Abstract The de Broglie wavelength describes wave-particle duality. The de Broglie wavelength formula and the Planck law seem to be contradicted in tunneling. Tunneling fast waves have longer wavelengths than "normal" waves. According to the de Broglie formula, a longer wavelength means smaller momentum (smaller energy). But fast waves have the same amount of energy as normal waves, since they can be transformed into each other.

This longer wavelength is not based on the refractive index of the barrier. The barrier in tunneling cannot be seen as an optical medium, rather a special kind of space made out of matter that other matter is able to use as space. Here we show that the 'rest actions', 'rest energies' of fast waves in different spaces can resolve the contradiction. This 'rest action' of the wave is a new concept that hasn't been considered. It is hidden in the Planck constant. In uncovering this part, we find that the Planck constant has two parts; one part shows the 'rest action', 'rest energy' of fast wave and another part shows the 'kinetic action', 'kinetic energy' of fast waves. The Planck constant seems to have a more general role than we have previously thought.

Fast waves are made out of normal waves (or particles). Fast wave is the same particle in a different form. The Fast Wave–Wave–Particle Triality describes a new kind of metamorphosis of matter— how tunneling electrons travel faster than light without violating special relativity. Using the Fast Wave–Wave–Particle Triality, we can realize that the speed of light is not a speed limit for particles with mass, since they can be transformed into fast
waves. The Fast Wave–Wave–Particle Triality shows the end of scope of the special theory of relativity, and opens a new worldview.

**Keywords:** space-matter theory, fast light, fast wave, kinetic energy of fast wave, rest energy of fast wave, de Broglie wavelength, Planck law, Planck constant

**Wave–particle duality**

Wave–particle duality is the concept that all matter can exhibit two behaviors—a particle-like behavior and a wave-like behavior. In other words, every elementary particle or quantic entity may be partly described in terms not only of particles, but also of waves. The well-known de Broglie wavelength\(^1\) shows the connection between the momentum of the given particle \((p)\) and the Planck\(^2\) constant \((h)\). See Eq. (1).

\[
\lambda = \frac{h}{p}
\]

In general, the momentum of a particle that has mass is \(p = m \times v\), where \(m\) is the object's mass, and \(v\) is its velocity. The momentum of a particle that has no mass, e.g. a photon, is written in Eq. (2).

\[
p = \frac{E}{c},
\]

where \(E\) is the photon's energy and \(c\) is the speed of light in a vacuum. Theoretically vacuum is space void of matter. To be more precise, space’s vacuum is a medium from where everything is taken out that can be taken out and the “rest” remains there. When the medium is not the vacuum, Eq. (3) is used in calculations of phase-matching in nonlinear optics.

\[
p = n \frac{E}{c},
\]
where in general $n = \frac{c}{v}$, where $c$ is the speed of light in a vacuum, is the refractive index of a transparent optical medium, also called the index of refraction of the material in which the signal propagates. The index of refraction is the factor by which the phase velocity $v_{\text{phase}}$ is decreased relative to the velocity of light in a vacuum: $v_{\text{phase}} = \frac{c}{n}$. The phase velocity describes the velocity of the crests of the wave.

Phase velocity is given as $v_{\text{phase}} = \frac{\sigma}{k}$, where $\sigma$ is the angular frequency of the wave and $k$ is the wave number. From Eq. (2) and the Planck law showed in Eq. (4)

$$E = h \times f,$$

so Eq. (5) is the following.

$$p = \frac{E}{c} = \frac{h \times f}{c} = \frac{h}{\lambda},$$

where $f = \frac{\sigma}{2\pi}$ is the frequency of the wave. Eq. (2) and (5) show that there is a close connection between the Planck law, the Planck constant and the de Broglie wavelength. In evaluating the photon momentum in a given medium the phase velocity $v_{\text{phase}}$ is used.

Let’s see a strange $n$ in the fast light experiment carried out at the University of Rochester USA. In this experiment a normal light impulse travels on an optical medium and a fast light impulse $v_{\text{fl}} > c$ travels on normal light. Fast light? has a longer wavelength than normal light $\lambda_{\text{fl}} > \lambda_{\text{normal}}$ and a measurable superluminal velocity: $v_{\text{fl}} > c$. This velocity of impulse is group velocity (envelop), so the velocity of fast light $v_{\text{fl}} = v_{\text{group}} \neq v_{\text{phase}}$. The envelop (fast impulses) is built out of a spread of optical frequencies: out of sinusoidal (sine, cosine) component waves; their velocities are called phase velocities $v_{\text{phase}}$. 
The wavelength of the fast light increased compared to the wavelength of the normal light in a vacuum. It means that the wavelengths and velocities of its spectral component waves increased, compared to the wavelengths and velocities of spectral component waves of normal light. The velocities of the components waves of fast light are also superluminal velocities, that is, \( v_{\text{phase}} > c \). It means that the velocities of photons of the components waves of fast light are superluminal velocities, too. How can \( n \) shows the superluminal velcity? Here \( n - 1 < 0 \), and \( n \) can be a great number with negative sign. This value of \( n \) is not useable in Eq. (3)—without additional mathematical methods.

