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# The Spacetime of Noninertial Frame of Reference without Gravitation

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**Abstract.** We have discussed the characteristics of the space-time of non-inertial frame of reference (NFR) and proposed the Requirement of General Completeness (RGC) and the Principle of Equality of All Frames of Reference (PEAFR) in present paper. The RGC is that the physical equations used to describe the dynamics of matter should include the descriptions that the matter rest and move (uniformly rectilinearly and acceleratedly) relative to a frame of reference, and consider the structure of the space-time of frame of reference. The PEAFR is that any frame of reference can be used to describe the motion of matter and the equations of General Completeness in all frames of reference are identical. According to the RGC and the PEAFR, we can deduce that the general complete physical equations are covariant. The descriptions of the space-time of NFR can be dealt without gravitation. The space-time of NFR is inhomogeneous and/or anisotropic caused by the accelerating motion. The inertial force is the manifestation of the deformed space-time. The Riemann curvature tensor of the space-time of NFR equals zero, but the affine connection does not vanish no matter what coordinate system be selected in the NFR. Mach's principle is incorrect.

**PACS.** 03.30.+p Special relativity – 04.20.-q Classical general relativity – 04.20.Cv Fundamental problems and general formalism – 04.20.Gz Spacetime topology

## 1 Introduction

Regarding the space-time of non-inertial frame of reference (NFR), the most physicists hold following opinions: 1) the inertial force can be regarded as a kind of gravitation (the principle of equivalence); 2) a NFR can be considered as an inertial frame of reference (IFR) with a gravitational field (the premise of the principle of general relativity); 3) the source of the inertial force field is far away galaxies (Mach principle) [1–6]. Some authors even obtained the spacetime metric of the NFR by solving the Einstein's field equations in vacuum [7,8]. Do we have another better option to describe the space-time of NFR? Can we describe the space-time of NFR without gravitation?

The problems of that Special Relativity (SR) could not easily be extended to NFR and of that the law of Newton gravity did not satisfy Lorentz covariance headed Einstein, at that time, toward to propose above opinions and to develop General Relativity (GR). People often think that this is Einstein's masterpiece, killing two birds with one stone. Now we know the reason of that the law of Newton gravity does not satisfy Lorentz covariance is that Poisson equation of gravitational field is not "Special Complete" physical equation [9]. The Requirement of Special Completeness (RSC) is that the physical equations used to describe the dynamics of matter and/or fields should include the descriptions that not only the matter and/or fields are at rest relative to an IFR, but also they move relative to this frame. The covariance is a basic characteristic of Special Complete physical equations. The main reason of that Einstein could not extend his Special Relativity without difficulty to establish the theory of space-time of NFR was that he placed the propagation property of light in the central position of his theory because it is very difficult to synchronize the clocks in a NFR. Now a new approach to Special Relativity without light has been proposed by G. Liu [9]. The theory of Lorentz covariance has had a solid foundation without light. It is time to re-investigate the problems of that Einstein met at that time and reconsider the opinions of that Einstein proposed.

The problem of a NFR and the problem of gravitation are two separate problems and should not be considered together. In principle, the problem of gravitation in an IFR can be solved by finding the gravitational field equations of Lorentz covariance [10], while the problem of space-time of NFR can be solved independently. The accelerating motion is absolute and is relative to all IFR. Distinguishing between an IFR and a NFR is a simple task. For instance, we can discover the rotation of the earth through observing the motion of a pendulum. When we are sitting in a vehicle,

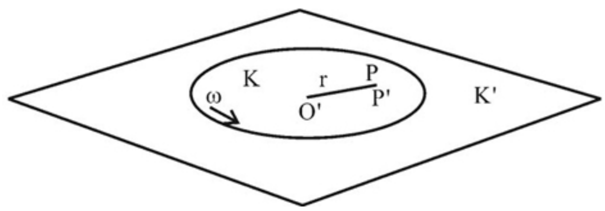


**Fig. 1.** On the frame of reference K and $K'$, to describe an event.

we can sense the acceleration of the vehicle. Moreover, in GR, it is also necessary to distinguish whether there is a gravitational field in the frame of reference. This is merely another way to distinguish an IFR and a NFR. An IFR is a special situation of NFR and is also a more abstract frame of reference. It can be predicted that an IFR is more superior at observing and describing the motion of matter. The frame of reference at the Sun's center of mass is more superior to the frame of reference at the earth's center of mass. This can be seen by comparing the complexity of Ptolemy's geocentric model with Copernicus's heliocentric model. We will see that a NFR can also be used to describe the motion of matter, while do not need to assume there is a gravitational field and use Mach principle. Inertial force is not gravitation. Gravitational field is real field, while the inertial force is the manifestation of the deformation of space-time due to the accelerating motion of the NFR relative to all IFR.

