From 'cogito ergo sum' to $E = mc^2$

Alexandre Harvey-Tremblay
ahtremblay@gmail.com

May 18, 2017

Using the cogito ergo sum as a starting point, I introduce a methodology which allows non-trivial knowledge to be obtained without the formal introduction of axioms. Using this methodology which I formalize, I then obtain a theory of everything (ToE) in physics. This method allows me to avoid the self-referential problem of a ToE which I argue is the core difficulty of any ToE built upon formal axioms. The theory obtained has enough generality to recover both general relativity and quantum field theory. Some of the new physics derived herein include a hard arrow time, an explanation of the quantum mechanical measurement, and a thermodynamic proof that the speed of light is maximal.

Contents

1 Philosophy

SINCE THE RED SHIFT MEASUREMENTS of distant galaxies made by Hubble half a century ago, it has been determined that the universe is expanding as it ages. This observation implies that the universe was smaller in the past than it is today. And at one point in time the whole of the universe was concentrated to a microscopic point in space - the beginning. This idea of a universe with an actual origin dating back to approximately 13.7 billion years has replaced the idea of an infinite and eternal universe that prevailed before the 19th century.

A universe with a beginning leads to an inescapable question. What caused the beginning? Why did it happen and could it have been different? Perhaps the universe is cyclical and involves an infinite sequence of big bangs and big crunches. Or perhaps the universe resides in a infinite meta-universe of interacting strings and branes. But neither of these possibilities provides the full answer. Invoking a meta-universe simply pushes back the ultimate question. Indeed, where would this meta-universe come from?

The most fundamental question of physics is why does the universe exists rather than not exist? And then, after explaining its existence, why does our universe has the laws it does, and not some other laws. Such an answer would be a complete theory of everything. Such an answer must also be unique, otherwise we would still need a better theory to explain which of the possible ones is the real one.

Many believe that such an answer cannot be formulated. Gödel's incompleteness theorem comes to mind as a reason why. Not only is explaining the origin of the universe a difficult problem in and of itself, but an even stronger difficulty arises because the answer, if it exists, must also be a part of the universe, the existence of which it is trying to explain. At its worst the answer could be circular or tautological.

1.1 The self-referential problem

I would argue that the most significant problem standing in the way to a complete formulation of the theory of everything (ToE for short) is caused by its self-referential nature.

Definition 1.1 (The self-referential problem)**.** *We, as intelligent observers, are trying to formulate a ToE, but we reside in the universe explained by this ToE. This creates a fundamental limit on its axiomatic formulation.*

Trying to prove the axioms of the ToE while residing in the universe explained by it is the equivalent of asking a mathematician to prove arithmetic using only Peano's axioms of arithmetic. This is an impossible task.

Indeed for axioms of first-order arithmetic or stronger, Kurt Gödel showed that it was impossible to prove the consistency of said axioms. Paraphrasing, Gödel manages to write an equation that states "If a proof that arithmetic is consistent exists, then *a contradiction* is implied". Hence for arithmetic to actually be consistent, it must be the case that we cannot find a proof of such within arithmetic itself. Hence, if arithmetic is consistent, it must be incomplete. Any ToE of the universe will have to at least contain arithmetic and hence will have the same incompleteness vulnerability.

Definition 1.2 (Theory of everything in physics)**.** *A theory of everything or final theory, ultimate theory, or master theory is a hypothetical single, all-encompassing, coherent theoretical framework of physics that fully explains and links together all physical aspects of the universe. Finding a theory of everything is one of the major unsolved problems in physics.* [1]

[1] Wikipedia. Theory of everything. https://en.wikipedia.org/wiki/Theory_of_everything, 2017c. [Online; accessed 15-May-2017]

In this document I will provide a novel methodology which solves the self-referential problem. The solution of the problem produces a unique mathematical equation which is the ToE. From the equation, I prove traditional notions of physics such as the existence and conservation of energy, thermodynamics, quantum information and the arrow of time. Building on these notions and from the single equation, I then reproduce both quantum field theory and general relativity.

1.2 Formal systems

Definition 1.3 (Axiom)**.** *An axiom or postulate as defined in classic philosophy, is a statement (in mathematics often shown in symbolic form) that is so evident or well-established, that it is accepted without controversy or question.* [2]

[2] Wikipedia. Axiom. https://en.wikipedia.org/wiki/Axiom, 2017a. [Online; accessed 15-May-2017]

The logical methods of obtaining new knowledge by starting from pre-existing notions and applying agreed upon rules of inference was first used by the ancient Greeks. With the exception of Tautologies, it is understood that "nothing is deduced if nothing is assumed". And so, a starting point is required.

Certainly any modern and formal theory of physics and mathematics is formulated using axioms, whilst making them very useful, are not without problems. Here are the major problems of using axioms.

1. What does it mean to accept something without controversy? With 7.4 billion people on Earth, there will always be one person who disagrees with everything and anything. So every axiom is potentially controversial.

2. There are infinitely many sets to choose from. As a result we usually end up with half a dozen well researched sets each competing with one another. Ex.: Propositional logic vs. Intuitionist logic, Category theory vs. Set theory, etc.

3. By the nature of logic, it is such that axioms form the simplest possible starting point for a theory. Hence, axioms are unprovable. In fact, if a simpler starting point were to be found, it means we should start from that point instead.

4. And finally, specifically to obtain a ToE, the major problem is the self-referential problem previously discussed.

1.3 *Cogito ergo sum*

To find the ToE, we must obtain knowledge without axioms. We must accept that this goal, however difficult it may be, is the only way to circumvent the self-referential problem and obtain a ToE. To do this, we will start at the very beginning of reason with finding the first statement that we can prove.

In the 1630s, Descartes became obsessed with finding a statement that he would consider to be necessarily true. He understood that it is not easy to find because with enough diligence almost any statement can be doubted of. But he remained hopeful. His rationale was that if he managed to find a statement that survived the most intense doubt process then it surely must be true.

He recognized that his senses often deceived him, hence he could not rely on them for finding the truth. But what about his mind? Even something as simple as geometry has many errors that are only found years after publication. Can we then really trust this field at the time considered the most fundamental and trustworthy of the fields of logic? As he continued his process of doubting he realized that he could doubt absolutely everything except that he could not doubt that he doubted. After-all, if one doubts everything, then one cannot doubt that he doubts. And to doubt, he must use his mind. And because he uses his mind, he must exist.

The fuller form of cogito ergo sum, is dubito, ergo cogito, ergo sum: Since I doubt, I think; since I think I exist. In other words, I cannot use my mind to prove that I do not exist, for using my mind is in itself a proof that it exists. I think, therefore I am, is the correct conclusion that a mind can reach in regards to its own existence.

Theorem 1.4 (Cogito ergo sum)**.** *I think, therefore I exist.*

Proof. I can find a reason to doubt any statements that I formulate. However, if I concede that I doubt everything, I cannot doubt that I doubt. And if I doubt, I must perform a mental action. Therefore I must exist. □

Descartes however missed two consequences of his discovery that he could have used to mathematically formalize it. The following statements can be proven as a consequence of cogito ergo sum. As a result, they hold the same level of certainty as the original statement.

Theorem 1.5. *The statement cogito ergo sum only exists if a language sufficiently descriptive to express it also exists.*

Proof. This is a tautology. Assuming that no language sufficiently descriptive exists to express it, it follows that the cogito ergo sum cannot be expressed. And if it cannot be expressed, it cannot be the subject of a proof. □

Theorem 1.6. *There exists two or more true statements.*

Proof. Theorem 1.4 and 1.5 are both true, hence 1.6 is also true. □

If either theorem 1.5 or theorem 1.6 is false, then cogito ergo sum is also false. If cogito ergo sum is true, then both theorems are true. Therefore all three theorems 1.4, 1.5 and 1.6 come as a single package of equal validity.

We have not had to introduce any axioms to prove these three theorems. They are proven before any axioms can be defined for the reason that a language must first exist for an axiom to be defined. The theorems are proven by contradiction as a result that one cannot consistently deny his own existence. Since we have obtained knowledge without axioms, we can therefore use this knowledge to avoid the self-referential problem. We will see that theorem 1.5 and 1.6 forms the basis of all that is needed to obtain the ToE.

1.4 Existential logic

To proceed forward, we must introduce a few definitions, such as symbols, language, sentences and theorems.

Definition 1.7 (Symbol)**.** *A symbol is a unique distinguishable identifier. Example:* 0, 1.

Definition 1.8 (Language)**.** *A language L is defined by a finite ordered list of distinct symbols, called the alphabet of the language. The specific order of the alphabet is its alphabetical order. For example, binary code has two symbols,* 0 *and* 1 *as its alphabet.*

Definition 1.9 (Sentence). *A sentence of L is defined by an ordered finite list of symbols taken from the alphabet of L. Repetitions of symbols are allowed. The empty sentence has zero symbols and is denoted with ϵ. For example, the sentence* 000101101 *is a binary sentence.*

Definition 1.10 (Theorem). *A sentence of L is a theorem if it is considered to be provable according to some rules of inference. The rules of inference are the transformations that are theorem-preserving; i.e. when applied to existing theorems produce new theorems.*

Remark 1.11. *Many works on formal language also define the concept of a well-formed sentence. This would be useful if we wanted to add further restrictions on the possible truth values that apply to sentences. For example, we might want to work in a language where every theorem is either provable, or its negation is provable. To achieve that, we need a set of rules to build well-formed sentences that preserves this requirement.*

In the general case where every sentence of the language is permitted, a sentence that cannot be proven does not necessarily imply that its negation is provable. It could never halt, or be a paradox such as the 'barber paradox' which can neither be true nor false. The point of introducing the concept of well-formed sentences is to reduce the complexity of the system by: removing never halting questions, removing paradoxes, removing self-referential statements, removing unprovable statements, etc. However, these complexities will prove absolutely necessary to derive the ToE, hence we will avoid the issue altogether and simply consider that all sentences are well-formed sentences.

Theorem 1.12. *Any language that is non-binary can be expressed as a binary language for which the symbols are* 0 *and* 1*.*

Proof. Suppose a language L with n symbols. Each symbol of L will be represented with $\lceil \log_2 n \rceil$ bits - the ceiling of the logarithm of n in base 2. For example, for the ASCII language with its 128 symbols, the first symbol is 7 repetitions of the bit 0, or 0000000. The second symbol is 0000001, and so on. Finally the last symbol of ASCII encoded in binary is 1111111. □

There exists a language simpler than binary called unary. It comprises only of varying-length repetitions of the symbol 1. The valid sentences are the set $L = \{1, 11, 111, 1111, 11111, ...\}$. Although it is simpler, we will not use this language here because we will soon introduce the prefix-free universal Turing machine. A Turing machine reading an input must know when to stop reading before it reads an undefined bit. Therefore, a second bit acting as a termination bit is required at the end of every unitary-encoded sentence to allow proper reading termination. Since this cannot be done in unary, binary will be the simplest language we can work with.

