assigns a prefix-free code to each program according to an arbitrary algorithm.

Adding a time conjugate to a Gibbs ensemble (such as the frequency $f$) adds a whole new dynamic to a thermodynamic system. The system now becomes aware of future, past and present entropy and can translate from time to space and from space to time for an entropic cost and provided that various limits are respected. By studying thermodynamic cycles involving space and time, I was able to look at what happens to the entropy when a system is translated forward or backward in time and draw conclusions in regards to the arrow of time. In the model presented, space serves as an entropy sink that encourages a forward arrow of time, the future is non-computable and the past is singular.

As these new conjugate pairs can be added to any Gibbs ensemble and will produce a similar system, the bits of Ω serve as the microscopic interpretation of the entropy for the system. A forward translation in time increases the number of bits of Ω that are known, which simultaneously lowers the entropy and increases the quantity of information encoded by the past. As multiple choices of prefix-free code exist to encode Ω for a given UTM, this provides an entropy sink available to offset the reduction in entropy produced by a forward translation in time - this is the role of the second observable. The main result is that for an arbitrary prefix-free encoding, the limitations and costs associated with this sink are identical to those required to derive special and general relativity from an entropic perspective. Furthermore, quantum mechanics will be shown to result from the non-computable entropy of Ω.
3.2 Algorithmic thermodynamics

The halting probability is similar to a Gibbs ensemble of statistical physics. In fact, this similarity has been noted by other authors before. Indeed, the Gibbs ensemble compares to the halting probability as follows:

\[
Z = \sum_x e^{-\beta (E + pV + Fx)} \quad \text{and} \quad \Omega = \sum_p 2^{-E(p) - |p|} \tag{3.1}
\]

To be upgraded to a full-fledge Gibbs ensemble I only need to add a conjugate variable to the halting probability analogous to the temperature \(\beta\). As suggested by Tadaki, I multiply the terms of the exponential by a compression factor \(D\) adjusting the packing density of the bits of \(\Omega\). I get

\[
Z'_{\Omega} = \sum_p 2^{-\beta E(p) + D|p|} \tag{3.2}
\]

\(E(p)\), as it is either 0 or \(\infty\) will absorb \(\beta\), so its contribution to \(\Omega\) remains the same. For \(\beta D|p|\), the effect is to "compress" or "decompress" the bits of \(\Omega\). If \(\beta D > 1\), no bit erasure takes place. To fix the idea, I unpack the sum taking \(\beta D = 2\) as an example.

\[
\begin{align*}
&= 2^{-2\times1} + 2^{-2\times2} + 2^{-2\times3} + 2^{-2\times4} + 2^{-2\times5} + \\
&= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} + 2^{-10} + \\
&= 0.01_b + 0.0001_b + 0.000001_b + 0.000000001_b + \\
&= 0.0101010101..._b
\end{align*} \tag{3.3}
\]

The result is that some zero-valued bits have been injected between the bits of \(\Omega\). To recover \(\Omega\), it suffices to eliminate the extra bits. No halting information is lost. As a result \(Z'_{\Omega}\) is "informationally" equivalent to \(\Omega\).

3.3 Statistical physics

<table>
<thead>
<tr>
<th>Observable</th>
<th>Conjugate variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (E)</td>
<td>Temperature (\beta = 1/(k_B T))</td>
</tr>
<tr>
<td>Volume (V)</td>
<td>Pressure (\gamma = p/(k_B T))</td>
</tr>
<tr>
<td>Number of particles (N)</td>
<td>Chemical potential (\delta = -\mu/(k_B T))</td>
</tr>
</tbody>
</table>

Before continuing to the next section, I will do a quick recall of
statistical physics. In statistical physics we are interested in the distribution that maximizes entropy

\[ S = -k_B \sum_{x \in X} p(x) \ln p(x) \]  \hspace{1cm} (3.7)

subject to the fixed macroscopic observables. The solution is the Gibbs ensemble. Taking the observables listed in Table 3 as examples, the partition function becomes

\[ Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \]  \hspace{1cm} (3.8)

The probability of occupation of a micro-state is;

\[ p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \]  \hspace{1cm} (3.9)

The average values and their variance for the observables are;

\[ \mathcal{E} = \sum_{x \in X} p(x) E(x) \] \hspace{1cm} \[ \mathcal{E} = -\frac{\partial \ln Z}{\partial \beta} \] \hspace{1cm} \[ \frac{\partial^2 \ln Z}{\partial \beta^2} \]  \hspace{1cm} (3.10)

\[ \mathcal{V} = \sum_{x \in X} p(x) V(x) \] \hspace{1cm} \[ \mathcal{V} = -\frac{\partial \ln Z}{\partial \gamma} \] \hspace{1cm} \[ \frac{\partial^2 \ln Z}{\partial \gamma^2} \]  \hspace{1cm} (3.11)

\[ \mathcal{N} = \sum_{x \in X} p(x) N(x) \] \hspace{1cm} \[ \mathcal{N} = -\frac{\partial \ln Z}{\partial \delta} \] \hspace{1cm} \[ \frac{\partial^2 \ln Z}{\partial \delta^2} \]  \hspace{1cm} (3.12)

The laws of thermodynamics can be recovered by taking the following derivatives

\[ \frac{\partial S}{\partial E} \Bigg|_{V,N} = \frac{1}{T} \] \hspace{1cm} \[ \frac{\partial S}{\partial V} \Bigg|_{E,N} = \frac{p}{T} \] \hspace{1cm} \[ \frac{\partial S}{\partial N} \Bigg|_{E,V} = -\frac{\mu}{T} \]  \hspace{1cm} (3.13)

which can be summarized as

\[ dE = TdS - p dV + \mu dN \]  \hspace{1cm} (3.14)

This is known as the state equation of the thermodynamic system. We are now ready to continue.

### 3.4 An "entropic UTM"

A UTM can attempt to calculate \( \Omega \) by starting each program in dovetail, and as they halt, add their contribution to \( \Omega \). After an infinite amount of time, \( \Omega \) will indeed be recovered. However, the calculation does not converge towards \( \Omega \) as it progresses and discontinuously yields \( \Omega \) only at infinity. To see why, consider the case where the first zero-valued bit of \( \Omega \) is at position \( i \). Since the general non-halting
problem is unsolvable, at most the calculation of \( \Omega \) differs from the
real value of \( \Omega \) by \( 2^{-i} \). The error rate does not decrease during the
calculation and only vanishes at infinity when all halting programs
are known.

To make the laws of physics come out I must adjust the calculation
so that it converges towards \( \Omega \) even during the calculation. In other
words, the error rate must be made to monotonically decrease during
the calculation. This can be done with entropic dovetailing.

**Definition 3.15 (Dovetailing).** Dovetailing is a program execution
strategy for a Turing machine to guarantee that progress will be made
on arbitrarily-many programs even in the presence of non-halting programs.

**Definition 3.16 (Standard dovetailing).** Consider the case of standard
dovetailing. First, we start the shortest program and perform one iteration.
Then, we start the second program and perform one iteration on the first and
second programs. Then, we start the third program and perform one itera-
tion on the first, second and third programs. And so on. Using dovetailing,
progress will eventually be made on every program and no program will
cause the TM to hang.

To convert the halting probability into a dovetailing calculation
of \( \Omega \), it suffices that I add to the sum the action observable \( S \) conjugated with the frequency \( f \). Also, note that \( f \) is related to the time \( t \)
via \( f^{-1} = t \). It yields,

\[
Z_{\Omega} = \sum_p e^{-\beta (\ln 2) [E(p) + 2\pi S f + D|p|]} \quad \text{Partition function (3.17)}
\]

Here, the \( 2\pi \) factor is added to recover the physical definition of \( S \) as
related to the angular frequency. Having a partition function in base
2 instead of the natural base \( e \) is equivalent to performing a change in
temperature from \( \beta' \) to \( \beta \ln 2 \). Hence \( T' = T / \ln 2 \). The state equation
therefore is

\[
dE = \frac{1}{\ln 2} T dS - 2\pi S df - D d|p| \quad \text{State equation (3.18)}
\]

What are these program observables and why am I allowed to
add them? Recall that \( Z'_{\Omega} \) and \( Z_{\Omega} \) are Gibbs ensembles. As a result,
observables of program properties can be added. I will now look
at \( 2\pi S f \) into more detail to understand the impact it has on the
calculation of \( Z_{\Omega} \). Starting with an example, suppose the following
values of \( S \) for the first three programs,

\[
S_1 = \frac{5}{2\pi} \quad S_2 = \infty \quad S_3 = \frac{5}{2\pi} \quad (3.19)
\]
Note that I do not, nor am I trying to, escape the non-computability of \( \Omega \). Indeed, \( S \) is non-computable because \( S \) bears the solution to the general non-halting problem. \( Z_\Omega \) simply shifts the non-computability from \( E(p) \) to \( S \). In my example the sum \( Z_\Omega \) becomes

\[
Z_\Omega = 2^{-1-\frac{5}{3}} + 2^{-2-\frac{4}{3}} + 2^{-3-\frac{7}{3}} + \ldots
\]  

(3.20)

As an example consider these values of \( Z_\Omega(t) \) for specific values of \( t \) along with the error rate \( \xi(t) = \Omega - Z_\Omega(t) \)

<table>
<thead>
<tr>
<th>estimation of ( \Omega )</th>
<th>error</th>
<th>bound (on error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega = 0.101..._b )</td>
<td>( \xi = 0 )</td>
<td>( \xi = 0 )</td>
</tr>
<tr>
<td>( \lim_{t \to 0^+} Z_\Omega(t) = 0 )</td>
<td>( \xi = \Omega )</td>
<td>( \xi \leq 2^{-0} )</td>
</tr>
<tr>
<td>( Z_\Omega(1) = 0.00010001..._b )</td>
<td>( \xi = 0.1001101..._b )</td>
<td>( \xi \leq 2^{-1} )</td>
</tr>
<tr>
<td>( Z_\Omega(5) = 0.010101..._b )</td>
<td>( \xi = 0.010101..._b )</td>
<td>( \xi \leq 2^{-2} )</td>
</tr>
<tr>
<td>( Z_\Omega(10) = 0.011001000100..._b )</td>
<td>( \xi = 0.001011101101..._b )</td>
<td>( \xi \leq 2^{-5} )</td>
</tr>
<tr>
<td>( Z_\Omega(1000) = 0.100110101000011..._b )</td>
<td>( \xi = 0.00001010110100..._b )</td>
<td>( \xi \leq 2^{-\infty} )</td>
</tr>
<tr>
<td>( \lim_{t \to \infty} Z_\Omega(t) = 0.0101..._b )</td>
<td>( \xi = 0 )</td>
<td>( \xi \leq 2^{-\infty} )</td>
</tr>
</tbody>
</table>

As I grow \( t \) from 0 to \( \infty \), the error rate monotonically diminishes until it eventually vanishes. I will now prove two theorems; 1) \( Z_\Omega \) can recover \( \Omega \) at \( t \to \infty \) and 2) \( Z_\Omega \) calculates \( \Omega \) through time with a monotonically decreasing error rate.

