Conjecture on the pairs of primes obtained inserting n with digit sum 12 after the first digit of twin primes

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Abstract. In this paper I conjecture that for any pair of twin primes [p, q], p ≥ 11, there exist a number n having the sum of its digits equal to 12 such that inserting n after the first digit of p respectively q are obtained two primes (almost always twins, as in the case [1481, 1483] where n = 48 is inserted in [11, 13], beside the case that the first digit of twins is different, as in the case [5669, 6661] where n = 66 is inserted in [59, 61]).

Conjecture:

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The least pairs of primes obtained from twins [p, q], p ≥ 11:
(inserting n with digit sum 12 after the first digit of p respectively q)

: [1481, 1483], [1487, 1489], [2399, 3391], [4481, 4483], [5669, 6661], [71471, 71473], [13901, 13903], [125507, 125509], [15737, 15739], [16649, 16651], [17579, 17581], [16691, 16693], [13997, 13999], [27527, 27529], [27539, 27541], [216569, 216571], [26681, 26683], [334511, 334513], [34847, 34849], [425519, 425521], [49331, 49333], [43961, 43963], [521821, 521823], [517469, 517471], [55799, 65701], [65717, 65719], [614741, 614743], [631859, 631861], [84809, 84811], [815621, 815623], [86627, 86629], [84857, 84859], [822881, 822883], [1291019, 1291021], [1156031, 1156033], [157049, 157051], [148061, 148063], [1174091, 1174093], [148151, 148153], [157229, 157231] (...)

The corresponding values of [p, q, n] for the forty terms above:

[11, 13, 48], [17, 19, 48], [29, 31, 39], [41, 43, 48], [59, 61, 66], [71, 73, 147], [101, 103, 39], [107, 109, 255], [137, 139, 57], [149, 151, 66], [191, 193, 66], [197, 199, 39], [227, 229, 75], [239, 241, 75], [269, 271, 165], [281, 283, 66], [311, 313, 345], [347, 349, 48], [419, 421, 255], [431, 433, 93], [461, 463, 39], [521, 523, 218], [569, 571, 174], [599, 601, 57], [617, 619, 57], [641, 643, 147], [659, 661, 318], [809, 811, 48], [821, 823, 156], [827, 829, 66], [857, 859, 48], [881, 883, 228], [1019, 1021, 291], [1031, 1033, 156], [1049, 1051, 57], [1061, 1063, 48], [1091, 1093, 174], [1151, 1153, 48], [1229, 1231, 57].

Note that n is not greater than 345 (which is just the 29th highest number having the digit sum 12) for any pair above!

Note that the conjecture implies that that the set of twin primes is infinite: for [11, 13] the least pair of twins obtained by the method above is [1481, 1483] but the conjecture says that for [1481, 1483] we also have such a pair of twins (indeed, the least one is [1291481, 1291483] obtained for n = 291) an so on (the least one for [1291481, 1291483] is [1219291481, 1219291483] obtained for n = 219; the least one for [1219291481, 1219291483] is [1129219291481, 1129219291483] obtained for n = 129; the least one for [1129219291481, 1129219291483] is [157129219291481, 157129219291483] obtained for n = 57).