Now that our binary language is defined, we can enumerate all of its sentences in alphabetical order. We start by enumerating the empty sentence ϵ. Then, we enumerate each sentence of one symbol in alphabetical order. Then each sentence of two symbols and so on.

Usually the next step would be to start finding theorems. To achieve this it is common to define a list of axioms and some rules of inference. But, when we apply this process to the search for a ToE, this methodology fails because of the self-referential problem. Instead, we will reverse the problem. We will identify the theorems first - albeit imprecisely in the beginning -, then we will ask if there are rules of inference and axioms which satisfy the list of theorems that we predefine.

In this methodology, the search for theorems (given certain axioms) becomes a search for axioms (given certain theorems).

Definition 1.13 (Alphabetical enumeration of a language)**.** *The alphabetical enumeration of a language L consists of enumerating, in order, all possible sentences of the language. We start with the empty sentence ϵ, then sentences of* 1 *symbol, followed by sentences of* 2 *symbols, etc. The list is sorted according to its alphabetical order from left to right, shortest sentence comes first.*

Definition 1.14. *We define S(i) as the i^{th} sentence of L according to its alphabetical enumeration.*

Because of theorem 1.6 we know that some sentences are provable, but we do not know which ones. To represent this formally, we will define a two-state identification function $T(i)$ as

Definition 1.15. *For each sentence $S(i)$*

$$T(i) = \begin{cases} 0 & S(i) \text{ is a theorem} \\ \infty & S(i) \text{ is not a theorem} \end{cases}$$

The reason for the choice of ∞ or 0 as the states, as opposed to say 0 and 1, will become clear when we start to write equations in the form of a sum.

Definition 1.16 (Unitary encoding)**.** *Unitary encoding is a map between the alphabetical enumeration of a language L and the following prefix-free encoding. The empty sentence ϵ is mapped to* 0*. Each other sentence of L is mapped to a new sentence of the form* 1...0*. The first sentence after ϵ is mapped to* 10*. The following sentence to* 110*, and so on.*

As an example, consider Table 1. The number of possible sentences grows unbounded from $i = 0$ to $i \to \infty$. As i increases, the length of $S(i)$ grows correspondingly. An arbitrary example of the theorems is shown in the last column. Each value of $T(i)$ is either 0 or ∞. We do not pretend to know exactly which sentence is a theorem and which is not, nor will we need to. We simply acknowledge that some must be theorems. The dynamics of these theorems and how they 'play' with each other through program-length and program-runtime will be enough to derive the ToE.

To derive the ToE, we will start with a simple equation then build upon it.

i	$S(i)$	$T(i)$
0	0	∞
1	10	0
2	110	0
3	1110	0
4	11110	0
...		

Table 1: The alphabetical enumeration of the sentences of the binary language encoded in unitary. In this example the values of $T(i)$ are chosen arbitrarily.

Theorem 1.17. *If we take the number i corresponding to each sentence and weigh it according to a power probability distribution in base 2, we can write*

$$1 = \sum_{i=1}^{\infty} 2^{-i} \tag{1.18}$$

Proof. Expanding the sum into binary, we get

$$\begin{aligned}\sum_{i=1}^{\infty} 2^{-i} &= 2^{-1} + 2^{-2} + 2^{-3} + \ldots \\ &= 0.1 + 0.01 + 0.001 + 0.0001 + \ldots \\ &= 0.\overline{1} \\ &= 1\end{aligned}$$

□

Theorem 1.19. *Further refining this equation we add the term $T(i)$ to capture only the theorems while excluding all other sentences from the sum,*

$$\Omega = \sum_{i=1}^{\infty} 2^{-T(i)} 2^{-i}$$

Proof. Again expanding the sum into binary, we get

$$\begin{aligned}\Omega &= \sum_{i=1}^{\infty} 2^{-T(i)} 2^{-i} \\ &= 2^{-\infty}2^{-1} + 2^{-0}2^{-2} + 2^{-0}2^{-3} + 2^{-0}2^{-4} + 2^{-\infty}2^{-5} + \ldots \\ &= 0 + 0.01 + 0.001 + 0.0001 + 0 + \ldots \\ &= 0.01110\ldots\end{aligned}$$

We obtain a number Ω where its bits are in a one-to-one correspondence with the sentences given by $S(i)$. If $S(i)$ is a theorem of L, the i^{th} bit of Ω is 1, otherwise it is 0. The reason why $T(i)$ uses 0 and ∞ as its two states is now clear. It is to remove the terms that are not theorems from the sum by making them tend to 0. □

In 1927, Emile Borel [3] introduced the concept of the know-it-all number. He suggested listing all the possible yes/no questions of a language. There exists infinitely many such questions. Borel's know-it-all number is a real digit between zero and one. If the i^{th} digit is a 0 then it means the answer to the corresponding question is false. If it is 1, the answer is yes.

[3] Gregory Chaitin. How real are real numbers? arXiv:math/0411418 [math.HO], 2004

Theorem 1.20. *Ω can be reduced to an Emile Borel's know-it-all number.*

Proof. Take all sentences of L that are provable. These are all of the yes/no questions of L. From this set, construct a binary number between 0 and 1 whose i^{th} digit after the decimal corresponds to the i^{th} element of the set. If the answer to the i^{th} question is yes, the i^{th} bit is 1. Otherwise, it is 0. □

The difference between Ω and Borel's know-it-all number is that the 0 digits of Ω represents sentences the provability of which might be false or unknown. Ironically, Borel's know-it-all number knows less than Ω as it is only aware of sentences that are theorems.

Remark 1.21 (The Universe)**.** *The number Ω represents all the knowledge obtainable for a language L according to some rules of logic. If we define the universe by its knowledge content, there is a Ω_U corresponding to our universe. This definition will be made explicit in the next chapter.*

This section is named existential logic because we pose the reverse problem of logic. From theorem 1.6 we know that true statements must exist. We may not know which ones they are specifically and, as we will see, to derive the ToE we do not need to. From theorem 1.5 we also know that language must exist or there would be no statements at all.

The name existential logic is a play on a similar term taken from existential philosophy, that existence precedes essence. We accept that things exist first and foremost (theorems) before their nature or essence can be explained (axioms and rules of inference). Adapted to this work the proposition "existence precedes essence" would read "theorems precede rules of inference". What exists in our universe exists regardless of our choice of axioms, not the other way around. Our methodology for finding the ToE should reflect that.

1.5 The universal Turing machine

According to the Church-Turing thesis, we would add that Ω must be calculated by a universal Turing machine (UTM) using a finite set of instructions. The definitions adopted so far are, for our purposes, equivalent to the Turing representation. However, switching to it at this stage gives us the benefit of unlocking the formalism of algorithmic information theory. Since we are now working with Turing machines it is more appropriate to use the word 'program' instead of 'sentence', and to use the word 'halts' instead of 'is a theorem'. For example, instead of saying 'this sentence is a theorem', we will say 'this program halts'. It is equivalent but more appropriate when referring to Turing machines.

Theorem 1.22. *Ω, if calculated from a UTM, is the halting probability of a prefix-free UTM.*

Proof. The proof will be divided into 2 lemmas.

Lemma 1.23. *Ω is a probability therefore $0 \leq \Omega \leq 1$.*

Proof. If all programs halt, $\Omega = 0.\overline{1} = 1$. And if no program halts, $\Omega = 0$. Since the possible values are a real number between 0 and 1, it meets the definition of a probability. $\square$

Lemma 1.24. *The program encoding is prefix free*

Proof. In the case of unitary encoding, it is easy to see that no program is the prefix of another. A UTM reading unitary encoding will end at the first 0 it encounters and will assume this is the full program. Since the encoding enforces a single 0 bit per program at the very end, this guarantees that the UTM cannot mistake one program for another. $\square$

$\square$

Definition 1.25. *Ω is a Chaitin omega number [4] (or halting probability). Each halting probability is a normal and transcendental real number that is not computable, which means that there is no algorithm to compute its digits. Indeed, each halting probability is Martin-Löf random, meaning there is not even any algorithm which can reliably guess its digits.*

[4] Ming Li and Paul Vitanyi. An introduction to kolmogorov complexity and its applications. Springer, 1997; and Wikipedia. Chaitin's constant. https://en.wikipedia.org/wiki/Chaitin%27s_constant, 2017b. [Online; accessed 15-May-2017]

Why is Ω not computable? Because of the halting problem. Many sentences exist that do not contradict the rules of inference, but cannot be proven by them. When attempting to prove these sentences, the UTM will run forever without halting.

1.6 A note on mathematics without axioms

So far, we have made use of arithmetic as defined by the Peano axioms. Yet, we have claimed that we will derive the ToE without introducing axioms. Have we not contradicted ourselves? The answer is no because arithmetic can be derived from the existence of a language. Indeed, any language will have symbols. By adding symbols or subtracting symbols, we can form sentences of varying length. The length of these sentences changes via addition or subtraction according to arithmetic. This addition or subtraction is required for language to work and without it, all languages would fail. As a result, arithmetic is part of the package of true statements derivable from the cogito ergo sum. The Peano axioms become theorems instead of axioms and they are proved by contradiction against the cogito ergo sum.

The same argument applies to a Turing machine. As the cogito ergo sum can only be derived after some linguistic manipulation, a process must exists to work with any language - otherwise theorems cannot be proven. For the sake of generality we define that process to be a Turing machine.

This line of reasoning cannot be used to prove everything and ends here however. For example, we could not derive geometry from this because a language need not occupy space. Therefore, introducing geometry immediately would be reducing it to a list of axioms and using it we would encounter the self-referential problem.

1.7 *The general halting partition*

I have chosen to sum the sentences as an exponential distribution of base 2 in equation 1.18 because it is the most conceptually simple way to do it. It is not however the only way as other choices which preserve the information are available.

In this final subsection of this section, we generalize the halting probability to the halting partition. We note that key properties of the halting probability are preserved under the following generalization.