**Theorem 3.28.** To prove that \( Z_\Omega \) recovers \( \Omega \) at \( t \to \infty \), I will show that \( \Omega \) is computable from \( Z_\Omega \) and that

\[
\lim_{t \to \infty} Z_\Omega \to Z'_\Omega
\]

**Proof.** A program \( p \) can have any value of \( S_p \) within \([0, \infty]\). If the program halts immediately, \( S_p = 0 \). If it never halts, \( S_p = \infty \). If it halts after a certain time, \( S_p \in \mathbb{N} \). A program that never halts will not contribute to the halting partition. This will be the case if \( S_p = \infty \). This yields,

\[
\lim_{f \to 0^+} 2\pi f S_p = \lim_{t \to \infty} \frac{2\pi S_p}{t} = \begin{cases} 
0 & \text{p halts} \\
\infty & \text{otherwise} 
\end{cases}
\]  

(3.29)

As this is the definition of \( E(p) \) we obtain

\[
\lim_{t \to \infty} \frac{2\pi S_p}{t} = E(p)
\]  

(3.30)

**Lemma 3.31.** \( E(p) + E(p) = E(p) \)
Proof. \( E(p) \) is either 0 or \( \infty \). Since \( 0 + 0 = 0 \) and \( \infty + \infty = \infty \), the lemma holds.

Therefore,

\[
\lim_{t \to \infty} Z_\Omega = \lim_{t \to \infty} \left( \sum_p 2^{-\beta[E(p)+2\pi Sf+D|p|]} \right) \tag{3.32}
\]

\[
= \sum_p 2^{-\beta[E(p)+E(p)+D|p|]} \tag{3.33}
\]

\[
= \sum_p 2^{-\beta[E(p)+D|p|]} \tag{lemma 3.31}
\]

\[
= Z'_{\Omega} \tag{3.34}
\]

Is knowing \( Z'_{\Omega} \) enough to compute \( \Omega \)? The answer is yes as I just need to remove the zero-valued bits inserted inbetween the bits of \( \Omega \).

\textbf{Theorem 3.35.} To show that equation (3.17) dovetails programs, it suffices to show the following. For \( 0 < t < \infty \), the partition function \( Z_\Omega \) is

\[
Z_\Omega(t) = \Omega - 2^{-k(t)}
\]

where \( 2^{-k(t)} \) is an error rate that is monotonically decreasing to 0 as \( t \to \infty \). As a result of increasing the time, the calculation of \( Z_\Omega \) produces an ever more precise estimation of \( \Omega \).

\textit{Proof.} Using a similar argument as the one provided by John C. Baez and Mike Stay, I argue that as \( S \) exponentially suppresses programs with long halting time, there will always be a time \( t \) such that the contribution of programs that have not yet halted will be less than \( 2^{-k(t)} \).

As a result, the partition function \( Z'_{\Omega} \) produces a monotonically improving estimation of \( \Omega \) over time. The fact that the error rate is able to decrease monotonically implies that the calculation does not hang. Hence it is a type of dovetailing. Furthermore, since I have defined the calculation with a Gibbs ensemble, I am guaranteed that the calculation maximizes the entropy during the calculation.

In the end, what I have described is a dovetailing algorithm which maximizes the entropy in the calculation of \( \Omega \).

3.5 \quad \textit{Prior and related work}

How then are the laws of physics recovered from (3.17)? To recover the laws of physics, I will make use of the properties of the Gibbs
ensemble notably by studying its state equation. Although not necessary, it helps to give (3.17) a physical interpretation of its observables. Let me start by considering the prior work.

Many authors have discussed the similarity between physical entropy \( S = -k_B \sum p_i \ln p_i \) and the entropy in information theory \( S = -\sum p_i \log_2 p_i \).

John C. Baez and Mike Stay suggest an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs are considered to be observables. Starting from Gregory Chaitin’s \( \Omega \) number, the halting probability

\[
\Omega = \sum_{\text{halt}} 2^{-|p|} \tag{3.36}
\]

is extended with algorithmic observables to obtain

\[
\Omega' = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{3.37}
\]

Noting the similarity between the Gibbs ensemble of statistical physics (3.8) and (3.37), they suggest an interpretation where \( E \) is the expected value of the logarithm of the program’s runtime, \( V \) is the expected value of the length of the program and \( N \) is the expected value of the program’s output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper):

1. \( T = 1/\beta \) is the algorithmic temperature (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2. \( p = \gamma/\beta \) is the algorithmic pressure (analogous to pressure). This measures the tradeoff between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount, while holding the number of programs in the ensemble and their mean output fixed.
3. \( \mu = -\delta/\beta \) is the algorithmic potential (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

–John C. Baez and Mike Stay

From equation (3.37) they derive analogues of Maxwell’s relations and they consider thermodynamic cycles such as the Carnot cycle.
or Stoddard cycle. For this they introduce the concepts of algorithmic heat and algorithmic work.

Other authors have suggested other somewhat arbitrary correspondence ⁸.

### 3.6 Physical interpretation

I suggest to map the program-observables to physical-observables as follows. As I will show, this interpretation correctly maps the algorithmic thermodynamics interpretation of special relativity (and other laws) to its physical interpretation. Hence, it would appear to be the preferred mapping.

- The program-runtime is the number of iterations a UTM needs to perform until a program halts. It is therefore natural to associate it with the physical *Time* in seconds. Indeed, a program requiring more iterations to halt will also require more time to terminate. If a system performs iterations at a faster or slower rate, the observable associated with time, the *Power* in Watts, can be adjusted to account for this variation.

- The program-frequency, is associated with the reverse of the second, s⁻¹, and its associated observable is the *Action* in Joules-seconds.

- The program-size is expressed in number of bits. Writing the bits one after the other on any medium (paper, disk drive, etc.) will require a certain physical size for each bit. As the line is the lowest dimensional geometry to spread bits, the program-size is naturally associated with the physical *length* as its simplest case. Furthermore, if an encoding medium would allow greater or lesser "packing-tightness" of the bits, it can be modelled with its associated observable, the *Force* in Newtons, pushing the bits together or pulling them apart. If one wishes instead to investigate geometries of higher dimensions, one can use different units. For the 2D case it can be mapped to an *Area* in m² and its associated observable will be the *Surface tension* in N/m. For the 3D case the program-size can be mapped to a *Volume* in m³ and its associated observable will be the *Pressure* in N/m². Even higher dimensions could be used, but their physical interpretation, if any, would be less clear.

- Only the halting event remains. As it is the only quantity with no units, it is natural to map it to the *Energy* in Joules. Indeed, in the Gibbs ensemble the energy is the only observable not multiplied by a conjugate variable. Adding extra units to the halting event

only to have them cancelled out by a conjugate variable would be futile.

Summarizing the points above, I obtain Table 4 as the mapping of choice between *algorithmic thermodynamics* and *physical thermodynamics*.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Variable</th>
<th>Units</th>
<th>Conjugate</th>
<th>Variable</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halting event</td>
<td>E</td>
<td>J</td>
<td>Temperature</td>
<td>T</td>
<td>K</td>
</tr>
<tr>
<td>Program-size (length)</td>
<td>x</td>
<td>m</td>
<td>Force</td>
<td>F</td>
<td>N</td>
</tr>
<tr>
<td>Program-size (area)</td>
<td>A</td>
<td>m^2</td>
<td>Stiffness</td>
<td>k</td>
<td>N/m</td>
</tr>
<tr>
<td>Program-size (volume)</td>
<td>V</td>
<td>m^3</td>
<td>Pressure</td>
<td>p</td>
<td>N/m^2</td>
</tr>
<tr>
<td>Program-frequency</td>
<td>f</td>
<td>1/s</td>
<td>Action</td>
<td>S</td>
<td>Js</td>
</tr>
<tr>
<td>Program-runtime</td>
<td>t</td>
<td>s</td>
<td>Power</td>
<td>P</td>
<td>W</td>
</tr>
</tbody>
</table>

Table 4: The preferred correspondence between *algorithmic thermodynamics* and *statistical physics*.

### 3.7 State equation

The state equation for the partition function (3.17) is,

**Definition 3.38** (Algorithmic state equation).

\[
dE = \frac{1}{\ln 2} TdS - 2\pi S df - Dd|p| \tag{3.39}
\]

This is analogous to the law of conservation of energy, interpreted as a law of conservation of halting information. I will now take the Taylor series of \(Dd|p|\). To do so I first pose \(L(p) := |p|\). Then I obtain,

\[
DL(p) = DL(0) + L'(0)p + \frac{1}{2} L''(0)p^2 + \frac{1}{6} L'''(0)p^3 + \ldots \tag{3.40}
\]

\[
DdL(p) = DL'(0)dp + DL''(0)pdp + DL'''(0)p^2 dp + \ldots \tag{3.41}
\]

switching the notation from \(p\) to \(x\), I get

\[
DdL(x) = DL'(0)dx + DL''(0)x dx + DL'''(0)x^2 dx + \ldots \tag{3.42}
\]

then further posing \(F := DL'(0), k := DL''(0), p := DL'''(0)\) (here \(p\) means the pressure, not a program), I get

\[
DdL(x) = Fdx + kxdx + px^2 dx + \ldots \tag{3.43}
\]

To recover the physical interpretation, it suffices that I replace \(Dd|p|\) with its Taylor expansion. The state equation, in the physical interpretation, is,
\[ dE = \frac{1}{\ln 2} TdS - 2\pi Sdf - \left( F + kx + px^2 + \ldots \right) dx \quad (3.44) \]

Further performing the replacements \( dA = xdx \) and \( dV = x^2dx \), we get

**Definition 3.45 (Physical state equation).**

\[ dE = \frac{1}{\ln 2} TdS - 2\pi Sdf - Fdx - kA - pdV - \ldots \quad (3.46) \]

and if a three-dimensional simplification cutoff is desired, we get

\[ dE = \frac{1}{\ln 2} TdS - 2\pi Sdf - Fdx - kA - pdV \quad (3.47) \]

\[ (3.48) \]

Solutions to this state equation yields entropic and computed “universes” generated by the calculation of \( \Omega \) with a prefix-free UTM. Note that as the Taylor expansion was taken, the physical state equation is only defined for smooth functions of \( L(p) = |p| \). Therefore a smoothness of the space of \( L(p) \) is implicitly assumed.
Part III

The Theory of Everything

We complete the halting partition by giving it an interpretation as the Theory of Everything in physics from the perspective of algorithmic information theory. The exponential terms are given the roles of macroscopic observables and are

1. $E(p)$, the halting event function, a bearer of halting information. It filters non-halting programs out of the sum.

2. $D|p|$, a "decompression" observable $D$ conjugated with the length $|p|$ of a program $p$ encoded with a prefix-free code.

3. $2\pi Sf$, the program-action observable $S$ conjugated with a frequency $f$. It is responsible for exponentially reducing the contribution to the sum of programs with long-halting times. This observable allows the estimation to converges towards $\Omega$ over the time of the calculation.

Grouping the conjugate pairs together into a partition function, we obtain the following equation.

$$Z_\Omega = \sum_p e^{-\beta \ln 2 \left[E(p) + 2\pi Sf + D|p|\right]}$$  \hspace{1cm} (3.49)

$$dE = \frac{1}{\ln 2} TdS - 2\pi Sdf - Dd|p|$$ \hspace{1cm} (state equation)

Remark 3.50. In the special case where $\beta = F = 1$ and $f = 0$, we recover the halting probability of a prefix-free UTM and $Z_\Omega = \Omega$.