In present paper, we will try to extend the Special Completeness to the General Completeness in which the structural characteristic of the space-time of NFR has considered and the equality of inertial frames of reference (EIFR) to the equality of all frames of reference (EAFR) by considering the structural characteristic of the space-time of NFR, then, establish the theory of the space-time of NFR without considering gravitation. In Chapter II, we will investigate the characteristics and structure of the space-time of NFR; In Chapter III, we will discuss Mach principle; In Chapter IV, the Requirement of General Completeness (RGC) and the Principle of Equality of All Frames of Reference (PEAFR) have been proposed; In Chapter V, we will discuss the covariance of the equations of General Completeness.

## 2 The Characteristic of Space-time in the NFR

### 2.1 The observations on a rotating plane

Fig.1, on the inertial plane $K'$, there is a rotating plane K which rotates around the center $O'$ at the angular speed $\omega$. The observations of the motions of matter on the inertial plane $K'$ satisfy the theory of Minkowski space-time. The observer on the plane $K'$ places the standard ruler and the standard clock at point $P'$. The observer at the point $P$ on the rotating plane $K$ has the identical standard ruler and clock with point $P'$ and rotates with the rotating plane $K$. During the infinitesimal time interval when point $P$ passes point $P'$, the infinitesimal local space around $P$ can be seen as an IFR. This is because every point in this local space has same velocity during this transient moment. In this local IFR $P$, we observe the standard clock and ruler at point $P'$ on plane $K'$. We suppose that the clock at point $P'$ on plane $K'$ runs past $d\tau$ and the length of standard ruler at point $P'$ which is perpendicular to $\overline{O'P'}$ is $dl_0$. According SR, we obtain

$$dt = d\tau/\sqrt{1 - v_p^2/c^2}, \tag{1}$$

$$dl = dl_0\sqrt{1 - v_p^2/c^2}. \tag{2}$$

Here $dt$ and $dl$ are respectively measured by the observer at point $P$. $c$ is the speed of light, $v_p$ is the speed of point $P$ relative to plane $K'$.

According to formula (1) and (2), we can see that $dt$ and $dl$ are the function of space-time. For the same $d\tau$ and $dl_0$, the $dt$ and $dl$ are not same at different points of space-time on the rotating plane because $v_p$ is a function of space-time.

## 2.2 The structure of space-time in the NFR

The observation of space-time on the rotating plane indicates that any point of the space-time in a NFR is a local IFR. The differences observed physical quantities between the local IFR and the corresponding point of the space-time of the IFR come down to the differences of space-time. The moving status of a NFR can be certain as soon as an IFR is selected, thereby the differences of space-time are certain between every local IFR of the NFR and the corresponding point of space-time of the IFR. The space-time of the NFR is the united entirety of infinite local IFRs. In each local IFR, the observations are different to the same length and time at the corresponding point of space-time of the IFR. Therefore, the space-time of a NFR is inhomogeneous and/or anisotropic.

We cannot establish a homogeneous coordinate system in a whole NFR, and just can introduce a soft curvilinear coordinate system. The coordinate transformation can only be nonlinear transformations between the NFR and the IFR. We suppose that there is a NFR $Q$ and an IFR $Q'$, the coordinate system in the frame of reference $Q$ is $x^\mu$, the coordinate system in the frame of reference $Q'$ is $x'^i$. The transformation of coordinate of the two space-times is

$$x'^i = x'^i(x^\mu). \tag{3}$$

Here, $i$ and $\mu$ take 0, 1, 2, 3. In frame of reference $Q'$, the space-time is Minkowski space-time and the metric of space-time is

$$ds^2 = \eta_{ij} dx'^i dx'^j, \tag{4}$$

here $\eta_{ij}$ is the metric tensor of Minkowski space-time.