Theorem 1.26. *A multiplication factor $\beta \in [0, \infty[$ can be added to $T(i)$ without changing the result of the sum*

$$Z = \sum_{i=1}^{\infty} 2^{-\beta T(i)} 2^{-i}$$

Proof. expanding the sum into binary, we get

$$\begin{aligned} Z &= \sum_{i=1}^{\infty} 2^{-\beta T(i)} 2^{-i} \\ &= 2^{-\beta\infty} 2^{-1} + 2^{-\beta 0} 2^{-2} + 2^{-\beta 0} 2^{-3} + 2^{-\beta 0} 2^{-4} + 2^{-\beta\infty} 2^{-5} + \ldots \\ &= 0 + 0.01 + 0.001 + 0.0001 + 0 + \ldots \\ &= 0.01110\ldots \end{aligned}$$

The constant Z does not change because β it is always multiplied by ∞ or 0 in the sum, which erases its effects. $\square$

Theorem 1.27. *A multiplication factor, $F \geq 1$ can be added to i while preserving the non-computable nature of the sum, as well as the halting information content of each sentence*

$$Z = \sum_{i=1}^{\infty} 2^{-\beta[T(i)+Fi]}$$

Proof. Tadaki has shown [5] that Gregory Chaitin's constant can be extended to include a compression term F on $2^{-i} \implies 2^{-Fi}$ such that the Takadi constant Ω^F remains non-computable F-random. Furthermore, he goes to show that Ω^F's first n bits contain $|n - F|$ halting bit. For example, take the case where $F = 2$, then expanding the sum into binary, we get

[5] K. Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. http://arxiv.org/abs/nlin/0212001, 2002

$$\begin{aligned}
Z &= \sum_{i=1}^{\infty} 2^{-Fi} \\
&= 2^{-2\times 1} + 2^{-2\times 2} + 2^{-2\times 3} + 2^{-2\times 4} + 2^{-2\times 5} + ... \\
&= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} + 2^{-10} ... \\
&= 0.01 + 0.0001 + 0.000001 + 0.000000001 + 0.0000000001 + ... \\
&= 0.0101010101...
\end{aligned}$$

The compression factor F "decompresses" the information by inserting some 0 in between the bits. It does not erase data. For the full proof, refer to Takadi's paper. □

A partition function allows us to extract algorithmic observables from the set of theorems of the language L. Each macroscopic property average such as program output, program length, program runtime, program memory usage, etc. forms an observable $C_k(i)$ of the partition function and has a conjugate variable σ_k. Add these final observables we now have a partition function and will use the symbol Z to denote its value.

Theorem 1.28. *The key properties of the halting probability are preserved when generalized to the halting partition*

$$Z = \sum_{i=1}^{\infty} e^{-(\ln 2)\beta[T(i)+Fi+\sum_k \sigma_k C_k(i)]}$$

When referring to Ω_U, the halting partition is called the general halting partition.

In the special case where $\beta = 1$, $F = 1$ and $-\sum_k \sigma_k C_k(i) = 0$, we recover the halting probability and $Z = \Omega$.

2 *Thermodynamics*

It has been said that thermodynamics is the most general of all the disciplines of physics. Hence it is expected to be the first derived from a ToE.

> A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts.
>
> –Albert Einstein

Introduction. In statistical physics, we are interested in the distribution that maximizes entropy

$$S = -\sum_{x\in X} p(x) \ln p(x) \tag{2.1}$$

subject to the fixed macroscopic observables. The solution is the Gibbs ensemble. As an example we take Table 2 as the observables.

Observable	Conjugate variable
Energy E	Temperature $\beta = 1/(k_b T)$
Volume V	Pressure $\gamma = p/(k_b T)$
Number of particles N	Chemical potential $\delta = -\mu/(k_b T)$

Table 2: Typical observables of statistical mechanics.

then the partition function becomes

$$Z = \sum_{x\in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{2.2}$$

The probability of occupation of a micro-state is

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{2.3}$$

the average values and their variance for the observables are

$$\overline{E} = \sum_{x\in X} p(x)E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{2.4}$$

$$\overline{V} = \sum_{x\in X} p(x)V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{2.5}$$

$$\overline{N} = \sum_{x\in X} p(x)N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{2.6}$$

The laws of thermodynamics can be recovered from the partition function by taking the derivatives

$$\left.\frac{\partial S}{\partial E}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial V}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial N}\right|_{E,V} = -\frac{\mu}{T} \tag{2.7}$$

We summarize these equations to

$$dE = TdS - pdV + \mu dN \tag{2.8}$$

Related work on algorithmic thermodynamics. In their paper [6], John C. Baez and Mike Stay, suggest an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs as considered to be observables. Starting from Gregory Chaitin's Ω number, the halting probability

[6] John C. Baez and Mike Stay. Algorithmic thermodynamics. arXiv:1010.2067 [math-ph], 1997

$$\Omega = \sum_{p \text{ halts}} 2^{-|p|} \tag{2.9}$$

is extended with algorithmic observables to obtain

$$\Omega' = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{2.10}$$

Noting the similarity between equation 2.2 and 2.10, they suggest an interpretation where E is the expected value of the logarithm of the program's runtime, V is the expected value of the length of the program and N is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

1. $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.

2. $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the tradeoff between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount, while holding the number of programs in the ensemble and their mean output fixed.

3. $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

From equation 2.10, they derive analogues of Maxwell's relations and they consider thermodynamic cycles such as the Carnot cycle or Stoddard cycle. For this they introduce the concepts of *algorithmic heat* and *algorithmic work*.

The authors then claim that the choice of correspondance between thermodynamic observables and algorithmic observables is somewhat arbitrary and reference other authors [7] who have used completely different correspondances.

[7] Ming Li and Paul Vitanyi. An introduction to kolmogorov complexity and its applications. Springer, 1997; and K. Tadaki. A statistical mechanical interpretation of algorithmic information theory. https://arxiv.org/pdf/0801.4194.pdf, 2008

In this work, we study *algorithm thermodynamics* for the purpose of explaining thermodynamics. We will claim that there does exist a preferred correspondance between algorithmic observables and statistical mechanical observables. This preferred choice is set by the units of the observables. In a physical partition function, each observable must have the units of energy and is divided by $\beta = 1/(k_b T)$ the units of which are also energy, leaving no units in the exponential. Therefore, in our replacements, the conjugate variables used must convert the units of its associated observable to an energy. Examples of allowed conjugate-observable pairs are listed in table 3.

Observable	Variable	Units	Conjugate	Variable	Units
Halting event	E	J	Temperature	$\beta = 1/(k_b T)$	$1/(J/K \times K)$
Program length	x	m	Force	$\gamma = F/(k_b T)$	$N/(J/K \times K)$
Running frequency	τ	$1/s$	Action	$\alpha = \mathcal{S}/(k_b T)$	$J \times s/(J/K \times K)$
Runtime	t	s	Power	$\kappa = P/(k_b T)$	$W/(J/K \times K)$

Table 3: The preferred correspondance between *algorithmic thermodynamics* and *statistical physics* based on units matching.

Theorem 2.11. *The general halting partition is the Boltzmann distribution used in statistical physics.*

Proof. We try to explain thermodynamics and physics starting from *algorithmic thermodynamics*. First, we recall theorem 1.28. We recover thermodynamics by introducing an energy associated with each halting event, denoted by E. Adding the rest of the conjugate-observable pairs, we obtain

$$Z = \sum_x e^{-(\ln 2)\beta(E+Fx+\mathcal{S}\tau+\dots)}$$

where the triple dots represent other possible observables. We interpret the program x as a micro-state of the set of all prefix-free programs that are run on the UTM. It is easy to see that the function for Z is the partition function of the Gibbs ensemble of thermodynamics.

□

Since both the running frequency and the runtime are associated with time, we only need to select one of them as our conjugate-observable pair. In this section, $\mathcal{S}\tau$ leads to conceptually simpler results and so is used here, whereas in the chapter on spacetime, Pt will be preferred.

2.1 *Energy*

Theorem 2.12. *The law of conservation of energy.*

Proof. Since we have recovered a thermodynamic partition function, we can define a conserved energy quantity. This is a direct consequence of taking the thermodynamic state equation of the partition function.

$$dE = TdS - Fdx - \mathcal{S}d\tau - \ldots \tag{2.13}$$

□

Theorem 2.14. *When the values of the halting bits of Z are not known to an observer, each bit of information has the following energy*

$$E = k_B T \ln 2$$

Proof. Consider an observer not aware of the bit values of Z. To the observer, Z looks like

$$Z_N = 0.\omega_1\omega_2\omega_3 \ldots \omega_N \tag{2.15}$$

There are $W = 2^N$ different possibilities, or micro-states. Since each bit has two possible values, the entropy of the system is $S = k_B \ln 2^N$. Adding or removing a bit changes the entropy and the energy by

$$\begin{aligned}
\Delta S &= S_{N+1} - S_N \\
&= k_B \ln 2^{N+1} - k_B \ln 2^N \\
&= k_B \ln 2 \\
\Delta E &= T\Delta S \\
&= T\left(k_B \ln 2\right)
\end{aligned}$$

□

This result agrees with the well known Landauer limit [8].

[8] R. Landauer. Irreversibility and heat generation in the computing process. http://worrydream.com/refs/Landauer%20-%20Irreversibility%20and%20Heat%20Generation%20in%20the%20Computing%20Process.pdf, 1961

2.2 Time

Posing $dE = 0$, we can look at a simplified subset of the general halting partition involving only the frequency and the length observable.

$$Z = \sum_x e^{-(\ln 2)\beta(Fx+\mathcal{S}\tau)} \tag{2.16}$$

where $\tau = 1/t$.

Theorem 2.17. *At the limit of $t \to \infty$, we recover Ω*

Proof. A program p can have any value of $\mathcal{S}$ within $[0, \infty]$. If the program halts immediately, $\mathcal{S} = 0$. If it never halts, $\mathcal{S} = \infty$. If it halts after a certain time, $\mathcal{S} = t_p$. A program that never halts will not be part of the halting partition. This will be the case if $\mathcal{S} = \infty$. As a result we obtain,

$$\lim_{t\to\infty} \frac{\mathcal{S}_x}{t} = \begin{cases} 0 & \text{x halts} \\ \infty & \text{otherwise} \end{cases}$$

This is the definition 1.15 of $T(i)$. Hence

$$\lim_{t\to\infty} \frac{\mathcal{S}_x}{t} = T(x)$$

Therefore,

$$\begin{aligned}\lim_{t\to\infty} Z &= \lim_{t\to\infty} \left(\sum_x e^{-(\ln 2)\beta(Fx+\mathcal{S}\tau)} \right) \\ &= \sum_x e^{-(\ln 2)\beta(Fx+T(x))} \\ &= \Omega\end{aligned}$$

At $t \to \infty$ the halting programs, i.e. $\mathcal{S} \in [0, \infty[$, have all halted, whereas the programs where $\mathcal{S} = \infty$ have not.

□

Theorem 2.18. *At the limit of $t \to 0^+$, we obtain initial conditions.*

Proof. We study the limit of $t \to 0^+$. We obtain

$$\lim_{t\to 0+} \frac{S_x}{t} = \begin{cases} 0 & S_x = 0 \\ \infty & \text{otherwise} \end{cases}$$

At that limit, the only programs that contribute to Z are those that halt immediately. These are the initial conditions. □

Theorem 2.19. *For $0 < t < \infty$, the partition function Z is*

$$Z(t) = \Omega - 2^{-k(t)}$$

where $2^{-k(t)}$ is an error rate that is monotonically decreasing to 0 as $t \to \infty$.