We can switch from the algorithmic interpretation to the physical interpretation by taking the Taylor expansion of the conjugate variable $|p|$ and by renaming $p$ to $x$. The physical partition function is,

$$Z_\Omega = \sum_x e^{-\beta \ln 2 \left[E(x) + 2\pi Sf + Fx + \frac{1}{2}kA + \frac{1}{p}V - \ldots\right]}$$  \hspace{1cm} (3.51)

$$dE = \frac{1}{\ln 2} TdS - 2\pi Sdf - Fdx - kdA - pdV - \ldots$$ \hspace{1cm} (state equation)

The significance is this equation is three-fold;

1. It is derived irrefutably in the sense given by Plato. Indeed, any theory which embeds universal reason must necessarily have the embedding bounded by this equation. The only way to change
this equation is to embed partial reason as opposed to universal reason. As the existence of universal reason is proven irrefutably via the existence of universal language, any ToE must be bound by this equation.

2. The equation represents the limits and entropic costs associated with encoding the state of a system using a series of yes/no questions. In the case of the universe, the number of questions required to encoded all of its content is of course astronomical. Nonetheless, the limits and costs do apply.

3. We conjecture that all laws of physics are emergent from the embedding bounded by this equation. In the next sections, we will explicitly show the emergence of $F = ma$, special relativity, general relativity, the holographic principle, the Schrödinger equation and the Dirac equation. Naturally, deriving both the Dirac equation and general relativity from the same theory is highly suggestive of a ToE candidate. This result motivates the ToE claim made in this paper.

As the derivation of the laws of physics will be done from this single equation and with no appeal to physical observation, they are derived from pure reason. This would imply that all universes which embeds universal reason share the same laws of physics.

**Remark 3.52.** We of course do not claim to derive all of current physics in this paper (although some new physics is indeed derived). For example, a derivation of the standard model of particles is not yet provided. We hope to show sufficient promising evidence to encourage members of the scientific community to further contribute to the theory herein proposed. As a result the results are tentative pending a full derivation of the standard model.

4 **Thermodynamics**

In this section I will consider the following approximations to be equivalent.

\begin{align*}
(dx = dA = 0) \land (dV > 0) & \iff F \gg (kx + px^2 + ...) \\
(dx = dV = 0) \land (dA > 0) & \iff kx \gg (F + px^2 + ...) \\
(dA = dV = 0) \land (dx > 0) & \iff px^2 \gg (F + kx + ...) 
\end{align*}

4.1 **Energy**

**Theorem 4.4.** The law of conservation of energy.
Proof. Since we have recovered a thermodynamic partition function, we can define a conserved energy quantity. This is a direct consequence of taking the thermodynamic state equation of the partition function.

\[ dE = \frac{1}{\ln 2} TdS - 2\pi Sdf - Fdx - kdA - pdV - \ldots \]  \hspace{1cm} (4.5) \]

The equation for the proposed ToE was obtained with no appeal to physical observations. As a result, it follows that any theorem derived from it necessarily is an experimental prediction. Therefore, deriving the law of conservation of energy is a prediction on the universe - it must have a law of conservation of energy or it cannot embed universal reason!

Remark 4.6. Note that the law of conservation of energy (despite its name) does not imply that the total energy must remain the same. For example, the total energy could increase (or decrease) with time. But if it does it will vary according to a sum over the individual contributions of each changing thermodynamics observables. In other words, all changes in energy are accounted for.

4.2 Time

Theorem 4.7. The state equation (3.45) implies a halting entropy decreasing with time.

Proof. Posing \( dE = dx = dA = dV = 0 \), I obtain

\[ dE = \frac{1}{\ln 2} TdS - 2\pi Sdf \]

0 = \((\ln 2)^{-1}TdS + 2\pi t^{-2}Sdt\)

0 = \((\ln 2)^{-1}TdS + Pdt\)

\[ \implies \frac{dS}{dt} = -\frac{P}{T} \]  \hspace{1cm} (decreasing entropy)  \hspace{1cm} (4.8) \]

\[ df = -t^{-2}dt \]  \hspace{1cm} (4.9) \]

\[ P = 2\pi t^{-2}S \]  \hspace{1cm} (4.10) \]

Definition 4.11 (Halting entropy). The halting entropy is the entropy exclusively associated with the calculation of \( \Omega \) over time. It is the entropy obtained when \( dE = dx = dA = dV = 0 \).

As time increases the entropy from the calculation of \( \Omega \) decreases according to the term \(-\frac{P}{T}\). Why does it decrease over time? Consider that at the beginning of the calculation none of the bits of \( \Omega(t) \) are known hence the error rate is at its maximum. Each bit
with an unknown value contributes $k \ln 2$ to the entropy. As the calculation progresses and the error rate is diminished, then each additional and correct bit that has been calculated becomes fixed and their entropy contributions are reduced to 0.

As a result, an arrow of time connected to the non-computability of $\Omega$ can be attributed to the system as follows. A forward translation in time is associated with an increase in halting information. Furthermore, since each bit of $\Omega$ is algorithmically random, then the future, which can only be described with more bits of $\Omega$, is guaranteed to be non-computable. While the past, which holds less bits than the present is guaranteed to be computable from the present. This corresponds more closely to our human experience, as we can remember and even deduce the past based on present evidence, but cannot precisely know the future until it happens.

Furthermore, as the entropy of the valid bits of $\Omega$ is exactly 0, then it means that the past of the system is fixed and cannot be changed. Again, this more closely matches our human experience as we cannot change our past, so why would its halting entropy be anything other than 0?

### 4.3 Exfoliation

As an entropy decreasing with time would violate the second law of thermodynamics, I suggest that an entropic exfoliation to space occurs so as to make the second law hold. In this scenario the entropy reduction from the calculation of $\Omega$ is compensated by an increase in entropy associated with the exfoliation observables. Consider the following theorem.

**Theorem 4.12.** The state equation (3.45), the second law of thermodynamics and theorem (4.7) imply an entropic exfoliation to space.

**Proof.**

\[
\frac{dE}{dt} = (\ln 2)^{-1} T dS - 2\pi S f - F dx - kdA - pdV - ...
\]

state equation \hspace{1cm} (4.13)

posing $dE = 0$ \hspace{1cm} (4.14)

\[
0 = (\ln 2)^{-1} T dS - 2\pi S f - F dx - kdA - pdV - ...
\]

\[
0 = (\ln 2)^{-1} T dS + P dt - F dx - kdA - pdV - ...
\]

\[
0 = (\ln 2)^{-1} T dS + P dt - F dx - kdA - pdV - ...
\]

\[
0 = (\ln 2)^{-1} T dS + P dt - F dx - kdA - pdV - ...
\]

\[
\frac{dS}{dt} = (\ln 2) \left( \frac{F dx}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} - ... \right) - (\ln 2) \frac{P}{T}
\]

(exfoliation) \hspace{1cm} (4.16)

\[
0 = (\ln 2) \left( \frac{F dx}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} - ... \right) - (\ln 2) \frac{P}{T}
\]

\[
\frac{dS}{dt} = (\ln 2) \left( \frac{F dx}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} - ... \right) - (\ln 2) \frac{P}{T}
\]

**Definition 4.17 (Exfoliation entropy).** The exfoliation entropy is the entropy contribution by the term \((\ln 2) \left( \frac{F dx}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} + ... \right)\) to the entropy.
To investigate this result, let us look at three cases:

\[
\begin{align*}
\frac{1}{T} \left( \int \frac{dF}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} + \ldots \right) &< \frac{P}{T} \quad \Rightarrow \quad \frac{dS}{dt} < 0 \quad \text{decreasing entropy (4.18)} \\
\frac{1}{T} \left( \int \frac{dF}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} + \ldots \right) &= \frac{P}{T} \quad \Rightarrow \quad \frac{dS}{dt} = 0 \quad \text{constant entropy (4.19)} \\
\frac{1}{T} \left( \int \frac{dF}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} + \ldots \right) &> \frac{P}{T} \quad \Rightarrow \quad \frac{dS}{dt} > 0 \quad \text{increasing entropy (4.20)}
\end{align*}
\]

At (4.19) a shift occurs in the direction of the production of entropy over time. It is the point at which the exfoliation entropy overtakes and exceeds the reduction in halting entropy. The second law of thermodynamics which states that \( dS/dt \geq 0 \) will hold for (4.19) and (4.20), but will be violated for (4.18). In any case, if \((\ln 2) \frac{1}{T} \left( \int \frac{dF}{dt} + k \frac{dA}{dt} + p \frac{dV}{dt} + \ldots \right) > 0\) then the second law of thermodynamics applicable to the exfoliation observables will be observed.

This derivation more closely matches human experience. Indeed,

1. at the beginning of time the future of the system is un-actualized, hence the possibilities are endless. To reflect this, the halting entropy is at its maximum at \( t = 0 \), and the exfoliation entropy is equal to 0. This matches our current belief that the exfoliation entropy at the Big Bang is very low.

2. during the evolution the future becomes past which is "set in stone". As the past is "set in stone", the halting entropy of the bits defining it are equal to 0. This is because we "remember" or "observe" only one past. This reduction in halting entropy is offset by a growth in exfoliation entropy, which is related to the size and complexity of the space encoded by the exfoliation observables. This growth in space entropy obeys the second law of thermodynamic.

3. at the end of time there is no future. The value of \( \Omega \) as been calculated, and the full history of the system is now "set in stone". The halting entropy is 0 and the exfoliation entropy is at its maximum. This matches the hypothesis of the heat death.

Note that contrary to the halting entropy, the exfoliation entropy of an observer’s past does not need to be equal to 0 as multiple exfoliated micro-states could be compatible with an observer’s present. Indeed, as per the second law of thermodynamics, the observer sees a monotonically increasing exfoliation entropy.

How then do we understand this result from the perspective of algorithmic information theory? The exfoliation variable represents the entropy in the choice of available prefix-free encodings for the
programs of the UTM. When no bits of $\Omega$ are known, it doesn’t make sense to speak of the ways to encode this information as there is nothing to encode. Hence the entropy should be 0. As more bits of $\Omega$ are known then more ways to encode this information exist and the entropy associated with the possible encodings increases.

4.4 Holographic principles

**Theorem 4.21.** The state equation (3.45) implies a holographic principle in the area-dominant regime, where $x dx$ is the dominant contributor to the exfoliation entropy.

**Proof.** Posing $dE = df = dx = dV = 0$, I get

$$0 = (\ln 2)^{-1} T dS - k dA$$

(state equation) (4.22)

$$T dS = (\ln 2) k dA$$

(4.23)

$$\int T dS = (\ln 2) \int k dA$$

(4.24)

$$TS = (\ln 2) k A + C$$

(4.25)

$$\implies S \propto A$$

(holographic principle)

The laws of physics which will be derived from the area-dominant approximation $dA \neq 0$ will necessarily contain a holographic principle linking the entropy to the area enclosing the volume. However, the holographic principle need not necessarily hold at other entropic growth scales, for example, where the volumetric entropy $dV$ is dominant. Indeed, state equation (3.45) would appear to suggest three different scales, each having a “holographic principle” of a different dimensional size.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Dominant force</th>
<th>Approximation</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>$F$</td>
<td>$dA = dV = 0$</td>
<td>$S \propto L$ (4.26)</td>
</tr>
<tr>
<td>2D</td>
<td>$kx$</td>
<td>$dx = dV = 0$</td>
<td>$S \propto A$ (4.27)</td>
</tr>
<tr>
<td>3D</td>
<td>$px^2$</td>
<td>$dx = dA = 0$</td>
<td>$S \propto V$ (4.28)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

In this scenario, the universe would be dominated by the linear scale at short distances then would be overtaken by the area scale and finally by the volume scale. In the next section, I will show that special relativity and the law of inertia are derivable in the $S \propto L$ scale, that general relativity is derivable at the $S \propto A$ scale and will provide citations suggesting that dark energy might be derivable from the $S \propto V$ scale.
4.5 Spacetime

Theorem 4.29. The state equation (3.45) implies a relation between space and time.