According to the formula (3), we obtain

$$dx'^i = \frac{\partial x'^i}{\partial x^\mu} dx^\mu, \tag{5}$$

$$dx'^j = \frac{\partial x'^j}{\partial x^\nu} dx^\nu. \tag{6}$$

Substituting above two formulas to (4), we gain

$$ds^2 = \eta_{ij} \frac{\partial x'^i}{\partial x^\mu} \frac{\partial x'^j}{\partial x^\nu} dx^\mu dx^\nu. \tag{7}$$

Introduce

$$\gamma_{\mu\nu} = \eta_{ij} \frac{\partial x'^i}{\partial x^\mu} \frac{\partial x'^j}{\partial x^\nu}, \tag{8}$$

$\gamma_{\mu\nu}$ is obviously the function of $x^\mu$. We have

$$ds^2 = \gamma_{\mu\nu} dx^\mu dx^\nu. \tag{9}$$

In any certain neighborhood of a point of space-time, $\gamma_{\mu\nu}$ is approximately a constant, so this neighborhood can also approximately be seen as a local Euclidean space-time. This is just the local IFR of a NFR. Now we can conclude that the structure of space-time in a NFR $Q$ is a Riemann space-time, its metric is formula (9), and the local Euclidean space-time in any point of space-time is the tangent space of the Riemann space-time.

We can see, in a NFR, its affine connection cannot be zero everywhere, even its Riemann curvature is zero, because we cannot establish a Cartesian coordinate system in the whole NFR to describe the motion of matter. This is the intrinsic characteristic of the deformed space-time of a NFR. It is worthy to clarify that the characteristic of space-time is decided by the motion of matter relative to the frame of reference, and the moving status of the frame of reference will naturally affect the observation and the description of the motion of matter, thereby, decide its structure of space-time. It does not relate to gravitation! We cannot find a transformation of coordinate in a NFR to eliminate the deformation of space-time. Although we can find a transformation of coordinate, for instance, one may use formula (3), to eliminate its deformation, but actually this transformation has already transferred from a NFR to an IFR. The structure of space-time in an NFR belongs to Riemann space-time, but its Riemann curvature is zero. It is also different from the structure of space-time in the IFR. We call this kind of space-time as Non-inertia Space-time, and call the flat, isotropic and homogeneous space-time as Inertia Space-time.

If one asks why a free body in an IFR is always at rest or maintains rectilinear uniform motion and a free body in a NFR move along with a curved line, the answer is because the space-time in the IFR is isotropic and homogeneous, while the space-time in the NFR is inhomogeneous and/or anisotropic. In Physics, the IFR is a special situation of the NFR in which acceleration is zero. In Mathematics, Minkowski space-time (or Inertia Space-time) in the IFR is a special situation of Non-inertia Space-time when the metric tensor $\gamma_{\mu\nu} = \eta_{ij}$. Both are the special situation of Riemann space-time.

## 3 The Objection of Mach Principle

"The idea that inertia represents the effects of interactions with faraway matter was first developed by Ernst Mach in the nineteenth century" (p.70) [5]. This idea called Mach Principle. No matter if this was Mach's real intention [11], anyway, "it was one of the powerful ideas that Einstein had in mind as he constructed his theory of gravitation" (p.70) [5]. "Einstein (1913b) wrote to Ernst Mach to express his appreciation for the inspiration that he had derived for his endeavors from Mach's ideas" (p.543) [6].

In a NFR, all free particles move with an acceleration even if there are no external forces acting on. The reason causing this accelerating motion, according to Mach principle, is that the free particle is acted on by the gravitation produced by the far away matter in universe. If we consider an empty space containing only an accelerating frame of reference and a testing particle, the inertial effect will occur. The accelerating motion of the testing particle would certainly not be affected by far away galaxies which do not exist completely. To say that there is a gravitational field is also false. Until now, no experiment supports Mach principle [12–14].

The gravitational field like the electromagnetic field and other fields is a real field which is related to the gravitational source, the tensor of energy and momentum [10]. If not having any matter and/or field there, where does the gravitational field come from? Furthermore, the accelerating motion of the testing particle relative to the NFR occurs instantaneously and the speed of propagation of gravitational force cannot be infinite.

We think that the accelerating motion of a free particle relative to a NFR can only be attributed to the deformation of space-time. This is the same as that the differences of the observed physical quantities between the IFRs are attributed to the differences of the space-time between the IFRs. In the Chapter II, we have already discussed that the space-time of a NFR is different from the space-time of an IFR. The space-time of a NFR is inhomogeneous and/or anisotropic. Certainly, the root cause of the non-homogenization or deformation of space-time is still the accelerating motion of frame of reference. The accelerating motion of frame of reference is relative to the any IFR. Newton's absolute space cannot be found. A convenient choice of the IFR is the one which is at rest relative to a faraway galaxy. At least, this is relatively better one. This probably is Mach's original idea [11]. "In his book, The Science of Mechanics, Mach [(1912), Chapter 2, section 6] had reasoned that it could not make sense to speak of the acceleration of a mass relative to absolute space. Anyone trying to clear physics of mystical ideas would do better to speak of acceleration relative to the distant stars (p.543)" [6].