Proof. [9]

Definition 2.20. *For any $k \geq 0$ and time $t \geq 0$, let $k(t)$ be the location of the first zero bit after position k in the estimation of Ω.*

Then because $-\frac{S_x}{t}$ is a monotonically decreasing function of the running frequency and decreases faster than $k(t)$, there will be a time step where the total contribution of all the programs that have not halted yet is less than $2^{-k(t)}$.

□

For example, say

$$\Omega = 0.0111100\ldots$$

To keep it simple we consider, in isolation, a single program and assume that all other programs have long halted (at $t \to 0^+$). Let us take the values $x = 5$ and $\mathcal{S}_x = 50$ for this program. We obtain,

$$\begin{aligned} Z_x(t) &= 2^{-x}2^{-\frac{S_x}{t}} \\ Z_5(t) &= 2^{-5}2^{-\frac{50}{t}} \\ &= 0.00001 \times 2^{-\frac{50}{t}} \end{aligned}$$

The halting probability Ω is,

$$\Omega = 0.0111000 \cdots + Z_5(t)$$

Let us look at what happens as we vary t.

1. If $t \to 0^+$, then $Z_5(0^+) = 0$. Z differs from Ω by the maximum uncertainty of 2^{-5}. Therefore $\Omega - Z_5(0^+)$ is accurate only in its first 5 bits.

2. As $t \to \infty$, then $Z_5(\infty) = 0.00001$.

3. Between 0 and ∞, $Z_5(t)$ varies from 2^{-5} at $t = 0$ to 0 at $t \to \infty$. Since $-(\mathcal{S}_5/t)$ is monotonically decreasing, the uncertainty $2^{-k(t)}$ must decrease monotonically to 0 as t increases.

4. At distances further than $2^{-k(t)}$, the partition function contains bits of programs that have yet to halt. So, in a sort, a reversal of time occurs where halting information is available before the time t is long enough for the program to have halted. [10]

[9] Here, we reproduce the definition of $k(t)$ and the proof provided by John C. Baez and Mike Stay in their paper on *algorithmic thermodynamics*.

[10] In the chapter on spacetime we will see that this is sufficient to derive special relativity.

2.3 *Hard arrow of time*

We now have the tools required to define a hard arrow of time.

Theorem 2.21. *The far future ($t \to \infty$) is non-computable.*

Proof. As proven in theorem 2.17 ,

$$\lim_{t \to \infty} Z(t) = \Omega$$

Ω, as the halting probability of a random program read by a UTM, was proven to be non-computable by Gregory Chaitin. Likewise, the halting problem guarantees that the final days of the universe cannot be precisely calculated. We must wait to see it unfold. □

Theorem 2.22. *The future ($t > t_0$) is not pre-computable.*

Proof. Without loss of generality suppose that,

$$Z(t_0) = 0.001....$$
$$Z(t_1) = 0.101....$$

$Z(t_0)$ differs from $Z(t_1)$ at digit d_1. Here we ask if and how can an observer at t_0 calculate the future at t_1. To calculate the future, an observer knowing $Z(t_0)$ must prove at time t_0 that $d_1 = 1$. However the observer can never succeed and here is why. If the observer does prove that $d_1 = 1$, then because he is part of the universe, the universe becomes $Z(t_1)$. Hence he has not pre-calculated the future, but simply travelled in time to a time where this result is known. Precalculating the future is equivalent to traveling forward in it to a time when the pre-calculation is true.[11] □

[11] We will significantly strengthen this results in the subsection 'Experiments & observers'

Theorem 2.23. *The past ($t < t_0$) and the present ($t = t_0$) are immediately accessible.*

Proof. Without loss of generality suppose that,

$$Z(t_{-1}) = 0.001....$$
$$Z(t_0) = 0.101....$$

As time goes forward more bits of Z are turned from 0 to 1. Since 0 bits represent an unknown answer as well as a negative answer, there is no loss of information when a bit-flip occurs. An observer at t_0 can read $Z(t_0)$ and since $Z(t_0)$ does not erase halting information with respect to $Z(t_{-1})$, he will have read all the past and present knowledge of the universe. □

2.4 Quantum information

Why is representation theory so successful at describing physics? The short answer is that representation theory can easily be described from the general partition function. Each Lie group has a corresponding Lie algebra related via its exponentiation. This is the well known as the Lie group–Lie algebra correspondence,

$$G = e^A \tag{2.24}$$

Taking the simplest physically observed representation, $su(2)$ with its double cover over $SO(3)$, we can insert it as an observable of the general halting partition. In this construction, if the three dimensional rotations become the observable, then each halting bit becomes a spin the measurement of which is the random bit itself. We will now use that to obtain the spin.

First, we consider the case at $t \to \infty$. Since the halting bit at every position is either a 0 or 1 and is non-computable, we will denote each bit as the matrix $0, 1$ rather than fix a specific value. Naively, we first try the following sum

$$\Omega = 0.1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.01 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.001 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.0001 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \dots$$

However this would be a trap. Indeed, the matrices can be added and we would obtain the wrong result of

$$\Omega = 0.11111... \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

This would indicate that the possible results are either all 1 digits, or all 0 digits. This is impossible for a UTM since Ω must be a normal real number. Therefore we must break the "information destroying" additivity of the matrices. This can be done by multiplying each term with an independent term M_i.

$$\Omega = 0.1M_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.11M_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \dots$$

In the most general case, this independent term takes the form of a 2×2 matrix. The only restriction on this multiplicative matrix is that it must preserve the length of the determinant of the matrix it multiplies. Hence $\det\{M\} = 1$. This is the $SU(2)$ Lie group.

$$\Omega = 0.1 \begin{bmatrix} \alpha_1 & -\overline{\beta}_1 \\ \beta_1 & \overline{\alpha}_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.11 \begin{bmatrix} \alpha_2 & -\overline{\beta}_2 \\ \beta_2 & \overline{\alpha}_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \dots$$

where

$$SU(2) = \left\{ \begin{pmatrix} \alpha & -\overline{\beta} \\ \beta & \overline{\alpha} \end{pmatrix} : \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1 \right\}$$

This prevents the erasure of the bits in the function caused by the addition of the matrices. Furthermore, because of the existence of an exponential map between a Lie group and a Lie algebra, we can rewrite Ω in its Gibb's ensemble exponential form,

$$\Omega = \sum_x e^{-(\ln 2)\beta(Fx+\alpha_x M)}$$

where M is the Lie algebra correspondance to the Lie group $SU(2)$ and α_x is a real number. The presence of the real number α imposed by the linear-algeabric correspondance conveniently allows us to recover the statistical mechanical form of an observable multiplied by its conjugate variable. Since the exponential terms represent an energy, it follows that it must be a real number. The existence of a double cover from $SU(2)$ to $SO(3)$ allows the use of real numbers.

In this scenario, the thermodynamic observable is the three dimensional invariance over rotations and the effect on the partition function is that each halting bit becomes a spin.

We have now recovered the properties of the spin-1/2, with the added benefit that we obtain a rigorous definition of the quantum measurement. The measurement happens when the halting bit is fixed to a 1 or a 0. Since the halting probability is a normal number, it proves that the quantum measurement of a spin is algorithmically random.

2.5 *Experiments & observers*

A UTM executing a program to termination before starting working on the next one will hang at the first non-halting program. To avoid this, one might be tempted to start each program one by one, then run each program for one iteration, then each program for a second iteration and so on. Progress will be made on every program regardless of the existence of non-halting programs. However, if there are infinitely many program each potentially running for an infinite amount of time, the solution is to dovetail the programs.

Definition 2.25 (Dovetailing programs)**.** *Dovetailing is a method of computation where program execution in interweaved so that non-halting programs do not cause other programs to hang their execution. The first program is started then it is run for one iteration. Then, the second program is started. Then, the first followed by the second program are each ran for one iteration. Then, the third program is started. And so on.*

The general halting partition clearly executes its programs in a dovetail-like fashion - see theorem 2.19. At any time, shorter programs will have had a longer running period than longer programs. This raises a question. The bits of Ω obtained via dovetailing are compressible to a very short algorithm. Can an observer recover the halting bits by running this program, and therefore correctly predict the measurement outcome of a spin before it is measured? The answer is no, because an observer can only run experiments.

Definition 2.26 (An experiment)**.** *A finite state machine executing a script. The experiment will be completed in a finite time and will use a finite amount of memory*[12]*.*

[12] It may also provably never terminate, but this is not the halting problem.

If the experiment has less internal state than the script defining the UTM running the universe, the experiment will see an algorithmically random halting partition for all future times t [13]. Hence, its future will be non-calculable. The experiment's arrow of time is enforced by the unsolvability of the halting problem using finite-state machines. For observers that can only run experiments, the same applies to them.

[13] As we will see in the subsection on the universe, this implies a quantum mechanical universe.

2.6 The universe

The bits encoding halting information are the source of entropy of the general halting partition. The other thermodynamic observables are non-information bearing. [14] Since this represents Ω and because of the conservation of energy, it follows that all information present in the universe is reducible to the halting probability of its algorithmic formulation.

[14] If a macroscopic observer builds a computer out of the non-information bearing observables, would the computer not store information? Answer - yes, but this information would also be encoded in the entropy of the system. It is no more than duplicated information.

A halting probability of a UTM is the most compressed form possible for this information, and can be used to prove any sentence of the language. This can be seen in the following way. Suppose that we want to prove the Riemann hypothesis - a millenium problem with a reward of $1,000,000\$$. Knowing Ω, it is very easy. We simply write a program that will halt if the proof is found or run forever. Specifically, this program could alphabetically try proofs in dovetail fashion until one is found, then halt. Now, this program p will have a well defined length l_p. If we know Ω, we can prove the Riemann hypothesis like this. We run all programs from shortest to longest in dovetail fashion on the UTM. Every once in while we look at the estimation of Ω obtained and compare it to the real value of Ω. If the uncertainty is less than 2^{-l_p} and p has not halted, we know the proof does not exist and we claim the reward.

A universe at $t \to \infty$ is a universe that contains all the answers to all the questions than can be asked in it. Essentially, it has solved the halting problem. To not solve the halting problem, the universe must exist at a time $t \in [0, \infty[$. We can now define both time and the universe.

Definition 2.27 (Universe)**.** *A universe comprises*

1. *a language* L*,*
2. *its alphabetical enumeration (the list of all sentences in* L *in alphabetical order),*
3. *a halting probability* Ω *for the unitary coding of the alphabetical enumeration of* L *where the bit at position* i *corresponds to the halting status of the program of length* i*,*
4. *a halting probability given by;*

$$\Omega = \sum_x e^{-(\ln 2)\beta(E+Fx)}$$

This defines the universe as the sum of all the knowledge that exists within it for its entire history. From this definition and supplemented with the running frequency to action observable $\mathcal{S}\tau$, we also obtain a definition of the universe at a time t.