Proof. Posing \( dE = dA = dV = dS = 0 \), I get

\[
\begin{align*}
0 &= -2\pi S df - F dx \\
0 &= 2\pi t^{-2} S dt - F dx \\
0 &= P dt - F dx \\
F dx &= P dt \\
dx &= \frac{P}{F} dt
\end{align*}
\]

(state equation)

The units of \( P/F \) are meters per second. This implies that any system described by (3.45) will have a characteristic power and force that relates time to space. Indeed, if the system is the universe, then by taking the characteristic Planck power and force we do recover the speed of light,

\[
P \frac{1}{F} = \frac{c^5}{G} \left( \frac{G}{c^4} \right) = c
\]

(4.34)

In lieu of an appeal to the Planck constant we are permitted to pose \( c := P/F \) as a definition and rewrite the result as

\[
dx = c dt
\]

(4.35)

which is the fundamental relation of special relativity and \( c \) is a constant connecting space to time.

4.6 Limiting relations

Theorem 4.36. The state equation (3.45) implies a maximum speed.

Proof. Posing \( dE = dA = dV = 0 \), I get

\[
\begin{align*}
0 &= (\ln 2)^{-1} T ds - 2\pi S df - F dx \\
0 &= (\ln 2)^{-1} T ds + 2\pi t^{-2} S dt - F dx \\
0 &= (\ln 2)^{-1} T ds + P dt - F dx \\
F dx - P dt &= (\ln 2)^{-1} T ds \\
\frac{dx}{dt} - \frac{P}{F} &= \frac{1}{\ln 2} \frac{T ds}{dt}
\end{align*}
\]

(state equation)

(maximum speed)

\[
df = -t^{-2} dt \\
P = 2\pi t^{-2} S
\]

(4.38)

(4.39)
To see why this implies a maximum speed, first consider that the units of this equation are meters per second. Second, consider the following three cases;

\[
\frac{dx}{dt} = \frac{P}{F} \implies \frac{dS}{dt} = 0 \quad (4.41)
\]

\[
\frac{dx}{dt} < \frac{P}{F} \implies \frac{dS}{dt} < 0 \quad (4.42)
\]

\[
\frac{dx}{dt} > \frac{P}{F} \implies \frac{dS}{dt} > 0 \quad (4.43)
\]

To prove that the speed \( \frac{P}{F} \) is a maximum it suffices to note the presence of a reversal of the second law of thermodynamics at the \( \frac{P}{F} \) barrier. Furthermore, as the irreversibility of the second law of thermodynamics is well established, it follows that the barrier cannot be overcome. A system evolving faster than \( c \) will experience a reversal of the second law compared to a system slower than \( c \) (and vice-versa), but neither will be able to cross \( c \) and flip its direction.

Please note that in standard physics the speed of light is accepted as an axiom and is not derived from more fundamental principles. Here the speed of light is a direct consequence of (3.45).

**Theorem 4.44.** The following relations each characterize a maximum quantity.

approx.

\[
\begin{align*}
\text{none} & \quad \frac{1}{\ln 2} \frac{T}{F} \frac{dS}{dt} = \frac{dE}{dt} - \frac{P}{F} & \text{maximum power (J/s)} \\
S \propto L & \quad \frac{1}{\ln 2} \frac{T}{F} \frac{dS}{dt} = \frac{dx}{dt} - \frac{P}{F} & \text{maximum speed (m/s)} \\
S \propto A & \quad \frac{1}{\ln 2} \frac{T}{k} \frac{dS}{dt} = \frac{dA}{dt} - \frac{P}{k} & \text{maximum viscosity (m}^2/\text{s)} \\
S \propto V & \quad \frac{1}{\ln 2} \frac{T}{p} \frac{dS}{dt} = \frac{dV}{dt} - \frac{P}{p} & \text{max. vol. flow rate (m}^3/\text{s)}
\end{align*}
\]

**Proof.** Each relation can easily be obtained from (3.45) by posing the other observables to 0. To prove that the quantities are a maximum, it suffices to notice that they each correspond to the point at which the second law of thermodynamics is reversed.

**Theorem 4.49.** When the values of the halting bits of \( Z_{\Omega} \) are not known to an observer, each bit of information has the following energy

\[
E = k_B T \ln 2
\]
Proof. Consider an observer not aware of the bit values of $Z$. To the observer, $Z$ looks like

$$Z_N = 0.\omega_1\omega_2\omega_3 \ldots \omega_N$$ (4.50)

There are $W = 2^N$ different possibilities, or micro-states. Since each bit has two possible values, the entropy of the system is $S = k_B \ln 2^N$. Adding or removing a bit changes the entropy and the energy by

$$\Delta S = S_{N+1} - S_N$$ (4.51)
$$= k_B \ln 2^{N+1} - k_B \ln 2^N$$ (4.52)
$$= k_B \ln 2$$ (4.53)
$$\Delta E = T \Delta S$$ (4.54)
$$= T (k_B \ln 2)$$ (4.55)

This result agrees with the Landauer limit.

**Theorem 4.56.** The partition function (3.17) implies a discrete halting entropy with a minimum step.

This theorem has a stronger requirement than the previous two theorems on maximum quantities. It is not enough to just prove an extremum value, but a minimum value that is also discrete.

Proof. To prove it, recall how $Z_{\Omega}$ calculates an estimation of $\Omega$ valid within a monotonically deceasing error rate $\zeta$. Knowing the precise value of $\zeta$ is equivalent to knowing $\Omega$ as $\Omega$ can simply be recovered by adding $\zeta$ to $Z_{\Omega}$. The implication is that the bits of $\zeta$ must also be non-computable. As one bit of $\zeta$ is enough to recover one bit of $\Omega$, it follows that, as $\zeta$ is the error rate, no bits of $\zeta$ can be known beyond the position of its first one-valued bit.

Second, let us define a non-divergent entropy for the system $\Omega = Z_{\Omega} + \zeta$. As the system is infinitely complex, its entropy will be convergent only for the first $N \in \mathbb{N}$ bits. The bits of $Z_{\Omega}$ have an entropy of 0, and the bits of $\zeta$ have an entropy of $N_\zeta k_B \ln 2$.

$$S = N_{Z_{\Omega}} k_B \ln 1 + N_\zeta k_B \ln 2$$ (4.57)
$$= N_\zeta k_B \ln 2$$ (4.58)

As a result, the smallest entropy of the system $S_0$ is $k_B \ln 2$. Furthermore, the entropy increases by steps of $k_B \ln 2$, as $N_\zeta$ is a natural number. This proves the theorem.
Theorem 4.59. Exfoliation observables and exfoliation conjugates are discrete as per the discrete halting entropy.

Proof. Consider the following relations connecting the halting entropy to the exfoliation variables, \( dS = (\ln 2)dE, \overline{dS} = (\ln 2)(Fdx + k dA + pdV + \ldots) \) and the halting variables \( dS = (\ln 2)2\pi df \) and \( dS = -(\ln 2)Pdt \). If \( dS \) is discrete, then it implies that these variables \( dE, S, F, k, p, df, P, dx, dA, dV \) and \( dt \) are also discrete.

Without loss of generality, consider the pair \( Sdf \). Since \( dS \) is discrete (4.56), then both \( S \) and \( df \) must be discrete. This reasoning can be applied to all exfoliation and halting variables. Why must both \( S \) and \( df \) be discrete? Suppose that either \( S \) or \( df \) are real. Then a real multiplied by a real is a real, and a real multiplied by a whole number is also a real. Hence for \( dS \) to be a whole number, both \( S \) and \( df \) must both be whole number.

Another way to see it is that the entropy of a real number can be infinite, but we are only allowed an entropy \( Nk_B \ln 2 \), hence none of these variables can be arbitrary real numbers. \( \square \)

The discretization of observables, notably the action observable, has been used since the early days of quantum physics to justify and explain it. Indeed, the explanation of the photoelectric effect and the black body radiation, two early successes of quantum physics were both explained via the discretization of what I refer in here as the exfoliation variables. Could the discretization of exfoliation variables imply a certain quantum character applicable to the partition function? We will return to this point in the section on quantum mechanics.

5 Relativity

5.1 Light cones as thermodynamic cycles

In this section, I look at the thermodynamic cycle of the system transiting through time and space starting at \( O \) to \( A \) to \( B \) and back to \( O \) as illustrated on Figure 1. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles such as those involving pressure and volume but also of relativistic light cones.

I pose that \( dE = 0 \) and the \( S \propto L \) approximation throughout the cycle.

\[
\frac{1}{\ln 2} TdS = Fdx - Pdt
\]

\text{(5.1)}
**O to A:** As $O$ is translated forward in time to $A$ while keeping the distance constant ($dx = 0$), the halting entropy must decrease over time to compensate.

\[
\left. \frac{1}{\ln 2} TdS = Fdx - Pdt \right|_{dx=0} \quad (5.2)
\]

\[
\Rightarrow \frac{dS}{dt} = -\left(\ln 2\right) \frac{P}{T} \quad (5.3)
\]

A forward translation in time causes the system to know more bits of $\Omega$. This reduces the halting entropy. Conversely, a backward translation in time causes the system to erase bits from its pool of information so as to increase its halting entropy. A backward translation in time is equivalent to erasing halting information about the system’s present.

**A to B:** As $A$ is translated forward in space to $B$ while keeping the time constant ($dt = 0$), the exfoliation entropy must increase over space to compensate.

\[
\left. \frac{1}{\ln 2} TdS = Fdx - Pdt \right|_{dt=0} \quad (5.4)
\]

\[
\Rightarrow \frac{dS}{dx} = \left(\ln 2\right) \frac{F}{T} \quad (5.5)
\]

I conclude that the further away from $A$ a region is, the higher its exfoliation entropy will be. Since $dt = 0$, no change in time is experienced.

**O to B:** As $O$ is translated forward both in time and in space to $B$ while keeping the entropy constant ($dS = 0$), the system has a velocity at the speed $c$.

\[
\left. \frac{1}{\ln 2} TdS = Fdx - Pdt \right|_{dS=0} \quad (5.6)
\]

\[
\Rightarrow \frac{dx}{dt} = \frac{P}{F} = c \quad (5.7)
\]

I conclude that an object travelling at speed $c$ is neither encouraged nor discouraged by entropy. However, the type of entropy changes. The rate $P/F$ is the rate of conversion of halting entropy to exfoliation entropy. At $O$ the system is comprised exclusively of halting entropy as its future is not yet determined. As the system evolves towards $B$, its halting entropy is decreased over time as the system replaces its future entropy with a singular past. Its exfoliation entropy however is increased over space to offset the reduction.
As a backward translation in time erases the most recently calculated bits of $\Omega$, I conclude that the system "forgets its future" during the backward translation.