**Superluminal velocities in tunneling**

Quantum tunneling refers to the quantum mechanical phenomenon where a particle (with or without mass) tunnels through a barrier that it classically could not surmount. Here \( n \) refractive index doesn’t play any role in the velocities of particle. Particles that travel with superluminal velocities in tunneling will be called fast waves in the following.

First Nimtz\(^7\), Enders and Speiker measured superluminal tunneling velocity with microwaves in 1992. According to them, the puzzle is that the jump of the particle over the barrier has no time (it spends zero time inside the barrier) and the particle is undetectable in this condition. Where is the particle? The tunneling does take time, so this time can be measured.

\( \psi(x) \) is the phase wave function of the tunneling particle outside the barrier. According to Nimzt, the particle cannot spend time inside the barrier, because the wave function has no missing part (and no missing time). The tunneling method of the particle is unknown and immeasurable. If the wave doesn’t spend time inside the barrier, what is the tunneling time?
Nimtz supposes that the measured barrier traversal time is spent at the front boundary of the barrier.

The second riddle in tunneling: experiments show\(^8\) that the tunneling particles are faster than light, and these facts are not compatible with the theory of relativity. The growing velocity of the particle with a rest mass (for example electron) causes growing mass according to the theory of relativity, and if \(v \to c\), then \(m \to \infty\). Since the mass (of electron) won't be \(\infty\), and the tunneling is fact, we have to suppose that \(v = c\) never occurs. There is a discrete jump in the velocities, and after \(v < c\) occurs \(v > c\). How is it possible?

Nimzt\(^6\) measured that the tunneling time \(\tau\) approximately equals the oscillation time \(T\),

\[
\tau \approx T = \frac{1}{f_{\text{tun part}}},
\]

where \(f_{\text{tun part}}\) is the frequency of the tunneling particle. (The tunneling time equals approximately the reciprocal frequency of the wave of the particle.) Eq. (7) shows how the barrier traversal time is connected with energy

\[
\tau \approx \frac{h}{E_{\text{tun part}}},
\]

where \(E_{\text{tun part}}\) is the energy of the tunneling particle. That is, the bigger the energy of the particle, the higher its velocity, and the shorter its tunneling time.

We know that in tunneling there are more kinds of fast wave. Here photons without mass and electrons with mass travel with superluminal velocities. That is, the superluminal velocity in a given space made out of matter is possible.

**Wavelengths in tunneling**

If \(L\) is the length of the barrier, then the velocity of the tunneling particle can be given as
\[ v_{\text{tunnel}} = f_{\text{tunnel}} \times \lambda_{\text{tunnel}} = \frac{L}{\tau} \]  

(8)

\[ \frac{1}{T} \times \lambda_{\text{tunnel}} = \frac{1}{\tau} \times L \]  

(9)

\[ \lambda_{\text{tunnel}} \approx L \]  

(10)

Eg. (10) shows that the wavelength of the tunneling particle \( \lambda_{\text{tunnel}} \) is as long as the length of the barrier. It means, the tunneling particle has one wave inside the barrier. (We can also see something like this in the fast light experiment.)

The tunneling can be explained with the following. First we need three new definitions:

- Space is that the given matter is able use as space; matter is that the given space accepts as matter.
- Space waves. Space waves has been measured by LIGO. Matter uses waves of the given space as signal of reference.
- Space doesn’t work without time. Time is the action-reaction phenomenon between space and matter. Time appears for matter as the wave of space. Every space has its own time.

The space of the tunneling fast wave \( \psi_{\text{fw}}(x) \) is different from our Space, since its space is inside the barrier, or to be more precise: the barrier is its space. From our viewpoint the barrier is matter. In tunneling we cannot consider the barrier as an optical medium, since the barrier has a “normal” refractive index \( n > 1 \). \( \psi_{\text{fw}}(x) \) uses the matter (mass) as space, where space made out of matter has very long "space wavelengths". \( \lambda_{\text{mass}} \) is a very special data; in this case, this is the wavelength of space made out of matter, that is, the barrier acts as space this way. On the other hand, the \( \psi_{\text{fw}}(x) \) is a "normal" wave, which means there are no half (or part) waves inside the barrier.
In tunneling a given photon or electron particle makes two metamorphoses—first from a normal wave condition into an unknown condition (“it disappears via the tunneling”)\textsuperscript{12}, and after the tunneling it reappears as the same photon or electron it was. If the particle travels in the barrier, we cannot measure it, but it doesn’t disappear; it has a fast wave form, since its velocity is superluminal, travelling in a special kind of space.