Definition 2.28 (Universe at time t)**.** *The universe as defined by 2.27, at time* t *has a halting partition given by*

$$Z = \sum_x e^{-(\ln 2)\beta(E+Fx+\mathcal{S}\tau)}$$

At time $t \in [0, \infty[$*,* Z *replaces* Ω*.*

Describing a time forward or backward is respectively the same as adding or removing the appropriate 1-valued bits to or from Z. Analogous to Ω, Z represents the probability that a program has halted within a certain time frame.

2.7 *Discussion*

From the perspective of an observer running finite-state experiments, access to the full listing of Ω is prohibited due to memory and time constraints. As a result, the observer must account for the uncertainty (e.g. superposition) of each bit of Ω that are unknown to him. From these considerations, quantum mechanics follows as we have previously seen with the spin and, in the next section, we will derive Schrödinger's equation from the same considerations.

The quantum measurement and simultaneous wave function collapse occurs when the halting bit becomes known to 'all possible observers' [15]. If the halting bit is known only to one observer, he becomes entangled with the system. This entangled system finally collapses when the information propagates to other observers.

[15] By 'all possible observers' we mean here that the information related to the halting bit value propagates macroscopically in the system such that it loses its entanglement and quantum character.

The halting probability experienced by an observer who lack complete knowledge of Z, is obtained when we take each possible halting probability and combine them in a degenerate partition function. Successful quantum measurement and wave function collapse reduces the degeneracy of the partition function experienced by the observer.

This is easily formalized from statistical mechanics by considering that Z is a canonical ensemble. A micro-canonical ensemble can be constructed by taking 2^N canonical ensembles. The maximally entangled ensemble seen by an observer with no halting knowledge is then given by

$$Z' = \frac{1}{A}\sum_{i}^{2^N}\left(\sum_{x_i} e^{-(\ln 2)\beta(E+Fx)}\right) \tag{2.29}$$

where the inner sum is iterated over the set of all possible Ω (for all UTMs), and A is a normalization constant. Taking a very long yet finite-length prefix of Ω, we simplify the problem by assuming that each bit arrangement is equally likely. This is why we were able to derive the energy per bit of $k_B T \ln 2$ in theorem 2.14.

Why is the future not pre-calculable? It is possible to approximate the future to a high degree of certainty. For example, we know that the Sun will shine for about 4 billion years more. However, this is not guaranteed. A mini black hole could swallow it in say 1 billion years. To calculate, with certainty, if the Sun will still shine in 4 billion years, we must rule out all of these edge cases. This can only be done if we solve the full universe.

In a relaxed sense we equate the philosophical problem of a frustrator to the halting problem of computer science.

Definition 2.30 (Frustrator in philosophy)**.** *A frustrator in philosophy is an agent having free will who, knowing his future, makes the choice which guarantees the known future will not occur.*

For example, a lottery player who knows he will win the next draw might simply decide not to play. This will frustrate the prediction. Since a free agent can be a frustrator to any future event, the inescapable conclusion is that no free agent can know their future. The frustrator problem in philosophy is usually taken as a strong suggestion that the future cannot be predicted with certainty.

In our derivation the future is presented as the solution to the halting problem. Since the observer cannot solve the halting problem, he must wait for the future to unfold in order to see what it is.

3 Space

WHY DOES SPACE HAVE THREE DIMENSIONS? We have derived a rotational invariance as the thermodynamic observable associated with the spin of the bits of a halting probability. In this chapter, we will extend this result to define a linear position away from an origin. This result will be strong enough to derive black holes, the force of gravity and Schrödinger's equation.

3.1 Maximum entropy

Theorem 3.1. *The maximal entropy of a section of a halting probability represented in space by spins is*

$$S = k_B \frac{c^3 A}{4G\hbar}$$

where A is the area of the sphere enclosing the volume used for the measurements.

Proof. In the previous section, we have seen that the spin of a halting bit leads to the statistical mechanic observable of a rotation in $3D$. It is therefore natural to relate the spin to the sphere. We start with an empty sphere and add spins one by one, until we reach a maximum. We ask; how many can we fit?[16]

[16] We take all spins to be centred at the origin. Introducing a position away from the origin will require bits to describe it. As we introduce a proper definition of position in the next subsection, we will see that this balances the entropy to eliminate any gains.

Lemma 3.2. *The spin requires four degrees of freedom to be fully represented.*

Proof. Two degrees are taken up by the yawn and pitch angle of the wave-function pointing to a point on the top-half surface of the Bloch sphere. These are respectively angle ϕ and θ shown on Figure 1. The Bloch sphere represents a pure quantum 2 state system with wave-function

$$\begin{aligned} |\psi\rangle &= \alpha\,|0\rangle + \beta\,|1\rangle\text{ , where} \\ \alpha &= \cos\frac{\theta}{2} \\ \beta &= e^{i\phi}\sin\frac{\theta}{2} \end{aligned}$$

The third degree is associated with the result for the measurement value of the spin along the z axis. Finally, the fourth degree is associated to the measurement value along the xy plane. [17] This completes the full description of a spin, including all observables and measurement outcomes.

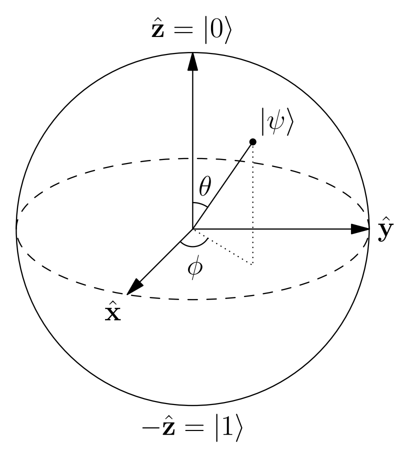

Figure 1: The Block sphere

[17] The third and the fourth degrees are not available from the wave-function but are instead read by a quantum measurement.

Remark 3.3. *Why must the measurement outcome of orthogonal quantum states both be described pre-measurement, if only one of them can be measured? After measurement and wave-function collapse, the system has one degree of freedom. The pure-state is destroyed and is replaced with a classical bit. However, the system does not know which axis will be measured first. Therefore they must both be described. The perturbation of the measurement resets the bit values associated with the spin.*

□

The two degrees of freedom of the yawn and pitch angle of the wave-function are real numbers and require up to infinitely many classical bits to be expressed to the desired precision. It would be a bad idea to encode the orientation of spins using real numbers because there is no limit to their precision. A single spin could consume arbitrarily many bits to describe just one of its real numbered degrees of freedom.

However, we consider the Pauli exclusion principle, and note that a non-integer spin cannot occupy the same quantum state. We also note that the precision required to define a spin is wasted beyond the precision of the instrument measuring it. This instrument cannot be precisely rotated with an angle smaller than the Planck angle. As a result, the spins need not be expressed more accurately than the Planck angle.

Lemma 3.4. *Since the distinguishable rotation angles are finite, there exists an efficient way to encode spins in a sphere, where one classical bit maps to one degree of freedom.*

Proof. We associate classical bits to a position on the surface of the enclosing sphere. It is natural to pose that the number of bits that can fit on its surface is given by

$$N = \frac{A}{L_p^2} = \frac{c^3 A}{G\hbar}$$

where L_p^2 is the Planck length. We can maximize the number of spins in a volume, and therefore maximize the entropy, if we minimize the number of classical bits required to express each spin. Figure 2 shows a spherical geodesic divided in equal areas, each occupied by a classical bit. Each bit on the surface encodes the presence or absence of a pure state wave-function pointing to the bit and centered at the origin.

From the Bloch representation, we easily see that only half of the bits on the surface are actually needed to express all permissible spin orientations in the volume. For each second bit, a one-valued bit

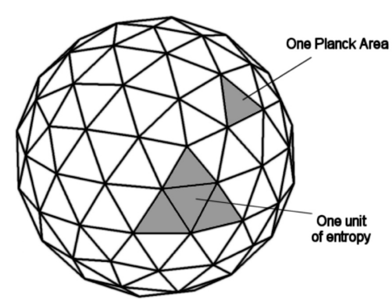

Figure 2: A circle that is divided in squares. Notice how the discrete radial symmetry is restricted to angles of 90°.

on the surface of the sphere corresponds to the presence of a wave-function $|\psi\rangle$ pointing to it, while a zero-valued bit indicates an empty spot. Figure 3 illustrates the encoding of the orientation in $2D$.

The remaining half of bits are used for the third and fourth degrees of freedom. This encoding is maximally efficient and each classical bit encodes a single degree of freedom.

□

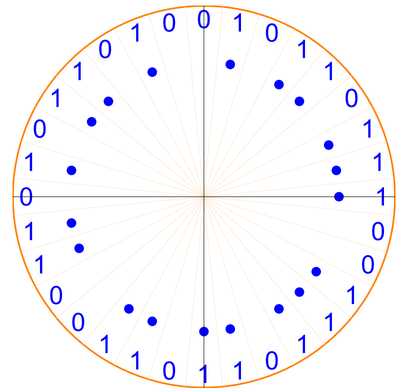

Figure 3: A one-valued bit indicates that a spin exists and is pointing to its position on the circle. A zero-valued bit indicates a vacant orientation. Each bit maps to a single spin.

We now want to determine the maximum entropy of the spin system describable by those bits. Since the general halting partition has a factor $\ln 2$ on the exponential term, the derivative dS/dE gives us a pseudo-entropy related to the real entropy by a factor $1/\ln 2$.

$$\begin{aligned} \frac{dS'}{dE} &= \frac{\ln 2}{k_B T} \\ dE &= k_B T \frac{dS'}{\ln 2} \\ &= k_B T dS \qquad \text{(where } S \ln 2 = S'\text{)} \end{aligned}$$

The factor $\ln 2$ is obtained because the statistical mechanics partition is derived from the base-2 sum of the halting probability. This implies that,

$$\begin{aligned} S &= \frac{1}{\ln 2} S' \\ &= \frac{1}{\ln 2} k_B \ln W \end{aligned}$$

where W is the number of micro-states for the system. Since each micro-state (e.i. spin) requires four classical bits to be described, a factor of $\frac{1}{4}$ is added, such that

$$\begin{aligned} W &= 2^{\frac{1}{4}N} \\ &= 2^{\frac{c^3 A}{4G\hbar}} \\ \ln W &= \frac{c^3 A}{4G\hbar} \ln 2 \\ S &= \frac{1}{\ln 2} k_B \frac{c^3 A}{4G\hbar} \ln 2 \end{aligned}$$

Simplifying the constants, we get

$$S = k_B \frac{c^3 A}{4G\hbar}$$

which is the Bekenstein-Hawking entropy.