5.2 Lorentz’s transformation

To recover the Lorentz’s factor $\gamma$, let us consider figure 2. Two observers start at the origin $S$ and travel in spacetime respectively to $O$ and $O'$. We regard $O'$ as traveling at speed $|v|$ in $O'$’s reference frame. From standard trigonometry, I derive the following values for the length of the segment:

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SO'$</td>
<td>$L$</td>
</tr>
<tr>
<td>$OO'$</td>
<td>$L \sin \theta$</td>
</tr>
<tr>
<td>$SO$</td>
<td>$L \cos \theta$</td>
</tr>
</tbody>
</table>

I start with the Pythagorean theorem and solve for $\cos \theta$.

$$|SO|^2 = |OO'|^2 + |SO'|^2$$  \hspace{1cm} (5.11)

$$L^2 = (L \sin \theta)^2 + (L \cos \theta)^2$$  \hspace{1cm} (5.12)

$$1 = (\sin \theta)^2 + (\cos \theta)^2$$  \hspace{1cm} (5.13)

$$\sqrt{1 - (\sin \theta)^2} = \cos \theta$$  \hspace{1cm} (5.14)

I consider that the distance between two observers moving at constant speed is simply $vt$. Hence, $|OO'| = vt$. Solving for $\sin \theta$, I obtain

$$|O_1O_2| = vt = L \sin \theta$$  \hspace{1cm} (5.15)

$$\Rightarrow \sin \theta = \frac{vt}{L}$$  \hspace{1cm} (5.16)

From equation (5.14) and (5.16), I get the reciprocal of the Lorentz factor,

$$\sqrt{1 - \left(\frac{vt}{L}\right)^2} = \cos \theta = \gamma^{-1}$$  \hspace{1cm} (5.17)

$$\Rightarrow \gamma = \frac{1}{\sqrt{1 - \left(\frac{vt}{L}\right)^2}}$$  \hspace{1cm} (5.18)
Finally, I consider that $L$ is the distance travelled in time by $O$ in its own reference frame. This is given via the relation $dx = cdt$. Hence $L = ct$. I obtain,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5.19)$$

which is the well known Lorentz factor and is the multiplication constant connecting $|SO|$ to $|SO'|$.

### 5.3 1D-Holographic principle (law of inertia)

First, let us derive a relation between $dS$ and $dN$.

**Theorem 5.20.** $dS = (\ln 2)k_BdN$

**Proof.**

$$S = Nk_B \ln(2) \quad \text{from 4.58} \quad (5.21)$$

$$\implies dS = (\ln 2)k_BdN \quad \text{(binary entropy)}$$

$\square$

Second, let us look at the implications of the first term, $Fdx$ in the $S \propto L$ regime.

**Theorem 5.22.** The $S \propto L$ scale implies the law of inertia, $F = ma$.

**Proof.** First, consider that as the entropy $S$, is related to the bits, then $dS = (\ln 2)k_BdN$ where $N$ is the number of bits. Second, I will derive the equation for an entropic force.

Under the approximation $F \gg (kx + px^2 + ...)$, the state equation is,

$$dE = \frac{1}{\ln 2}TdS - 2\pi Sdf - Fdx \quad \text{state equation approx.} \quad (5.23)$$

$$0 = (\ln 2)^{-1}TdS - Fdx \quad \text{posing } dE = df = 0 \quad (5.24)$$

$$Fdx = (\ln 2)^{-1}TdS \quad \text{add } Fdx \quad (5.25)$$

$$F = (\ln 2)^{-1}T \frac{dS}{dx} \quad \text{divide } dx \quad (5.26)$$

$$F = (\ln 2)^{-1}T \frac{(\ln 2)k_BdN}{dx} \quad \text{binary entropy} \quad (5.27)$$

$$F = Tk_B \frac{dN}{dx} \quad \text{entropic force} \quad (5.28)$$

As my goal is to recover $F = ma$, I must link $T$ to the acceleration. To do so I will use the Unruh temperature\textsuperscript{10} experienced by a body,

undergoing constant acceleration as suggested by Erik Verlinde in
11. The existence of a well defined temperature allows me to con-
clude that the system is described at thermodynamic equilibrium,

\[
F = \left( \frac{\hbar}{2\pi c k_B} \right) k_B \frac{dN}{dx} \quad \text{Unruh temperature (5.29)}
\]

\[
F = \left( \frac{1}{2\pi c} \frac{dN}{dx} \right) a \quad \text{clean up (5.30)}
\]

Finally, the equation \( F = ma \) can be recovered provided that the ratio \( dx/dN \) is the reduced Compton wavelength within one multiplication constant.

\[
\Rightarrow 2\pi \frac{dx}{dN} = \frac{\hbar}{mc} \quad (5.31)
\]

What does this means from the algorithmic perspective and why is \( (2\pi)dx/dN \) the reduced Compton wavelength? \( dx/dN \) is the ratio between the position of an object and the number of bits required to express such a position. It implies that each increment of an object’s position by its reduced Compton wavelength must use one additional bit of entropy. The algorithm to encode position used by the UTM is of the form \( x = n\lambda \), where \( n \) indicates the number of times its reduced Compton wavelength is repeated to reach its position. The entropy usage is optimized as the position of an object does not need to be specified more accurately than its reduced Compton wavelength.

5.4 Note on the Schwarzschild radius

As we have seen, the first term of the Taylor expansion is associated with the inertial mass as it implies \( F = ma \). We have also seen that the first term further implies a one dimensional version of the holographic principle. As a result, we would expect that the mass in the universe is bounded linearly. Is that the case?

Consider the Schwarzschild radius,

\[
R = \frac{2GM}{c^2} \quad (5.32)
\]

As we can see, the radius grows linearly with the mass. Hence the one dimensional holographic principle associated with the inertial mass holds.
5.5 2D-Holographic principle (General relativity)

In this section, I will show how the state equation (4.45) suggests that general relativity is an emergent entropic phenomenon attributable to the second term, \( kx dx \), of the Taylor expansion of \( d|p| \).

**Theorem 5.33.** The area-dominant regime implies general relativity.

**Proof.** My goal in this proof is to derive the Einstein field equation of general relativity starting from the holographic principle

\[
\frac{1}{\ln 2} T dS = k x dx
\]

\[
\Rightarrow S \propto A
\]

\[
\Rightarrow dE = \gamma dA
\]

This has indeed been done before in the literature, notably by Ted Jacobson in 12, then later (and differently) by Erik Verlinde in 13.

Furthermore, Christoph Schiller in 14 argues that a maximum power (4.45) implies the Field equation. Here, I will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

Jacobson, starting from \( dE = T dS \), first connects \( dE \) to an arbitrary coordinate system and energy flow rates,

\[
dE = \int T_{ab} k^a d\Sigma^b
\]

Here \( T_{ab} \) is an energy-momentum tensor, \( k \) is a killing vector field and \( d\Sigma \) the infinitesimal elements of the coordinate system. Jacobson then shows that, assuming that the holographic principle holds (and in here it does according to 4.21), the right part of (5.36) can be rewritten to

\[
dA = \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b
\]

where \( R_{ab} \) is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

\[
\int T_{ab} k^a d\Sigma^b = \gamma \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b
\]

which can only be satisfied if

\[
T_{ab} = \gamma \frac{c^2}{a} \left[ R_{ab} - \left( \frac{R}{2} + \Lambda \right) g_{ab} \right]
\]

Here, the full field equations of general relativity are recovered including the cosmological constant (as an integration constant).
5.6 Dark energy

Associating dark energy to a volumetric entropy has been suggested and discussed by other authors before\(^{15}\). Here, I suggest that dark energy provides the physical interpretation for the third term of the Taylor expansion.

Posing \(dE = df = dx = dA = 0\), I get

\[
0 = (\ln 2)^{-1}TdS - pdV \quad \text{state equation (5.41)}
\]

\[
TdS = (\ln 2)pdV \quad \text{dark energy (5.42)}
\]

To determine the value of the pressure \(p\) associated with volumetric entropy, we consider the case of an entropic force. In this case, the pressure relates to the force as

\[
F = -pA \\
\Rightarrow p = -\frac{F}{A} = -\frac{F}{4\pi x^2} (5.44)
\]

The sign of the force is negative because the force points in the direction of increased entropy which is oriented outward the enclosing area.

To determine \(x\), it suffices to notice that \((F + kx + px^2)dx\) encodes the informational content of the universe up to a boundary given by \(x\) which is common to all terms of the Taylor expansion. Physically it makes sense to connect this bound to the Hubble horizon as it defines an event horizon applicable to the "instantaneous system". As it is an event horizon, its temperature is given by De Sitter’s temperature and is constant at the horizon. Therefore, an entropic force is expected. To obtain the magnitude of the force, it suffices to calculate the entropic force as per the Bekenstein-Hawking entropy and the De Sitter temperature, both applicable to event horizons.

\[ dS = 2\pi \frac{k_B c^3}{\hbar} x \, dx \]  
Bekenstein-Hawking entropy (5.45)

\[ T = \frac{\hbar H}{k_B 2\pi} \]  
De Sitter temperature (5.46)

\[ F = T \frac{dS}{dx} \]  
entropic force (5.47)

\[ \implies F = \left( \frac{\hbar H}{k_B 2\pi} \right) \left( 2\pi \frac{k_B c^3}{\hbar} x \right) \]  
clean up (5.48)

\[ = \frac{c^3}{G} H x \]  
(5.49)

As \( x \) is the radius of the Hubble horizon \( x = c/H \), we obtain the final value of the force \( F = c^4/G \), the Planck force. Finally, the pressure is given by:

\[ F = \frac{c^4}{G} \]  
Planck force (5.50)

\[ \implies p = -\frac{F}{A} = -\left( \frac{c^4}{G} \right) \left( \frac{1}{4\pi(c/H)^2} \right) \]  
(5.51)

\[ p = -\frac{c^2H^2}{4\pi G} \]  
(negative pressure)

This is close to the current measured value for the negative pressure associated with dark energy. As we can see, the entropic derivation of dark energy first suggested by\(^{16}\) applies to the third term of the Taylor expansion.

6 Characteristic units

My goal in this section is to show how the definition of the Planck units naturally flows from the state equation (3.45). To do so, I must first obtain definitions for \( G \), \( c \) and \( \hbar \) by deriving from (3.45) known laws of physics which contain them. I start by obtaining the gravitational constant \( G \) from Newton’s law of gravitation.