Let’s take a look now at a tunneling photon in the space made out of matter (barrier). Every light wave works using the basic law of Eq. (11):

\[ v = f \times \lambda, \quad (11) \]

where \( v \) velocity depends on the space where the light propagates, in our space’s vacuum \( v = c \). Note using different spaces we don’t use different refracting indices, we use different phase velocities of photons (or electrons). The frequencies of the waves are not affected by above mentioned:

\[ f_\text{normal} = f_{fw}. \quad (12) \]

Now see Eq. (13):

\[ \lambda_\text{normal} < \lambda_{fw}. \quad (13) \]

Photons have two metamorphoses, so their energy must be the same in both cases—as a normal photon and as a fast wave in tunneling.

\[ E_\text{normal} = E_{fw}, \quad (14) \]

But we know that in the tunneling their wavelengths grow, so

\[ p_\text{normal} = \frac{\hbar}{\lambda_\text{normal}} > p_{fw} = \frac{\hbar}{\lambda_{fw}}. \quad (15) \]

Note that I don’t use velocities here; I use the wavelengths of photons. Eq. (15) shows that the two conditions of a photon don’t have the same momentum. But they must have the same
momentum (energy), since this is the same photon and a photon with larger amount of energy cannot be built out of a photon with less energy. How do we solve the problem?

**Fast wave–wave–particle triality**

Did $p_{\beta}$ and/or $h$ change?

1. $p_{\beta}$ mustn't change, since the law of conservation of momentum must remain true.

2. $h$ is a constant; we don't accept that it changes.

Now we can conclude that the de Broglie formula is not applicable to fast waves.

Or can we rewrite the de Broglie and Planck formulas in new ways that work with fast waves? Yes, we can. See the following equations. Eq. (12) is true, so $f \times \lambda_{fw} = v_{fw}$ and $f \times \lambda_c = c$; now we study two different spaces:

$$\frac{c}{\lambda_c} = \frac{v_{fw}}{\lambda_{fw}}.$$  \hspace{1cm} (16)

$$\lambda_c = \frac{c}{v_{fw}} \times \lambda_{fw}. $$ \hspace{1cm} (17)

Now we can rewrite Eq. (16) and (17),

$$h = \frac{c}{v_{fw}} \times \lambda_{fw},$$  \hspace{1cm} (18)

$$\lambda_{fw} = \frac{v_{fw} \times h}{c \times p} = \frac{v_{fw} \times h}{c \times \frac{1}{p}}.$$  \hspace{1cm} (19)

If $v_{fw} = c$, then we get back the original formula from Eq. (4).

What does Eq. (19) mean? First of all it means that $h$ exists in every space. It always appears as one unity, but it has two hidden parts. One part of it can grow in the case of fast light. Since $h$ is a constant, it needs to have another part that decreases in the same time with the same scale.
In plain English: the Planck constant has two parts that work together. One part of it depends on the velocity of the fast wave; this part is shown in Eq. (17). This is the part of the kinetic energy that increases \( h \) in the case of fast light. In different spaces, the two parts can change in different directions.

We know from the above-mentioned that all forms of a photon have the same amounts of energy. So, the Planck constant must have a part that makes this result possible. There must exist a factor that reduces this part of \( h \).

Saying this, we can rewrite the Planck law in this form:

\[
E_\beta = (f_{fw} \times (h \times \frac{c}{v_{fw}})) \times (\frac{v_{fw}}{c}) = f_{fw} \times h. \tag{20}
\]

\[
E_\beta = \frac{f_{fw}}{h} \times ((h \times \frac{c}{v_{fw}}) \times (h \times \frac{v_{fw}}{c})) = f_{fw} \times h, \tag{21}
\]

where \( h_{rest} = h \times \frac{c}{v_{fw}} \) is the rest energy part and \( h_{kinetic} = h \times \frac{v_{fw}}{c} \) is the kinetic energy part of the Planck constant—in the case of fast light.

\[
E_\beta = \frac{f_{fw}}{h} \times h_{rest} \times h_{kinetic} = f_{fw} \times \frac{h_{rest} \times h_{kinetic}}{h}. \tag{22}
\]

Eq. (20), (21) and (22) mean that every particle has a 'rest action', 'rest energy'\(^{13}\). The de Broglie formula and Planck’s law remain untouched, if \( v_{fw} = c \).