Einstein's opinion with regard to the inertial force is the precondition of that he proposed the principle of general relativity. Seemly, only the inertial force is regarded as gravitation, then, the IFR and the NFR can be regarded as being equal for describing the physical phenomena. Einstein's logical clue is that a NFR is regarded as an IFR with a gravitational field, then, according to the principle of special relativity, the both frames are equality [3,4]. This logic is incorrect! It makes the theory of the space-time of NFR complicated. We will see in following chapter that any frames of reference are equal to describe the motions of matter without considering gravitation, only considering the structure of space-time of the frame.

## 4 Basic Principles

In this chapter, we will discuss two principles. These are the premises that we establish the theory of the space-time of non-inertial frames of reference.

### 4.1 The reqirement of general completeness (RGC)

A new requirement to the physical equations, the Requirement of Special Completeness (RSC), has been proposed by G. Liu [9], namely, the physical equations used to describe the dynamics of matter and/or fields should include the descriptions that not only the matter and/or fields are at rest relative to an IFR, but also they move relative to this frame. Combining this requirement and the equality of inertial frames of reference (EIFR), G. Liu has proposed a new approach to the theory of Lorentz covariance without light. The covariance of physical equations is attributed to the Completeness of physical equations and the EIFR. Actually, according to above analysis we can see that, regardless of the IFRs or the NFRs, all of them can be used to observe and describe the motions of matter, their differences are the differences of characteristics of their space-time. To be exact, its geometrical characteristics of the space-time are different, while the Minkowski space-time is actually the special situation of the non-inertia space-time of NFR. Certainly the differences of the space-time are caused by the motion of matter relative to the frame of reference. The frame of reference moved in different status has different space-time. So called inertial force or field represents the effects of the deformation of space-time. Therefore, if we extend the Special Completeness to the NFR, a more general condition to replace "the inertial frame of reference" is that "the structural characteristic of the space-time of the frame of reference has been considered in the physical equations". Actually, in the statement of the Special Completeness, the condition, "the IFR", just requires us to consider the structural characteristic of Minkowski space-time in the equations. So we state the Requirement of General Completeness like this: the physical equations used to describe the dynamics of matter and/or fields should include the descriptions that the matter and/or fields rest and move (uniformly rectilinearly and accelerated) relative to frame of reference, and consider the structure of the space-time of frame of reference. The physical equations satisfied this requirement are called General Complete equations. We say these equations have the General Completeness.

### 4.2 The principle of equality of all frames of reference (PEAFR)

What is the relationship between the equations of General Completeness in each frame of reference? For one series of motions of matter, all observers in every frame of reference can establish themselves equations of General Completeness. We have already known, in the situation that metric tenser $\gamma_{\mu\nu}$ can be expressed to Minkowski metric $\eta_{ij}$, namely when the frame of reference is an IFR, the form of the equations of General Completeness (now we should call them the equations of Special Completeness) in every frame of reference is same. In this situation, because the metric tensor in every frame of reference is the same, the equations of General Completeness are not only identical in form, but also completely identical. For normal frames of reference, the metric tensors $\gamma_{\mu\nu}$ in every frame of reference are different, and that the metric tensors will affect its meaning of the equations of General Completeness. This is because the difference of structure of space-time between each frame of reference reflects in the difference of metric tensor and the covariant differential of tensor. Therefore, any frame of reference can be used to describe the motion of matter and/or field and the equations or laws of General Completeness in all frames of reference are identical in form. We call this as the Principle of Equality of All Frames of Reference.

## 5 Covariance of the Equations of General Completeness

Supposing there are a series of motions of same kind of matter, for instance, the electromagnetic motions, for two arbitrarily selected frames of reference, by making a series of observations, the observers in the frames of reference can establish their own equations of General Completeness. According to the PEAFR, we know that the equations of General Completeness are identical. Due to the difference of moving status of two frames of reference, their observations (for instance, the observations to a moving charger $q$) are different. Just like that has been pointed out in the paper [9], the differences of the observations finally only come down to the differences of space-time. Here show us again that space-time is the extensibility and the continuity of motion of matte relative to a frame of reference. If not having the frame of reference, the motion of matter could not be described. If not having the motion of matter, space and time will be meaningless. The differences of space-time embody in the transformation of coordinate of space-time between the frames of reference. Therefore, according to the PEAFR, we can reason that the equations of General Completeness remain identical under transformation of coordinate between all frames of reference. We call this as the Covariance of the Equations of General Completeness.