**Theorem 6.1.** The gravitational constant \( G \) is defined as \( c^3L^2/\hbar \).

**Proof.** A derivation of Newton’s law of gravitation from the entropic perspective has been done before in \(^{17}\). I work in the area-dominant regime where \( dx = dV = 0 \). This regime contains the holographic principle and, as a result, the entropy of the system grows via \( A \), an


area law. I further consider that the entropy of this area law is given by bits exclusively occupying a small area $L^2$ on the surface. In this case the total number of bits on the surface is given by

$$N = \frac{4\pi x^2}{L^2} \quad (6.2)$$

The equipartition theorem applies to energy terms of the partition function which are quadratic. The term $kdA$ is $\frac{1}{2} kx^2$ in the partition function. As a result its average energy is $E = \frac{1}{2} N k_B T$ as per the equipartition theorem.

$$E = \frac{1}{2} \left( \frac{4\pi x^2}{L^2} \right) k_B T \quad (6.3)$$

$$\implies T = \frac{L^2 E}{2\pi k_B x^2} \quad (6.4)$$

I obtain a constant temperature throughout the system indicating that it is at thermodynamic equilibrium. As my goal is to recover the gravitational constant, I inject this temperature in the entropic force relation.

$$F = Tk_B \frac{dN}{dx} \quad \text{entropic force (5.28)} \quad (6.5)$$

$$F = \left( \frac{L^2}{2\pi k_B x^2} \right) k_B \frac{dN}{dx} \quad \text{derived temperature (6.6)}$$

I then replace the ratio $dx/dN$ by the reduced Compton wavelength.

$$F = \left( \frac{L^2}{2\pi k_B x^2} \right) k_B \left( 2\pi \frac{mc}{\hbar} \right) \quad (6.7)$$

$$F = \left( \frac{L^2 c}{\hbar} \right) \frac{Em}{x^2} \quad \text{clean up (6.8)}$$

I then convert $E$ to its rest mass via $E = mc^2$.

$$F = \left( \frac{L^2 c^3}{\hbar} \right) \frac{Mm}{x^2} \quad (6.9)$$

I obtain the Newton’s law of gravitation along with a definition for $G$.

$$F = GMm \frac{1}{x^2} \quad (6.10)$$

$$\implies G = \frac{L^2 c^3}{\hbar} \quad (6.11)$$
which further implies that

\[ L = \sqrt{\frac{\hbar G}{c^5}} \]  

(Planck’s length)

\[ \square \]

**Theorem 6.12.** The speed of light \( c \) is defined by \( P/F \).

*Proof.* I refer to the proof for theorem 4.36 where \( P/F \) is a characteristic speed associated with an inversion in the direction of the second law of thermodynamics. Then, under the principle that the second law is irreversible, the speed \( P/F \) is a boundary and defines \( c \).  

\[ \square \]

**Theorem 6.13.** The action \( S \) is defined by \( \hbar \).

*Proof.* Posing \( dS = dx = dA = dV = 0 \), I get

\[ dE = -2\pi Sdf \]  

(state equation (6.14))

Switching to the angular frequency,

\[ dE = -Sd\omega \]  

\[ df = d\omega/(2\pi) \]  

(6.15)

\[ \int dE = -\int Sd\omega \]  

(6.16)

\[ E = -S\omega + C \]  

(6.17)

Posing \( C = 0 \) and flipping the axis for \( \omega \), this is the photon angular-frequency to energy relation \( E = \hbar \omega \implies S = \hbar \).  

\[ \square \]

I have now obtained a definition for three of the fundamental constants.

\[ \hbar = S \quad c = \frac{P}{F} \quad G = \frac{L^2 c^3}{\hbar} \]  

(6.18)

I can now define characteristic units applicable to the UTM,

\[ G = \frac{L^2 c^3}{\hbar} \implies L = \sqrt{\frac{\hbar G}{c^5}} \quad \text{(Planck’s length)} \]

\[ t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}} \quad \text{(Planck’s time)} \]

\[ E = S/t \implies E = \sqrt{\frac{\hbar c^5}{G}} \quad \text{(Planck’s energy)} \]

\[ P = t^{-2}S = \frac{c^5}{G} \quad \text{(Planck’s power)} \]

\[ \frac{P}{F} = c \implies F = \frac{c^4}{G} \quad \text{(Planck’s force)} \]

which agrees with the physical Planck units.
7 Quantum mechanics

7.1 Universal Brownian motion on conjugate variables

The conjugate variables of the partition function are $dx$ and $dt$. These conjugate variables are related to the entropy of the partition function which represents halting programs. It is known that the halting probability is algorithmically random and normal. As a result we expect it to have an equal number of 0 and 1 for its digits (for its infinite expansion). Furthermore, a thermodynamic system at equilibrium and with a non-zero entropy will continuously switch around its available micro-states. For the partition function, the available micro-states are the different possible bits of $\Omega$ along with the available prefix-free code.

The conjugate variables $dx$ and $dt$ respectively represent the program length and the halting time for a program $p$. Therefore, as non-halting programs are missing from the sum, the $dx$ cannot define a position for lengths and $dt$ cannot define a time for missing programs. As these bits encodes the position via $dx$ and time via $dt$, a small random walk is expected in time and space as the micro-states switch around.

In a previous section we have shown that $dx$ implies $F = ma$. Here, we will show that a random walk over $dx$ will transform $F = ma$ into the Schrödinger equation. Furthermore, we have previously shown that $dx$ and $dt$ implies special relativity. Here, we will show that a random walk over $dx$ and $dt$ will transform special relativity to the Dirac equation.

Please note that we are not suggesting a pilot-wave type of interpretation where the particle would exist punctually but would be undergoing Brownian motion until a measurement is made. Rather, we suggest that any positional or time information undergoes a "Dirac equation-like diffusion" so to make positional or time information perishable over time. We can imagine placing a mark at a position in space. After a certain time, the random walk will diffuse the position of the marker at any number of possible locations until its actual position is measured again. The future position of the marker evolves according to the Dirac equation.

Instead of being punctual, the marker could be continuous and weighted. The same diffusion-like behaviour will be observed. The Brownian behaviour applies generally to the axis system.

7.2 Schrödinger equation

In a previous section, we have used the program-size to entropy relation $TdS = Fdx$ to recover $F = ma$. In this section we use the
same relation but we extend it with the non-computable features of the UTM. Doing so will allow us to recover the Schrödinger equation.

We recall that a UTM encodes position via the $dx$ conjugate associated with program lengths. As a result, the UTM can only express a position if the program with the corresponding size is part of its partition function (e.i. it halts). In this section, we will argue that the missing non-halting programs are responsible for a universal Brownian motion in space applicable to the $dx$ variable. This will be enough to recover the Schrödinger’s equation.

**Theorem 7.1.** A position described with missing program-sizes will evolve in time according to Schrödinger’s equation.

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x,t) \right] \psi(x,t)$$

The proof is slightly more involved than the preceding theorems. First, here is a sketch of the proof.

1. We will show that non-halting programs leave holes in space such that a position cannot be expressed.
2. We will show that these holes are causing a Brownian motion of the encoded position.
3. We will derive its diffusion coefficient to be $\hbar/(2m)$.
4. We will consider that the presence of any external field is experienced as acceleration via $F = ma$.
5. Using the well known Brownian motion equations of Langevin, we show that the above reproduces Schrödinger’s equation exactly.

**Lemma 7.2.** A spatial encoding based on programs will leave holes in space corresponding to non-halting programs.

*Proof.* We recall the general halting partition, where $df = 0$ and $F \gg kx + px^2 + \ldots$.

$$Z = \sum_x e^{-\beta (ln 2) [E(x) + Fx]} \quad (7.3)$$

We have also seen that the conjugate $x$ denotes program lengths. However, not all programs halt hence some lengths are missing from the sum. These missing programs are holes in space the position of which cannot be expressed by the UTM’s positional algorithm. Since $\Omega$ is a normal number, we can expect the position of these holes to be algorithmically random.
Lemma 7.4. A particle in space will experience Brownian motion due to the holes.

Proof. We will calculate the average displacement $\Delta x$ of a particle subjected to entropic positioning and space holes. Since $Z$ is a normal number, we conclude that half of the program’s lengths are available to describe position and half are not. Therefore, to describe a particle at position $x$, there is a 50% chance there is a halting program available to express it. And in the case where there is no program at exactly $x$, then there is a 50% chance that there will be one at position $x + 1$, and so on. In other words, a particle at $x$ has 50% chance of being at $x$, 25% chance of being at $x + 1$, 12.5% chance of being at $x + 2$, etc. Expressed as a sum, we obtain

$$\Delta x = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$$

(7.5)

$$= \sum_{i=0}^{\infty} \frac{i}{2^{i+1}}$$

(7.6)

$$= 1$$

(7.7)

On average, as it moves through space, a position will shift by $\Delta x = 1$ at each iteration of the Brownian motion.

Lemma 7.8. The diffusion coefficient of the described Brownian motion is

$$D = \frac{\hbar}{2m}$$

Proof. It is well known that in general the diffusion coefficient of Brownian motion is given by

$$D = \frac{l^2}{2\tau}$$

(7.9)

where $l$ is the length of the random step and $\tau$ is the frequency of the occurrence of the steps. As we have previously connected the reduced Compton wavelength to $F = ma$ taking the role of the system’s characteristic length associated with positional encoding for a mass of bits, it makes sense to use it here as well. We get a scaling factor of

$$\lambda = \frac{\hbar}{mc}$$

(7.10)

Since entropic positioning can only express position as multiples of $\lambda$, we take it as the Brownian step of length $l$. The diffusion coefficient becomes

$$D = \left( \frac{\hbar}{mc} \right)^2 \frac{1}{2\tau}$$

(7.11)
This leaves us with the need to define $\tau$. For $\tau$, we take the characteristic frequency of the wave $E = \hbar \omega$. Solving for $\tau = 1/\omega$, we obtain

$$\omega = \frac{E}{\hbar} \quad (7.12)$$

$$\omega^{-1} = \frac{\hbar}{E} = \tau \quad (7.13)$$

Replacing $\tau$ in the equation for $D$, we obtain

$$D = \frac{\hbar^2}{m^2 c^2} \left( \frac{E}{2\hbar} \right) \quad (7.14)$$

Using $E = mc^2$, and reducing the constants, we obtain our final expression of $D$,

$$D = \frac{\hbar^2}{m^2 c^2} \left( \frac{mc^2}{2\hbar} \right) \quad (7.15)$$

$$= \frac{\hbar}{2m} \quad (7.16)$$

**Lemma 7.17.** The Langevin equations for Brownian motion with a diffusion coefficient of $\hbar/(2m)$ and an external inertial field experienced as $F = ma$ reproduces Schrödinger’s equation.

**Proof.** We recall the Langevin equation,

$$d [x(t)] = v(t) dt \quad (7.18)$$

$$d [v(t)] = -\frac{\gamma}{m} v(t) dt + \frac{1}{m} W(t) dt \quad (7.19)$$

where $W(t)$ is a random force and a stochastic variable giving the effect of a background noise to the motion of the particle.

From $F = ma$ and replacing the acceleration $d[v(t)]/dt$ with $F/m$, Edward Nelson is able to show that the Langevin equation becomes,

$$\nabla \left( \frac{1}{2} \vec{u}^2 + D \nabla \cdot \vec{u} \right) = \frac{1}{m} \nabla V \quad (7.20)$$

where $D$ is the diffusion coefficient of $\hbar/(2m)$ obtained in lemma 7.8, where $\vec{F} = -\nabla V$, where $\vec{u} = v \nabla \ln \rho$ and $\rho$ is the probability density of $x(t)$. For brevity, the proof of 7.20 is omitted here but can be reviewed in Nelson’s paper. Eliminating the gradients on each side and simplifying the constants, we obtain

$$\frac{m}{2} \vec{u}^2 + \frac{\hbar}{2} \nabla \cdot \vec{u} = V - E \quad (7.21)$$
where $E$ is the arbitrary integration constant. This equation in nonlinear because of the term $\vec{u}^2$ but it can be made linear by a change of dependant variable. To make it linear, let us pose

$$\vec{u} = \frac{\hbar}{m} \frac{1}{\psi} \nabla \psi$$

and replace it into equation 7.21, we obtain

$$V - E = \frac{m}{2} \left( \frac{\hbar}{m} \frac{1}{\psi} \nabla \psi \right)^2 + \frac{\hbar^2}{2m} \nabla \cdot \left( \frac{\hbar}{m} \frac{1}{\psi} \nabla \psi \right)$$

$$= \frac{\hbar^2}{2m} \frac{1}{\psi^{\prime 2}} (\nabla \psi \cdot \nabla \psi) + \frac{\hbar^2}{2m} \left[ \nabla \cdot \left( \frac{1}{\psi} \nabla \psi \right) \right]$$

$$= \frac{\hbar^2}{2m} \frac{1}{\psi^{\prime 2}} (\nabla \psi \cdot \nabla \psi) + \frac{\hbar^2}{2m} \left[ \frac{\psi \nabla \cdot \nabla \psi - \nabla \psi \cdot \nabla \psi}{\psi^2} \right]$$

(Identity)

$$= \frac{\hbar^2}{2m} \frac{1}{\psi^{\prime 2}} (\nabla \psi \cdot \nabla \psi) + \frac{\hbar^2}{2m} \left[ \frac{1}{\psi} \nabla \cdot \nabla \psi - \frac{1}{\psi^2} (\nabla \psi \cdot \nabla \psi) \right]$$

(7.25)

The first and the last terms cancel each other.

$$\frac{\hbar^2}{2m} \frac{1}{\psi^{\prime 2}} \nabla^2 \psi = V - E$$

(7.26)

Finally, it simplifies to

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V - E \right] \psi = 0$$

(7.27)

which is the time independent Schrödinger’s equation.