□

Theorem 3.5. *At most, half of the permissible orientations are occupied by spins.*

Proof. First, note that 4 classical bits are required to describe a spin. Second, note that 2 of these classical bits are used to mark occupied orientations. This means that half of the orientations must be vacant. □

The halting probability of a UTM is a normal real number that is algorithmically random. This implies that, statistically, it contains as many zeros as ones in its binary digit representation. Conveniently, the maximal entropy of the sphere occurs when the bits on its surface are composed of equally many zeros and ones.

3.2 Position

We recall the general halting partition,

$$Z = \sum_{x} e^{-(\ln 2)\beta(E+Fx+\mathcal{S}\tau+\dots)}$$

and we specifically focus our attention on the second term in the sum of the exponential Fx, where F is a force with units Joules per meter and x is the length of the program with units meter. This term has the same form and units as an entropic force of the form $Fdx = TdS$. For example, entropic forces also describe polymer tension, osmotic force, etc.

Since x represents the program's length in meters, this result strongly suggests that it is encoding spatial lengths. To encode length using bits we will consider an efficient algorithm with the following characteristics;

1. It must have the observer as its origin.
2. It must be able to express positions arbitrarily far away from the origin.
3. It must be able to express positions with arbitrary precision.

Naively, we might be tempted to use a sequence of bits to represent the quantity of discreet steps required to reach a point in space from the present location. For example, suppose the plank's length is the smallest unit of space. Then an object of 1m away from the origin would have its position encoded with the number 6.25×10^{34} in binary. A rather large bit requirement for something that is very simple to describe in layman's terms.

We can make significant better use of our bits by specifying a scale before listing the bits. For example, let's say we only care about specifying the position within a centimetre. We could initially specify a scale, then list the number of repetition of the scale unit to reach the location of interest.

The price to pay for such algorithmic compression is that the position of an object is definable up to a certain precision, and it must have a characteristic scale. I will conjecture that this is the most efficient position encoding algorithm that meets the requirements above.

Definition 3.6 (Positioning via program length, or PVPL for short)**.** *Suppose a Cartesian coordinate system with orthogonal axis x y z and an origin at $(0, 0, 0)$. The position of a point will be encoded via four digits. One scaling constant λ, and 3 scale-repetition values l_x, l_y and l_z. It's position is noted as $p = (\lambda, l_x, l_y, l_z)$. The point can be identified within an error margin of $\pm(1/2)\lambda$. The scale λ can be made as small as we want it by increasing the bit count of l_x, l_y and l_z.*

Theorem 3.7 (Conversion formulas)**.** *PVPL encoding can be converted back to cartesian coordinates by multiplying the scaling factor λ with the repetition value for each coordinate and keeping the uncertainties.*

$$
\begin{aligned}
x &= l_x \lambda \pm \Delta\lambda \\
y &= l_y \lambda \pm \Delta\lambda \\
z &= l_z \lambda \pm \Delta\lambda
\end{aligned}
$$

where $\Delta\lambda = (1/2)\lambda$.

For example, a point at cartesian coordinate $(0, 0, 1)$ with an error margin ± 0.1 is encodable in PVPL with the following 4-digits: $p_1 = (0.1, 0, 0, 1010_b)$. To double the precision without changing the position, we must add an extra bit to the repetition value in z and halve the scaling constant such that $p_2 = (0.05, 0, 0, 10100_b)$. Using PVPL to describe near objects of similar scales has the advantage of requiring a very low bit count.

Theorem 3.8 (Positional entropy)**.** *Using the positional encoding of definition 3.6, we can pose a relation between ΔS and Δx of*

$$\Delta S = 2\pi k_B \frac{1}{\lambda} \Delta x$$

Proof. Naively, we might be tempted to allow the PVPL values l_x, l_y and l_z to take any bit sequence. If we do so, doubling Δx from 10 meters to 20 meters (or in binary; 1010_b meters to 10100_b meters) increases the bit count by 1. Since this does not double the entropy, it contradicts theorem 3.8.

However, it would be a mistake. PVPL values are not the bit sequence themselves, but the length of the program. What is the entropy of a program of length L? A physical observer will measure the entropy of the program to be equal to its length. A physical observer does not and cannot know the precise prefix-free encoding because he sees a degenerate general halting partition. Hence, he must accept that all possible bit sequences could represent a program of length L. Since there are 2^L such sequences, $S = k_B L = k_B \Delta x$.

Why multiply Δx with $1/\lambda$? For example, suppose we have $\Delta x =$ 5 meters. If we encode the position using the meter scale, we will require a program of length 5. However, if we use the centimetre scale, the required program length will now be 500. This increases the entropy by a factor of 100. The smaller the scale the higher the entropy.

Why multiply Δx with a factor of 2π? Our previous result from theorem 3.1 shows that the number of bits depends on the area of the sphere. This results suggests that the bits rest of the surface of the sphere defining a volume in space.

Suppose a circle of radius r with N points uniformly distributed on its perimeter. The points on the perimeter maps to a segment on a line of length L. Keeping the scale intact, the length of the segment will be $L = 2\pi r$ and each point will be separated by a distance $d = 2\pi r/N$. The factor 2π multiplying Δx is a consequence of the scale preserving mapping of linear distances encoded by equidistant bits on a circle.

Why go to such lengths to avoid rescaling the line coordinates and to keep the factor 2π? We are not allowed to rescale the axis as a rescale would increase the precision of all positions and therefore consume more bits.

□

3.3 *Schwarzschild radius*

Theorem 3.9. *Using all the classical bits on the surface of a sphere encoding a halting probability, we encode, using entropic positioning, the longest distance that can be expressed for the black hole. We find that this distance is equal to the Schwarzschild radius,*

$$r_s = \frac{2GM}{c^2}$$

Proof. According to theorem 3.1, the maximum entropy in a volume of space is

$$S = k_B \frac{Ac^3}{4G\hbar} = k_B \frac{\pi r^2 c^3}{\hbar G}$$

We will convert all of these bits to entropic positioning so that we can express the point furthest away from the center of the sphere. We recall equation 3.14.

$$\Delta S = 2\pi k_B \frac{1}{\lambda} \Delta x$$

Equating the two entropies $(\Delta S = S)$, we get

$$k_B \frac{\pi r^2 c^3}{\hbar G} = 2\pi k_B \frac{1}{\lambda} \Delta x$$

Since we are dealing with a black hole, it should not be able to express programs of length longer than its horizon, or information might leak out. Therefore we pose $\Delta x = r$, and we obtain,

$$k_B \frac{\pi r^2 c^3}{\hbar G} = 2\pi k_B \frac{1}{\lambda} (r)$$

Solving for r, we obtain

$$r = \frac{1}{k_B} \frac{\hbar G}{\pi c^3} 2\pi k_B \frac{1}{\lambda} \tag{3.10}$$

Then, we take the algorithmic scaling factor λ to be, in fact, the Compton wavelength. We obtain

$$r = \frac{1}{k_B} \frac{\hbar G}{\pi c^3} 2\pi k_B \left(\frac{Mc}{\hbar} \right)$$

Reducing the constants, we obtain

$$r = \frac{2GM}{c^2}$$

Which is the Schwarzschild radius.

□

3.4 Hawking radiation

Theorem 3.11. *A sphere encoding bits of a halting probability will radiate at a temperature inversely proportional to its mass. The temperature will be consistent with Hawking radiation;*

$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

Proof. According to theorem 3.1, the maximum entropy in a volume of space is

$$S = k_B \frac{A c^3}{4 G \hbar} = k_B \frac{\pi r^2 c^3}{\hbar G}$$

Using the thermodynamic relation $dE = TdS$, we want to obtain the energy, then the mass via $E = mc^2$.

$$dS = 2k_B \frac{\pi r c^3}{\hbar G} dr$$

multiplying by T, we obtain the derivative of energy

$$dE = TdS = 2k_B T \frac{\pi r c^3}{\hbar G} dr$$

dividing by c^2, we obtain the mass

$$dM = \frac{1}{c^2} dE = 2k_B \pi T \frac{c}{\hbar G} r dr \tag{3.12}$$

posing r to be the Schwarzschild radius calculated in theorem 3.9 and dr to be its derivative with respect to the mass, we obtain

$$dM = 2k_B \pi T \frac{\left(\frac{2GM}{c^2}\right) c}{\hbar G} \left(\frac{2G}{c^2}\right) dM$$

Solving for T and reducing the constants, we obtain

$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

, the temperature of Hawking radiation.

□

3.5 *Newton's law of inertia*

Theorem 3.13. *The positional entropy leads to Newton's law of inertia and to the proof of the existence of inertia.*

$$F = ma$$

Proof. The laws of statistical physics state that we can take the derivative of the entropy with respect to x and obtain F/T. For an entropic force, the derivative is

$$\frac{\partial S}{\partial x} = \frac{F}{T}$$

In the quasi-static approximation, the derivative becomes

$$F\Delta x = T\Delta S$$

Let us replace ΔS with theorem 3.8. We obtain

$$F\Delta x = T\left(2\pi k_B \frac{1}{\lambda} \Delta x\right)$$

$$F = 2\pi k_B T \frac{1}{\lambda} \tag{3.14}$$

Inspired by Erik Verlinde's paper on entropic gravity [18] , we take T

[18] Erik Verlinde. On the origin of gravity and the laws of newton. arXiv:1001.0785v1 [hep-th], 2010

to be Unruh's temperature [19] and we obtain

$$F = 2\pi k_B \left(\frac{1}{2\pi k_B} \frac{\hbar a}{c} \right) \frac{1}{\lambda}$$

Then, we take the algorithmic scaling factor λ to be, in fact, the Compton wavelength ($\lambda = \frac{\hbar}{mc}$). We obtain

$$F = 2\pi k_B \frac{1}{2\pi k_B} \frac{\hbar a}{c} \left(\frac{mc}{\hbar} \right)$$

Finally, it reduces to

$$F = ma$$

□

[19] Unruh's temperature: $k_B T = \frac{1}{2\pi} \frac{\hbar a}{c}$. The use of Unruh's temperature is justified by the same reasons given by Erik Verlinde. Unruh has shown that an observer in an accelerated frame of reference will measure a vacuum temperature of the given form.