How to obtain the equations of General Completeness? We have known that tensor expression of physical equations is best way to manifest the Requirement of the General Completeness. Here, the tensor expression has included the considerations of the metric tensor of space-time. The covariant differential has two characteristics: first, it changes from tensor to tensor; second, when the frame of reference is inertial frame of reference and we select the Cartesian coordinates, so $\Gamma^{\mu}_{\nu\lambda}$=0, it becomes to normal differential. These characteristics enlighten us by using the following arithmetical method to obtain the equations of General Completeness: write the equations of Special Completeness in terms of tensor forms in IFR, then, replace $\eta_{ij}$ by using $\gamma_{\mu\nu}$, and replace all of the differentials by using the covariant differentials. The equations obtained in this way are covariant with respect to all transformations of coordinates of space-time between all frames of reference.

Both of changing the physical equations into the General Complete form and changing the physical equations into the general relativistic form are similar. We can borrow this kind of achievements of General Relativity, but it must be stressed that the equations of General Completeness do not have relationship with the principle of equivalence and gravitational field.

## 6 Summary

In present paper, we have discussed the characteristics of space-time of NFR. The space-time of NFR is deformed. It is inhomogeneous and/or anisotropic. The deformation of space-time is caused by the relativity of space-time. The inertial force is only attributed to the deformation of space-time. We must emphasize here: the moving status of the frame of reference affects the observation of the motion of matter, thereby, decides its structure of space-time, and it does not relate to gravitation! We cannot find a transformation of coordinate in a NFR to eliminate the deformation of space-time. The IFR and the NFR can be distinguished. Their differences are the differences of structure of space-time. In Mathematics, the deformation embodies on the metric tensor of space-time. Basing on the discussions about the space-time of NFR, we have proposed the RGC and the PEAFR. The tensor expression of physical equations is best way to manifest the RGC and the PEAFR. All physical equations should be written as the General Complete tensor equations. They are covariant. The accelerating motion is absolute. The superior frame of reference on the observation and description does exist. Mach principle is incorrect! A real gravitational field should been described in IFR and NFR.

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## References

1. A. Einstein, *The Meaning of Relativity, 5th edition*, (Princeton University Press, Princeton, New Jersey, 1970).
2. A. Einstein, *Relativity–the special and general Theory, Third Edition*, (METHUEN and GO. LTD., LONDON, 1920).
3. L. Liu, *General Relativity*, (Advanced Education Publishing Company, Beijing, 2004), P.8-12.
4. S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, (John Wiley and Sons, Inc., New York, 1972), p.17-20.
5. R. Feynman, *Feynman Lecture on Gravitation*, (Addison-Wesley Publishing Company, New York, 1995).
6. C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, (W. H. Freeman and Company, San Francisco, 1973).
7. J. Deng, X. Hu, Y. Ding, D. Xue, and Y. Feng, *Commun. Theor. Phys. (Beijing, China)*, **52**, 75(2009). doi:10.1088/0253-6102/52/1/16
8. C. Moller, *The Theory of Relativity*, (Clarendon, Oxford 1952), Chap. VIII.
9. G. Liu, *Physics Essays*, **27**, 173(2014). DOI: 10.4006/0836-1398-27.1.173
10. G. Liu, *International Journal of Astronomy and Astrophysics*, **3**, 8(2013). DOI: 10.4236/ijaa.2013.31002
11. J. B. Barbour and H. Pfister, *Mach's Principle: From Newton's Bucket to Quantum Gravity*, (Birkhauser, Boston 1995).
12. V. W. Hughes, H. G. Robinson, and V. Beltran-Lopezet, *Phys. Rev. Lett.*, **4**, 342(1960). DOI: 10.1103/PhysRevLett.4.342
13. E. V. Ivash, R. F. Sweet, *IL Nuovo Cimento*, **XXII**, (1961) 352. DOI 10.1007/BF02783025
14. L. Liu, *General Relativity*, (Advanced Education Publishing Company, Beijing, 2004), p.96.