We are now ready to derive the time dependent Schrödinger equation and prove theorem 7.1.

**Proof.** We use the same proof used by Edward Nelson in the same paper. Starting from the time dependent Schrödinger equation, we show that a replacement of $\psi = e^{R+iS}$ leads to the Langevin equation of Brownian motion.

$$\frac{\partial \psi}{\partial t} = i \frac{\hbar}{2m} \nabla^2 \psi - i \frac{1}{\hbar} V \psi$$

(7.28)

Replacing $\psi$ with $e^{R+iS}$, we obtain

$$\frac{\partial}{\partial t} \left( e^{R+iS} \right) = i \frac{\hbar}{2m} \nabla^2 \left( e^{R+iS} \right) - i \frac{1}{\hbar} V \left( e^{R+iS} \right)$$

(7.29)

Taking the derivatives and the gradients, we obtain

$$\left[ \frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} \right] \left( e^{R+iS} \right) = \frac{i \hbar}{2m} \left[ \nabla^2 R + i \nabla^2 S + (\nabla (R + iS))^2 \right] \left( e^{R+iS} \right) - i \frac{1}{\hbar} V \left( e^{R+iS} \right)$$

(7.30)
Eliminating $e^{R+iS}$ from each side and simplifying, we obtain

\[
\frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} = \frac{i\hbar}{2m} \left[ \nabla^2 R + i\nabla^2 S + (\nabla(R + iS))^2 \right] - i\frac{1}{\hbar} V
\]  

(eliminating $e^{R+iS}$)

\[
\frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} = \frac{i\hbar}{2m} \left[ \nabla^2 R + i\nabla^2 S + (\nabla R)^2 + 2i\nabla R \cdot \nabla S - (\nabla S)^2 \right] - i\frac{1}{\hbar} V
\]  

(taking the product)

\[
\frac{\partial R}{\partial t} + i \frac{\partial S}{\partial t} = \frac{\hbar}{2m} \left[ i\nabla^2 R - \nabla^2 S + i(\nabla R)^2 - 2\nabla R \cdot \nabla S - i(\nabla S)^2 \right] - i\frac{1}{\hbar} V
\]  

(distributing the $i$)

We obtain two equations by separating the real and the imaginary parts

\[
\frac{\partial R}{\partial t} = \frac{\hbar}{2m} \left[ -\nabla^2 S - 2\nabla R \cdot \nabla S \right] \tag{7.31}
\]

\[
\frac{\partial S}{\partial t} = \frac{\hbar}{2m} \left[ \nabla^2 R + (\nabla R)^2 - (\nabla S)^2 \right] - \frac{1}{\hbar} V \tag{7.32}
\]

This is equivalent to the Langevin equations with some replacements

\[
\frac{\partial \vec{u}}{\partial t} = -\frac{\hbar}{2m} \nabla(\nabla \cdot \vec{v}) - \nabla(\vec{v} \cdot \vec{u}) \tag{7.33}
\]

\[
\frac{\partial \vec{v}}{\partial t} = \frac{\hbar}{2m} \nabla(\nabla \cdot \vec{u}) + \frac{1}{2} \nabla(\vec{u}^2) - \frac{1}{2} \nabla(\vec{v}^2) - \frac{1}{m} \nabla V \tag{7.34}
\]

Lemma 7.35. Equation 7.31 with the replacements $\nabla R = (m/\hbar) \vec{u}$ and $\nabla S = (m/\hbar) \vec{v}$ produces 7.33

Proof.

\[
\frac{\partial R}{\partial t} = \frac{\hbar}{2m} \left[ -\nabla^2 S - 2\nabla R \cdot \nabla S \right] \tag{equation 7.31}
\]

(taking the gradient)

\[
\nabla \frac{\partial R}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\nabla^2 S - 2\nabla R \cdot \nabla S \right] \right)
\]

(7.36)

\[
\frac{\partial \nabla R}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\nabla \cdot \nabla S - 2\nabla R \cdot \nabla S \right] \right)
\]

(replacing $\nabla R$ and $\nabla S$)

\[
\frac{m \partial \vec{u}}{\hbar} = \nabla \left( \frac{\hbar}{2m} \left[ -\nabla \cdot \left( \frac{m}{\hbar} \vec{v} \right) - 2 \left( \frac{m}{\hbar} \vec{v} \cdot \left( \frac{m}{\hbar} \vec{v} \right) \right) \right] \right)
\]

(eliminating $m/\hbar$)

\[
\frac{\partial \vec{u}}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ -\nabla \cdot \vec{v} - 2 \frac{m}{\hbar} \vec{u} \cdot \vec{v} \right] \right)
\]

(equation 7.33)

\[
\frac{\partial \vec{u}}{\partial t} = -\frac{\hbar}{2m} \nabla(\nabla \cdot \vec{v}) - \nabla(\vec{u} \cdot \vec{v})
\]

□

Lemma 7.37. Equation 7.32 with the replacements $\nabla R = (m/\hbar) \vec{u}$ and $\nabla S = (m/\hbar) \vec{v}$ produces 7.34
Proof.

\[
\frac{\partial S}{\partial t} = \frac{\hbar}{2m} \left[ \nabla^2 R + (\nabla R)^2 - (\nabla S)^2 \right] - \frac{1}{\hbar} V \tag{equation 7.32}
\]

\[
\nabla \frac{\partial S}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ \nabla \cdot \nabla R + (\nabla R)^2 - (\nabla S)^2 \right] \right) - \frac{1}{\hbar} \nabla V \tag{taking the gradient}
\]

\[
m \frac{\partial \vec{\nu}}{\partial t} = \nabla \left( \frac{\hbar}{2m} \left[ \nabla \cdot \left( \frac{m}{\hbar} \vec{u} \right) + \left( \frac{m}{\hbar} \vec{u} \right)^2 - \left( \frac{m}{\hbar} \vec{v} \right)^2 \right] \right) - \frac{1}{\hbar} \nabla V \tag{replacing \nabla R and \nabla S}
\]

\[
\frac{\partial \vec{\nu}}{\partial t} = \frac{\hbar}{2m} \nabla (\nabla \cdot \vec{u}) + \frac{1}{2} \nabla (u^2) - \frac{1}{2} \nabla (v^2) - \frac{1}{m} \nabla V \tag{equation 7.34}
\]

This completes the proof of theorem 7.1.

7.3 Dirac equation

In a previous section, we have used \( TdS = Fdx \) to recover \( F = ma \). In another section, we have used \( TdS = Pdt + Fdx \) to recover special relativity. We have then used a random walk on \( dx \) to recover the Schrödinger equation which is the quantum analogue to \( F = ma \). Of course, the natural question to ask is, will using \( TdS = Pdt + Fdx \) and applying a random walk to both \( dt \) and \( dx \) be enough to recover the Dirac equation, the quantum analogue to special relativity? The answer is yes!

In this section, we will see that applying a random walk to both the \( dt \) and the \( dx \) variables is enough to recover the Dirac equation for relativistic quantum mechanics. Let us begin by answering why would there be a random walk on \( dt \).

First we consider that, as is the case with program length, program runtime varies from one UTM to the next. Programs that are difficult to solve on one UTM are likely to be difficult to solve on other UTMs. For example the travelling salesman problem is hard to solve on every UTM. The runtime of these programs will be randomly distributed and centred around a mean runtime.

Second, we consider an analogous argument to the one used to justify a random walk on \( dx \), but applied to \( dt \). On some UTM a program of size \( x \) might have halted and on others it might not have. Therefore a particle can be defined to be at a time \( t \) only if a program halting at time \( t \) is in the partition function. If there is no such available halting program at time \( t \), then the particle will be a time \( t \pm \Delta t \), the runtime of the next available halting program. Since the halting problem is algorithmically random and non-computable, we consider this behaviour as a random walk in time.
A connection between a random walk in time and space and the telegraphic equation has been linked to the Dirac equation before. D. G. C. McKeon and G. N. Ord proposes a random walk model in space and in time. Starting from the equation for a random walk in space, we have

\[ P_\pm(x, t + \Delta t) = (1 - a \Delta t) P_\pm(x \mp \Delta x, t) + a \Delta t F_\pm(x \pm \Delta x, t + \Delta t) \]  \hspace{1cm} (7.38)

then, D. G. C. McKeon and G. N. Ord extend this equation with a random walk in time. They obtain

\[ F_\pm(x, t) = (1 - a_L \Delta t - a_R \Delta t) F_\pm(x \mp \Delta x, t - \Delta t) + a_{L,R} \Delta t B_\pm(x \mp \Delta x, t + \Delta t) + a_{R,L} \Delta t F_\pm(x \mp \Delta x, t - \Delta t) \]  \hspace{1cm} (7.39)

where \( F_\pm(x, t) \) is the probability distribution to go forward in time and \( B_\pm(x, t) \), backward in time. They then introduce a causality condition such that \( F_\pm(x, t) \) and \( B_\pm(x, t) \) only depends on probabilities from the past.

\[ F_\pm(x, t) = B_\mp(x \pm \Delta x, t + \Delta t) \]  \hspace{1cm} (7.40)

From equation 7.3 and 7.40, they get

\[ B_\pm(x, t) = (1 - a_L \Delta t - a_R \Delta t) B_\pm(x \mp \Delta x, t + \Delta t) + a_{L,R} \Delta t B_\mp(x \pm \Delta x, t + \Delta t) + a_{R,L} \Delta t F_\pm(x \mp \Delta x, t - \Delta t) \]  \hspace{1cm} (7.41)

In the limit \( \Delta x, \Delta t \to 0 \) and with \( \Delta x = v \Delta t \), they get,

\[ \pm v \frac{\partial F_\pm}{\partial x} + \frac{\partial F_\pm}{\partial t} = a_{L,R} (-F_\pm + B_\pm) + a_{R,L} (-F_\pm + F_\mp) \]  \hspace{1cm} (7.42)

\[ \pm v \frac{\partial B_\mp}{\partial x} + \frac{\partial B_\mp}{\partial t} = a_{L,R} (-B_\mp + F_\mp) + a_{R,L} (-B_\mp + B_\pm) \]  \hspace{1cm} (7.43)

Posing these changes of variables,

\[ A_\pm = (F_\pm - B_\mp) \exp[(a_L + a_R)t] \]  \hspace{1cm} (7.44)

\[ \lambda := -a_L + a_R \]  \hspace{1cm} (7.45)

then 7.43 becomes

\[ v \frac{\partial A_\pm}{\partial x} \pm \frac{\partial A_\pm}{\partial t} = \lambda A_\mp \]  \hspace{1cm} (7.46)

Finally, they pose \( v = c, \lambda = mc^2/h \) and \( \psi = F(A_+, A_-) \), they get

\[ ih \frac{\partial \psi}{\partial t} = mc^2 c \psi - ihelmz \frac{\partial \psi}{\partial x} \]  \hspace{1cm} (7.47)

which is the Dirac equation in 1+1 spacetime.
8 Domains of physics

A proof pattern comes out of the results we obtained so far. Consequently, we can now map current and potential future research regarding this theory on a map corresponding to the observables of the partition function (Table 5).