3.6 *Newton's law of gravity*

Theorem 3.15. *Bits spread across the surface of a sphere will produce an entropic force governed by Newton's law of gravity. Namely, a force with the following formula.*

$$F = \frac{GmM}{r^2}$$

Proof. Here we abandon the volume entropy and consider that gravity is an entropic force created from the very bits that encode the universe at its most fundamental level. The number of classical bits on the surface is

$$N = \frac{c^3 4\pi r^2}{G\hbar}$$

From this equation, we can obtain the energy via the equipartition theorem which maps to the energy of a system to its number of degrees of freedom.[20]

[20] We will reproduce the proof by Erik Verlinde

$$\begin{aligned} \overline{E} &= \frac{1}{2} k_B T N && \text{(equipartition theorem)} \\ &= \frac{1}{2} k_B T \left(\frac{c^3 4\pi r^2}{G\hbar} \right) \end{aligned}$$

We can obtain the mass by dividing by c^2

$$M = \frac{1}{c^2} E = k_B T \frac{2\pi r^2 c}{\hbar G}$$

solving for T, we obtain

$$T = \frac{\hbar G M}{2\pi k_B c r^2}$$

We recall equation 3.14 and insert the T we just found in it and we obtain

$$F = 2\pi k_B T \frac{1}{\lambda}$$
$$F = 2\pi k_B \left(\frac{\hbar G M}{2\pi k_B c r^2} \right) \frac{1}{\lambda}$$

Again, posing that the algorithmic scale factor λ is in fact the Compton wavelength, we obtain

$$F = 2\pi k_B \frac{\hbar G M}{2\pi k_B c r^2} \left(\frac{mc}{\hbar} \right)$$

Note here that we introduce m as the mass of the Compton wavelength as opposed to M because we want to quantify the force felt between two objects of different masses. Finally, it reduces to

$$F = \frac{GmM}{r^2}$$

□

Erik Verlinde further generalizes entropic gravity to account for arbitrary matter distributions.

3.7 *Schrödinger's equation*

Theorem 3.16. *A position described by entropic positioning (theorem 3.8) will evolve in time according to Schrödinger's equation.*

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[\frac{-\hbar^2}{2m} \nabla^2 + V(x,t) \right] \psi(x,t)$$

The proof is slightly more involved than the preceding theorems. First, here is a sketch

1. We will show that entropic position encoding using the bits produced by the general halting partition leaves holes in space where a position cannot be expressed.

2. We will show that these holes are causing a Brownian motion of the encoded position.

3. We will derive its diffusion coefficient to be $\hbar/(2m)$.

4. We will consider that the presence of any external field is experienced as acceleration via $F = ma$.

5. Using the well known Brownian motion equations of Langevin, we show that the above reproduces Schrödinger's equation exactly.

Lemma 3.17. *Holes.*

Proof. We recall the general halting partition

$$Z = \sum_x e^{-(\ln 2)\beta(E+Fx)}$$

We have previously seen how positions can be encoded with PVPL. We have also seen that the observable x denotes program lengths. However, not all programs halt hence some lengths are missing from the sum. These missing programs are holes in space the position of which cannot be expressed by the general halting partition. Since Ω is a normal number, we can predict certain randomness related properties of these holes.

□

Lemma 3.18. *A particle in space will experience Nrownian motion due to the holes.*

Proof. We will calculate the average displacement $\overline{\Delta x}$ of a particle subjected to entropic positioning and space holes. Since Z is a normal number, we conclude that half of the program's lengths are available to describe position and half are not. Therefore, to describe a particle at position x, there is a 50% chance there is a halting program available to express it. And in the case where there is no program at exactly x, then there is a 50% chance that there will be one at position $x + 1$, and so on. In other words, a particle at x has 50% chance of being at x, 25% chance of being at $x + 1$, 12.5% chance of being at $x + 2$, etc. Expressed as a sum, we obtain

$$\begin{aligned}\overline{\Delta x} &= \frac{1}{2}0 + \frac{1}{4}1 + \frac{1}{8}2 + \frac{1}{16}3 + ... \\ &= \sum_{i=0}^{\infty} \frac{i}{2^{i+1}} \\ &= 1\end{aligned}$$

On average, as it moves through space, a position will shift by $\overline{\Delta x} = 1$ at each iteration of the Brownian motion. But will a stationary point also experience Brownian motion? The answer is yes. A stationary point will experience Brownian motion because of the degeneracy of the general halting partition. The general halting partition is degenerate to an observer because he does not know the specific Ω-value used by the universe. Therefore he must assume that all Ω are possible and that each forms a degenerate state. As the micro-states switch around the possible degeneracies, the holes are moved around. The probabilities are the same as the sum above and $\overline{\Delta x} = 1$ for a stationary point. □

Lemma 3.19. *The diffusion coefficient of the described Brownian motion is*

$$D = \frac{\hbar}{2m}$$

Proof. It is well known that in general the diffusion coefficient of Brownian motion is given by

$$D = \frac{l^2}{2\tau}$$

where l is the length of the random step and τ is the frequency of the occurrence of the steps. Entropic position uses the scale factor λ for each unit of length. When λ is the Compton wavelength, we get a scaling factor of

$$\lambda = \frac{\hbar}{mc}$$

Since entropic positioning can only express position as multiples of λ, we take it as the Brownian step of length l. The diffusion coefficient becomes

$$D = \left(\frac{\hbar}{mc}\right)^2 \frac{1}{2\tau}$$

This leaves of us with the need to define τ. For τ, we take the characteristic frequency of the wave $E = \hbar\omega$. Solving for $\tau = 1/\omega$, we obtain

$$\begin{aligned} \omega &= \frac{E}{\hbar} \\ \tau = \omega^{-1} &= \frac{\hbar}{E} \end{aligned}$$

Replacing τ in the equation for D, we obtain

$$D = \frac{\hbar^2}{m^2c^2}\left(\frac{E}{2\hbar}\right)$$

Using $E = mc^2$, and reducing the constants, we obtain our final expression of D,

$$\begin{aligned} D &= \frac{\hbar^2}{m^2c^2}\frac{(mc^2)}{2\hbar} \\ &= \frac{\hbar}{2m} \end{aligned}$$

□

Lemma 3.20. *The Langevin equations for Brownian motion with a diffusion coefficient of $\hbar/(2m)$ and an external field $F = ma$ reproduces Schrödinger's equation.*

Proof. We recall the well known Langevin equation,

$$d\left[x(t)\right] = v(t)dt \tag{3.21}$$

$$d\left[v(t)\right] = -\frac{\gamma}{m}v(t)dt + \frac{1}{m}W(t)dt \tag{3.22}$$

where $W(t)$ is a random force and a stochastic variable giving the effect of a background noise to the motion of the particle.

From $F = ma$ and replacing the acceleration $d[v(t)]/dt$ with F/m, Edward Nelson [21] is able to show that the Langevin equation becomes,

$$\frac{1}{2}\nabla u^2 + D\nabla^2 u = \frac{1}{m}\nabla V \tag{3.23}$$

where D is the diffusion coefficient of $\hbar/(2m)$ obtained in lemma 3.19, where $F = -\nabla V$, where $u = v\nabla \ln \rho$ and ρ is the probability density of $x(t)$. For brevity, the proof of 3.23 is omitted here but can be reviewed in his paper. Eliminating the gradients on each side and simplifying the constants, we obtain

$$\frac{m}{2}u^2 + \frac{\hbar}{2}\nabla u = V - E \tag{3.24}$$

where E is the arbitrary integration constant. This equation in non-linear because of the term u^2 but it can be made linear by a change of dependant variable. To make it linear, let us pose

$$u = \frac{\hbar}{m}\frac{1}{\psi}\nabla\psi$$

and replace it into equation 3.24 , we obtain

$$\frac{m}{2}\left(\frac{\hbar}{m}\frac{1}{\psi}\nabla\psi\right)^2 + \frac{\hbar}{2}\nabla\left(\frac{\hbar}{m}\frac{1}{\psi}\nabla\psi\right) = V - E$$

taking the gradients and the exponents, we obtain

$$\frac{\hbar^2}{2m}\frac{1}{\psi^2}\nabla^2\psi + \frac{\hbar^2}{2m}\left[-\frac{1}{\psi^2}\nabla^2\psi + \frac{1}{\psi}\nabla^2\psi\right] = V - E$$

The first two terms cancel each other.

$$\frac{\hbar^2}{2m}\frac{1}{\psi}\nabla^2\psi = V - E$$

[21] Edward Nelson. Derivation of schrodinger's equation from newtonian mechanics. `http://dieumsnh.qfb.umich.mx/archivoshistoricosMQ/ModernaHist/Nelson%20a.pdf`, 1966. Physical Review, Volume 150, Number 4

Finally, it simplifies to

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V - E\right]\psi = 0 \tag{3.25}$$

which is the time independent Schrödinger's equation.

□

We are now ready to derive the time dependent Schrödinger's equation and prove theorem 3.16.

Proof. We use the same proof used by Edward Nelson in the same paper. Starting from the time dependent Schrödinger's equation and show that a replacement of $\psi = e^{R+iS}$ leads to the Langevin equation of Brownian motion. We write the time dependent Schrödinger's equation, perform the replacement and obtain the Langevin equations of Brownian motion.

$$\frac{\partial\psi}{\partial t} = i\frac{\hbar}{2m}\nabla^2\psi - i\frac{1}{\hbar}V\psi \tag{3.26}$$

Replacing ψ with e^{R+iS}, we obtain

$$\frac{\partial\left(e^{R+iS}\right)}{\partial t} = i\frac{\hbar}{2m}\nabla^2\left(e^{R+iS}\right) - i\frac{1}{\hbar}V\left(e^{R+iS}\right)$$

Taking the derivatives and the gradients, we obtain

$$\left[\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t}\right]\left(e^{R+iS}\right) = \frac{i\hbar}{2m}\left[\nabla^2 R + i\nabla^2 S + (\nabla(R+iS))^2\right]\left(e^{R+iS}\right) - i\frac{1}{\hbar}V\left(e^{R+iS}\right)$$

Eliminating e^{R+iS} from each side and simplifying, we obtain

$$\begin{aligned}
\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t} &= \frac{i\hbar}{2m}\left[\nabla^2 R + i\nabla^2 S + (\nabla(R+iS))^2\right] - i\frac{1}{\hbar}V && \text{(eliminating } e^{R+iS}\text{)}\\
\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t} &= \frac{i\hbar}{2m}\left[\nabla^2 R + i\nabla^2 S + (\nabla R)^2 + 2i\nabla R\nabla S - (\nabla S)^2\right] - i\frac{1}{\hbar}V && \text{(taking the product)}\\
\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t} &= \frac{\hbar}{2m}\left[i\nabla^2 R - \nabla^2 S + i(\nabla R)^2 - 2\nabla R\nabla S - i(\nabla S)^2\right] - i\frac{1}{\hbar}V && \text{(distributing the } i\text{)}
\end{aligned}$$

We obtain two equations by separating the real and the imaginary parts

$$\frac{\partial R}{\partial t} = \frac{\hbar}{2m}\left[-\nabla^2 S - 2\nabla R\nabla S\right] \tag{3.27}$$

$$\frac{\partial S}{\partial t} = \frac{\hbar}{2m}\left[\nabla^2 R + (\nabla R)^2 - (\nabla S)^2\right] - \frac{1}{\hbar}V \tag{3.28}$$