Fast waves propagate in a different space compared to normal waves and not in a different (optical) medium. That is, light can use matter as space. The statement can be expressed in a more general form. Nowadays the barrier is seen just as a barrier made out of matter. But in barriers photons and electrons travel faster than light. They travel in the barrier as fast waves. In this case, they use a matter as space, and in this space they have new forms—fast waves.

Knowing Eq. (22), there is more than one space, and the Planck constant remains true in every space. The Planck constant seems to have a more important role than we thought.
Using the de Broglie formula and the Planck law in different spaces, we have a passage between particles and fast waves. So there is a 'fast wave–wave–particle triality' instead of the 'wave–particle duality'.

**When can particles or waves turn into fast waves?**

Particles or waves turn into fast waves, if the particle-space relation compels this state. We know, the fast wave comes into being if the space is made out matter. In this case the densities and the energies of particle and the given space are specified by Eq. (23)-(26).

If the density of an object is smaller than the density of space, this object can act as space from the viewpoint of a third object, and can act as matter from the viewpoint of another object. According to my Space-Matter Theory the density of space $D_{\text{Space}}$ can be calculated. It has the biggest density, matter $D_{\text{object}}$ has lower densities cf. Ref. 10. The tunneling works this way. The barrier is made out of matter, but electrons and photons use it as space. If

$$D_{\text{object}1} < D_{\text{Space}} \text{ and } D_{\text{object}2} < D_{\text{Space}},$$

then both objects are matter in Space. If Eq. (23), (24) are true:

$$d_{\text{min}} \geq \frac{D_{\text{object}1}}{D_{\text{object}2}} \geq d_{\text{max}},$$

then object$_2$ can use object$_1$ as space. The values of $d_{\text{min}}$ and $d_{\text{max}}$ will specify the relationship between matter and matter—making space out of one matter, if Eq. (25) is true

$$e_{\text{max}} \geq \frac{E_{\text{object}1}}{E_{\text{object}2}} \geq e_{\text{min}},$$

where $E_{\text{object}i}$ is the energy of the object$_1$ and object$_2$.

The original version of the de Broglie wavelength means that particle turns into wave, if

$$\lambda_{\text{wave}} \geq l_{\text{particle}},$$

where $l_{\text{particle}}$ is the size (length) of the particle. In this relationship the wavelengths of space is not involved. Actually we had to calculate with the wavelengths of space waves, but in our
normal circumstances Eq. (27) is always true, so Eq. (26) is enough to know studying the de Broglie wavelength.

\[ \lambda_{\text{space-wave}} \ll \lambda_{\text{particle}} \]  \hspace{1cm} (27)

In tunneling (and in some other cases) Eq. (27) is not true. Now we need to calculate with the wavelengths of space(s), too.

Particle and wave turn into fast wave, if Eq. (23)-(26) and Eq (28) fulfilled:

\[ \lambda_{\text{space-m}} \geq \lambda_{\text{particle}} \quad \text{and} \quad \lambda_{\text{space-m}} \geq l_{\text{particle}} \] \hspace{1cm} (28)

In Eq. (28) space is made out of matter and it wavelength is \( \lambda_{\text{space-m}} \). Eq. (29) shows it a more general form,

\[ \lambda_{\text{space}} \geq \lambda_{\text{particle}} \quad \text{and} \quad \lambda_{\text{space}} \geq l_{\text{particle}}, \] \hspace{1cm} (29)

where any kind of space made out of space or matter appears as space for the given particle (or set of particles).

The fast wave–wave–particle triality makes possible to understand how can matter suit itself to the given space.

What is the 'fast wave–wave–particle triality' good for?

This concept is able to explain how tunneling and spooky action work. In tunneling, a barrier made out of matter works as space. In this 'barrier space' the particles (for example photons, electrons) travel faster than \( c \). Nimtz, Enders and Spieker have measured superluminal tunneling velocities since 1992. The tunneling electrons travel in this 'barrier space' faster than light. They seem to violate the special relativity, states Nimtz. They don't. Why? The reason is very simply: there is an end of scope of the special theory of relativity. The special theory of relativity doesn’t valid in tunneling and cannot answer the question: How can particles with masses travel with superluminal velocity? The tunneling electron loses its mass
and acts as a fast wave. When it leaves the 'barrier space' and enters 'our normal space', it gets back its mass. Tunneling particles use the fast wave–wave–particle triality. Using fast wave–wave–particle triality we can see into a black hole giving new concepts and calculations. Black hole doesn’t mean a singularity any more; it means new kinds of space, matter and time we can study. This is the case in the dark matter, too, using fast wave–wave–particle triality.

References

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