<table>
<thead>
<tr>
<th>$dx = 0$</th>
<th>$Fdx$</th>
<th>$kxdx$</th>
<th>$px^2dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dt = 0$</td>
<td>Halting information</td>
<td>$F = ma$</td>
<td>General Relativity</td>
</tr>
<tr>
<td>$dt$</td>
<td>Entropic time</td>
<td>Special relativity</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$d\xi(x) = 0$</th>
<th>$Fd\xi(x)$</th>
<th>$kxd\xi(x)$</th>
<th>$px^2d\xi(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\xi(t) = 0$</td>
<td>Halting information</td>
<td>Schrödinger equation</td>
<td>Quantum general relativity*</td>
</tr>
<tr>
<td>$d\xi(t)$</td>
<td>Quantum time*</td>
<td>Dirac equation</td>
<td>-</td>
</tr>
</tbody>
</table>

1. Halting information: When $dt = dx = 0$, the only observable of the partition function is the halting event function representing the halting information of the system. As the $dx$ conjugate is set to zero, the system cannot encode the halting information using prefix-free codes, hence the partition function does not converge.

2. $F = ma$: When the $dx$ term is added to the partition function, an arbitrary prefix-free code can now be selected to encode the halting information in a convergent partition function. Since the prefix-free code is arbitrary, it helps to study it as a Taylor expansion. The first term of this expansion allows us to recover the law of inertia.

3. General relativity: The second term of the Taylor expansion allows us to recover Field equations of general relativity.

4. Schrödinger equation: An entropic system at equilibrium will constantly switch around the various available micro-states compatible with the macroscopic observables. Modelling this switch as a random walk on the conjugate variable yields the Schrödinger equation.

5. Dirac equation: Further applying the random walk model to the $dt$ variable yields the Dirac equation.

6. Entropic time: Here we find a new definition of time based on the progressive reduction of future possibilities and the creation of a singular past. This explains the arrow of time.

We can continue this pattern in Table 5 to identify possible new domains of physics. Indeed, applying the random walk model to the
domains, responsible for producing general relativity, "should" extend general relativity to the quantum domain. Again if the pattern holds, the same "should" be true of dark energy.
9 Discussion & Summary

I will now unpack the derivation and discuss it point by point.

9.1 Universal language

The first step is the derivation of a universal language sufficiently expressive to describe any axioms, any proof system and any theorem. The universal language is constructed out of first order sentences. Each sentence is provable by a direct construction of the object the existence of which it claims. Therefore the universal language is proven to exist irrefutably. As a result, any theorem of universal language is also irrefutable.

Remark 9.1. Note that theorems that do require axioms to prove are not direct theorems of universal language. However, the theorem including the axioms it requires, in the form of $k \vdash t$, are theorems of universal language. As this applies to all $k$ and all $t$, the language is universal.

9.2 Universal reason

The second step is the derivation is to show that universal language embeds universal reason.

Definition 9.2 (The existence of universal reason).

$$\forall k \forall t [k \vdash t \implies \text{ToE} \vdash (k \vdash t)]$$

Why this sentence? Let us consider all set of axioms and their corresponding theorems and we call that: universal reason. We know from the existence of $\Omega$-sentences that universal reason exists within universal language. As a result, the existence of universal reason within the universe is irrefutable. Therefore, it must be the case that universal reason is recoverable from the ToE, as the ToE presumably explains everything in the universe.

The theorem of universal reason states that for all sets of axioms $k$ and all theorems $t$, if $k$ proves $t$, then it implies that the theory of everything proves that $k$ proves $t$. The theorem of universal reason is a necessary consequence that universal reason is done in the universe.

As a result, any universe which contains universal reason must be bound by its existence. This is a statement of pure reason, perhaps the only one, which bounds what the universe can be. Since this statement is enough to construct a ToE and recover the laws of physics, it follows that the laws of physics are logically implied by the existence of universal reason.

The conjecture is that all laws of physics are emergent from the existence of universal reason. Indeed, it was explicitly shown for
inertia, special relativity, general relativity, the holographic principle, 
the Schrödinger equation and the Dirac equation.

9.3 Compressing universal knowledge

To make the laws of physics come out, it helps to rearrange universal 
reason into an equation that we can work with. To do so, I have 
shown how to convert universal reason into a sum which produces a 
binary real number. As they both have the same cardinality, this can 
be easily done by associating each sentence $k \vdash t$ to a natural num-
ber. Then, the sum translates universal knowledge to a binary real 
sentence according to:

$$
\Omega = \sum_{n=1}^{\infty} 2^{-E(n) - n} \quad \text{where } E(n) = \begin{cases} 
0 & [k \vdash t]_n \text{ is true} \\
\infty & \text{otherwise}
\end{cases} \quad (9.4)
$$

This is sufficient to connect universal reason to the halting proba-
bility of a prefix-free universal Turing machine.

9.4 Halting probability

As per algorithmic information theory, there exists many prefix-free 
code available to encode programs. As a result, we generalize the 
sum to account for any prefix-free code and not just natural numbers. 
We obtain,

$$
\Omega = \sum_{p} 2^{E(p) - |p|} \quad \text{where } E(p) = \begin{cases} 
0 & p \text{ halts} \\
\infty & \text{otherwise}
\end{cases} \quad (9.5)
$$

The second modification is to add the program-action observable 
conjugated with the program-frequency to the sum. Why add this? 
The problem is that it is not possible to increase our knowledge of $\Omega$ 
without it as the sum does not converges towards $\Omega$. Since $\Omega$ is non-
computable, a system without it would be limited to just a handful 
of yes/no questions until it hits the first non-halting program, then 
the error rate will seize to decrease. Only at $t \to \infty$ will the system 
discontinuously yield $\Omega$.

Adding the action to frequency pair allows a smooth conver-
gence towards $\Omega$ as time passes by. The system can therefore grow 
its knowledge of $\Omega$ to a level enough to describe any system of any 
complexity using yes/no questions. In the case of the universe, the 
number of yes/no question to encode it all is of course astronomical.

The $2\pi Sf$ variable is enough to define an entropic time in which 
the past is singular and the future is non-computable. This is the 
solution to the well known problem of the arrow of time.
The final sum obtained is

\[ Z_\Omega = \sum_p e^{-\beta (\ln 2)[E(p) + 2\pi S_f + D|p|]} \quad (9.6) \]

9.5 The universe

\( \Omega \) represents all information within the universe encoded as a series of yes/no questions. The questions are of the form "Is this sentence a theorem of the universe, yes or no?". With the entropic time observable, the universe is defined by the knowledge it contains at a given time which is associated to an estimation of the value of \( \Omega \). As time moves forwards, more knowledge of \( \Omega \) is obtained and the estimation is improved. The highly entropic future is represented by the unknown bits of \( \Omega \). As these bits are calculated, the future is replaced by a singular past and the entropy decreases. Furthermore, as the past cannot be changed, the entropy of the bits of \( \Omega \) defining it are 0. The second observable, entropic space, acts as an entropy sink to counteract the entropy reduction association with reducing the possible futures as time is translated forward.

A physical representation of the partition function can be obtained by taking the Taylor expansion of \( D|p| \) and replacing \( p \) with \( x \). We obtain this partition function

\[ Z_\Omega = \sum_x e^{-\beta (\ln 2)[E(x) + 2\pi S_f + F_x + \frac{1}{2} k x^2 + \frac{1}{3} p x^3 + \ldots]} \quad (9.7) \]

Which is given the interpretation as the ToE.

9.6 Classical & quantum physics

The equation for the ToE integrates physics within a very simply conceptual framework. First, classical physics is directly recoverable simply by solving the equation of state for the system under different regimes (e.g. for example \( df = 0 \)). Indeed, the law of inertia, special relativity and general relativity have been recovered in such a manner. Quantum physics is recovered simply by considering that the iteration over the possible micro-states are responsible for a universal brownian motion over the \( dx \) and \( dt \) variables.

We do not suggest a pilot-wave interpretation of quantum mechanics. Rather, the Schrödinger and the Dirac equation should be interpreted as diffusion effect over time for any positional or time markers. This applies to anything related to position such as punctual markers or probability densities, etc.
10 Conclusion

We note an affinity between a system which embeds universal reason and the laws of physics. The affinity occurs when we consider a UTM calculating its $\Omega$ number in a manner so as to maximize the entropy throughout the calculation. When the entropy is maximized, the halting probability becomes a Gibbs ensemble. As a result, additional program observables can be added to the halting probability while preserving its connection to $\Omega$. The laws of physics come out when a single observable is added, $2\pi S_f$, as it is enough to make the calculation converge towards $\Omega$ over time.

Understanding physics from the perspective of an entropic UTM holds several conceptual advantages. For one, we can now define a non-computable future with a computable singular past whose halting entropy is 0. This provides us with an arrow of time closely matching human experience. The entropy of the complete system (which includes future possibilities as well as an encoding scheme for the past) does stay constant over time as the change of entropy of one is offset by the other. The second law of thermodynamics, understood as an increase in entropy over time, is perceived in the exfoliation variables while the larger system, made to include future possibilities, has a constant entropy over time. In this system future possibilities are consumed to produce encoding possibilities. The second law of thermodynamics is therefore corrected to a law of conservation of entropy for the larger system comprised both entropic time and entropic space.

The decomposition of the program encoding scheme used by the UTM via a Taylor expansion produces terms which can be linked to a scale where a specific entropic force is dominant. For the first Taylor expansion term, we recover special relativity (speed of light (4.36), light-cones (figure 1) and the Lorentz’s factor (figure 2)) and the law of inertia (5.22). For the second term, we recover general relativity (5.33) and the holographic principle (4.21). Finally, the third term might be related to an entropic explanation of dark energy (5.42). As a Taylor expansion tends to produce a series where the first term dominates locally, we obtain an interpretation where inertia dominates at local distances until it is overtaken by general relativity at stellar scales and eventually by dark energy at cosmological scales. This agrees with our experimental observations. Quantum mechanics is recovered as a result of the random walk produced on $dx$ and $dt$ from the switch of available micro-state associated with the entropy of the partition function.

So why does the axiomaticless derivation work at all?

Sufficient reason, by virtue of which we consider that we can find no
true or existent fact, no true assertion, without there being a sufficient reason why it is thus and not otherwise, although most of the time these reasons cannot be known to us.

–Gottfried Wilhelm Leibniz

The principle of sufficient reason of Leibniz suggests that for any fact to be true, there must be a sufficient explanation for it. If the principle holds, then it follows that positing an axiom as a requirement to derive the ToE violates the principle of sufficient reason as axioms are assumed true without reason. Therefore, if the universe does exist, then there must be a minimalist derivation of the laws of physics.

The minimalist derivation in this paper is a constructive proof of Leibniz’s principle of sufficient reason applied to the existence of the universe.

This was the original argument which motivated my research.

References


21 An appeal to falsifiability is considered to be an insufficient reason.