This is equivalent to the Langevin equations with some replacements

$$\frac{\partial u}{\partial t} = -\frac{\hbar}{2m}\nabla^2 v - \nabla(v \cdot u) \tag{3.29}$$

$$\frac{\partial v}{\partial t} = \frac{\hbar}{2m}\nabla^2 u + \frac{1}{2}\nabla(u^2) - \frac{1}{2}\nabla(v^2) - \frac{1}{m}\nabla V \tag{3.30}$$

Lemma 3.31. *Equation 3.27 with the replacements $\nabla R = (m/\hbar)u$ and $\nabla S = (m/\hbar)v$ produces 3.29*

Proof.

$$\begin{aligned}
\frac{\partial R}{\partial t} &= \frac{\hbar}{2m}\left[-\nabla^2 S - 2\nabla R \nabla S\right] && \text{(equation 3.27)}\\
\nabla\frac{\partial R}{\partial t} &= \nabla\frac{\hbar}{2m}\left[-\nabla^2 S - 2\nabla R \nabla S\right] && \text{(multiplying by } \nabla\text{)}\\
\frac{\partial \nabla R}{\partial t} &= \nabla\frac{\hbar}{2m}\left[-\nabla\nabla S - 2\nabla R \nabla S\right] && \\
\frac{m}{\hbar}\frac{\partial u}{\partial t} &= \nabla\frac{\hbar}{2m}\left[-\nabla\left(\frac{m}{\hbar}v\right) - 2\left(\frac{m}{\hbar}u\right)\left(\frac{m}{\hbar}v\right)\right] && \text{(replacing } \nabla R \text{ and } \nabla S\text{)}\\
\frac{\partial u}{\partial t} &= \nabla\frac{\hbar}{2m}\left[-\nabla v - 2\frac{m}{\hbar}u \cdot v\right] && \text{(eliminating } m/\hbar\text{)}\\
\frac{\partial u}{\partial t} &= -\frac{\hbar}{2m}\nabla^2 v - \nabla(u \cdot v) && \text{(equation 3.29)}
\end{aligned}$$

□

Lemma 3.32. *Equation 3.28 with the replacements $\nabla R = (m/\hbar)u$ and $\nabla S = (m/\hbar)v$ produces 3.30*

Proof.

$$\begin{aligned}
\frac{\partial S}{\partial t} &= \frac{\hbar}{2m}\left[\nabla^2 R + (\nabla R)^2 - (\nabla S)^2\right] - \frac{1}{\hbar}V && \text{(equation 3.28)}\\
\nabla\frac{\partial S}{\partial t} &= \nabla\frac{\hbar}{2m}\left[\nabla\nabla R + (\nabla R)^2 - (\nabla S)^2\right] - \frac{1}{\hbar}\nabla V && \text{(multiplying by } \nabla\text{)}\\
\frac{m}{\hbar}\frac{\partial v}{\partial t} &= \nabla\frac{\hbar}{2m}\left[\nabla\left(\frac{m}{\hbar}u\right) + \left(\frac{m}{\hbar}u\right)^2 - \left(\frac{m}{\hbar}v\right)^2\right] - \frac{1}{\hbar}\nabla V && \text{(replacing } \nabla R \text{ and } \nabla S\text{)}\\
\frac{\partial v}{\partial t} &= \nabla\frac{\hbar}{2m}\left[\nabla u + \frac{m}{\hbar}u^2 - \frac{m}{\hbar}v^2\right] - \frac{1}{m}\nabla V && \text{(eliminating } m/\hbar\text{)}\\
\frac{\partial v}{\partial t} &= \frac{\hbar}{2m}\nabla^2 u + \frac{1}{2}\nabla(u^2) - \frac{1}{2}\nabla(v^2) - \frac{1}{m}\nabla V && \text{(equation 3.30)}
\end{aligned}$$

□

This completes the proof of theorem 3.16. □

4 Spacetime

WHY IS THERE A MAXIMUM SPEED IN THE UNIVERSE? In this section, we investigate the general halting partition augmented with the power-time observable $P \times t$.

$$Z = \sum e^{-\beta(E-Pt-Fx)} \tag{4.1}$$

The units of the term $P \times t$ are $(J/s) \times s = J$ and the units of $F \times x$ are $(J/m) \times m = J$. Here J is joules, P is a power in Watts, F is a force in newtons, t is a time in seconds and x is a distance in meters.

The fundamental thermodynamics state equation for this partition functions becomes,

$$dE = TdS - Pdt - Fdx \tag{4.2}$$

4.1 Light-cone

We work in the quasi static approximation

$$\Delta E = T\Delta S - P\Delta t - F\Delta x \tag{4.3}$$

We look at the thermodynamic cycle of the system transiting through time and space starting at P_x to P_0 to P_t to P_x as illustrated on Figure 4. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles such as those involving pressure and volume, etc.

We pose that $\Delta E = 0$ throughout the cycle.

P_x to P_0: As we translate P_x closer in space to P_0 while keeping the time fixed to $\Delta t = 0$, the entropy must decrease to compensate. Since entropy tends to increase, we conclude that objects have a tendency to resist being returned to the origin and are instead encouraged to expand away from each other.

$$\Delta S = \frac{F}{T}\Delta x \qquad (\Delta x \geq 0)$$

P_0 to P_t: As we translate P_0 forward in time to P_t while keeping the distance fixed to $\Delta x = 0$, the entropy must increase to compensate. We conclude that an object evolving forward in time is encouraged by entropic considerations.

$$\Delta S = \frac{P}{T}\Delta t \qquad (\Delta t \geq 0)$$

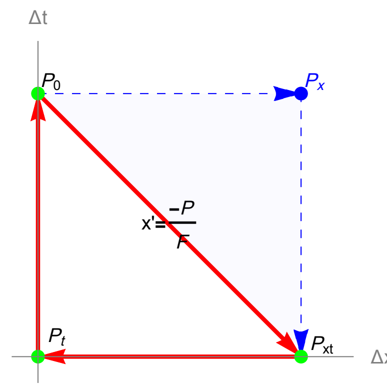

Figure 4: A thermodynamic cycle through space, time and entropy as observables.

P_x to P_t: As we translate P_x forward both in time and space to P_t, we have

$$\Delta S = \frac{P}{T}\Delta t + \frac{F}{T}\Delta x$$

and if $\Delta S = 0$, we have a translation at a constant speed given by

$$\frac{\Delta x}{\Delta t} = -\frac{P}{F}$$

We conclude that an object travelling at speed $-P/F$ is neither encouraged nor discouraged by entropic considerations. We will now look at this in more details.

4.2 *Speed of light*

Theorem 4.4. *The maximum speed of any object is a unique constant (c).*

Proof. Starting from equation 4.3 and posing $dE = 0$,

$$\begin{aligned} Fdx &= TdS - Pdt \\ \frac{dx}{dt} &= \frac{T}{F}\frac{dS}{dt} - \frac{P}{F} \\ v &= \frac{T}{F}\frac{dS}{dt} - \frac{P}{F} \end{aligned} \tag{4.5}$$

Note that the units for each term of equation 4.5 are meters per seconds. The equation therefore describes a speed.

Let us look at three cases:

1. If $|v| > |-P/F|$, then $dS/dt < 0$ and the entropy decreases with time. This violates the second law of thermodynamics.

2. If $|v| < |-P/F|$, then $dS/dt > 0$ and the entropy increases with time. This is fine.

3. If $|v| = |-P/F|$, then $dS/dt = 0$ and the entropy remains constant. This is also fine.

Since, according to the second law of thermodynamics, the average entropy cannot decrease with time, it follows that P/F is the fastest speed possible for a given system. Hence,

$$\frac{P}{F} = k$$

□

Taking the characteristic Planck units, we obtain

$$P\left(\frac{1}{F}\right) = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c$$

the speed of light - here proven to be an upper bound enforced by the second law of thermodynamics.

4.3 *Lorentz transformation*

As is well-known, a fixed maximum speed leads to special relativity and the Lorentz transformations. But what does that tell us about the entropy?

If we add two speeds together like this

$$\begin{aligned} v_1 + v_2 &= -c + \frac{T}{F}\frac{dS_1}{dt} - c + \frac{T}{F}\frac{dS_2}{dt} \\ &\neq v_f \end{aligned}$$

we do not get a faster speed, but instead two particles with two different speeds v_1 and v_2 forming a composite system. For the entropy to be additive, it must be the case that the two systems it describes must be statistically independent. Since this is not the present case, the entropies cannot be added linearly. Instead,

$$v_f = \gamma\left(v_1, v_2\right)$$

4.4 *General Relativity*

Analogous to the Newtonian case, Eric Verlinde shows that, assuming special relativity, the entropy derivation of gravity can be used to recover general relativity. Here we present a sketch of his proof. He starts with a gradient of bits over the surface of a sphere

$$dN = \frac{dA}{G\hbar}$$

The mass of these bits over a surface is obtainable via integration

$$\begin{aligned} M &= \frac{1}{2}\int_S T dN \\ &= \frac{1}{4\pi G}\int_S e^{\phi}\nabla\phi dA \end{aligned}$$

Where e^{ϕ} is the red shift that he derives earlier in his paper. This equation is known to be the natural generalization of Gauss's law to general relativity. He then argues that this is enough to recover the full description of general relativity. For brevity the proof will not be included. Instead we will refer to Eric Verlinde's paper for the rest.

4.5 *Quantum Field Theory*

Quantum field theory will be held to be recoverable using standard literature methods [22] and starting from theorem 3.16 (Schrödinger's equation) and theorem 4.4 (the speed of light as a maximum speed).

[22] Mark Srednicki. Quantum field theory. https://web.physics.ucsb.edu/~mark/ms-qft-DRAFT.pdf, 2006

5 Conclusion

In the previous sections we have seen that an agent able to prove his own existence via the cogito ergo sum will necessarily see a relativistic quantum universe described by a general halting partition. This demonstration was done by refusing to introduce axioms, thus avoiding the self-referential problem. Both general relativity and quantum field theory are derivable from the general halting partition. It seems therefore likely that the general halting partition is the theory of everything in physics.

5.1 Future research suggestions

- The PVPL encoding for position implies that distances far away from the origin have higher entropy than short distances. Therefore there should be a uniform push to expand positions away from each other. Could this be related to the expansion of the universe?
- Following Brownian motion along general relativistic geodesics and posing the field to be the acceleration of the particle might reveal a valid subset of quantum general relativity.
- The bit composition of Ω dictates quantum measurement results. Far in time (e.g. today), its composition should approach a normal distribution where a bit read is equally likely to a one or a zero. However, in the early universe, there should be significantly more zeros than ones in the estimation of Ω. Nuclear or molecular reactions that depend on quantum measurements in the early universe might be severely tilted away from the expected normal distribution of quantum measurement. Could this be related to the matter-antimatter asymmetry?

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