Providing an explanation of dark energy has proved difficult and elusive. Using the principles of Informativity, a model based on counts of the fundamental measures—length, time and mass—an understanding of the expansion of our universe is resolved. Several expressions—mass, density, age of the universe, Hubble’s constant, visible matter and dark energy—are presented that exemplify the approach and mathematical procedures. The postulates of Informativity change our understanding of space providing a framework with which to understand the phenomenon of dark energy.
1. INTRODUCTION

Informativity [1] is a model that presents postulates geometric in origin for the description of phenomena. Using Informativity we may present a fundamental expression for length, mass and time that is physically significant [1] (see equations 66-70). Informativity is most successful in expressing relations anywhere a fundamental boundary is found as defined by certain ratios of the fundamental measures: $l_f$, $m_f$ and $t_f$. For example, the expression $c = l_f/t_f$ is a fundamental boundary expression describing length frequency, an upper bound of length units per unit of time. The ratio may be recognized as the speed of light where $l_f$ is the smallest significant unit of length and $t_f$ is the smallest significant unit of time. A second and equally notable expression is $b = m_f/t_f$, a fundamental boundary expression describing mass frequency, an upper bound of mass units per unit of time. Mass frequency describes an upper bound to observable mass. Through the study of boundaries, observation relative to the inertial frame may be understood in a way that differs from the traditional approach.

The term frequency has been chosen to draw attention to the numerical nature of fundamental expressions. Whereas, for instance, a flow rate would lend well to the measurement of a specific phenomenon in time, a frequency is applicable to an event that repeats in a countable manner with respect to time. Frequency provides emphasis on counting. In the example of the speed of light, the focus is on a count of $l_f$ measures per unit of $t_f$. With this approach we may then understand observation as a system with count properties bound by whole-unit numerical constraints.

We will also investigate our understanding of measurement in relation to two frames of reference. By convention, measure is at best understood as a composite expression of other measures. Self-referencing measures for length, mass and time provide a framework for developing expressions of observed phenomena. But, when working with dark energy, there has been difficulty in understanding phenomena that are properties of the universe. This research recognizes that if measure is defined against a frame of reference within the system, then properties of the system cannot be resolved.

We will present expressions for measure defined relative to the system. The new framework considers what phenomena look like when the definitions of measure are presented as self-defining. Using the speed of light and the conservation of energy to constrain this approach, new expressions are presented as a demonstration of the approach and mathematical procedures, i.e. a calculation of Hubble’s constant, the mass and matter density of the universe and precisely how much matter is visible and how much is dark energy. Both the expansion of space and the presented mass/energy distributions are shown to match our best observations.

While the concept of an expanding space is not new, the mechanics and physical significance as to why space must expand has been elusive. Through our understanding of both the self-referencing and self-defining properties of length, mass and time we may present expansion as an inevitable consequence of the forward progression of time.

Although this paper will provide explanation for many Informativity expressions, further explanation is provided in the publication titled, 'Measurement Quantization Unites Classical and Quantum Physics’ [1].
2. METHODS

2.1. Understanding the Fundamental Measures

Before we begin, we will establish Informativity expressions for the fundamental measures [1] (see equations 20-22). Where \( \theta_{si} = 3.26239 \) radians [2] as described in Appendix A, then

\[
l_f = \frac{2G\theta_{si}}{c^3} = \frac{2 \times 6.67408 \times 10^{-11} \times 3.26239}{(299792458)^3} = 1.61620 \times 10^{-35} \text{ m},
\]

\[
t_f = \frac{2G\theta_{si}}{c^4} = \frac{2 \times 6.67408 \times 10^{-11} \times 3.26239}{(299792458)^4} = 5.39106 \times 10^{-44} \text{ s}.
\]

\[
m_f = \frac{2\theta_{si}}{c} = \frac{2 \times 3.26239}{299792458} = 2.17643 \times 10^{-8} \text{ kg},
\]

In some circumstances \( \theta_{si} \) is most appropriate in its radian form. In the case above, a unit analysis is appropriate where \( \theta_{si} \) is recognized as momentum \( \text{kg m s}^{-1} \). While \( \theta_{si} \) may be understood respective of Informativity or the Planck approach, each approach will produce similar values. The advantage of Informativity is that the measurements used in resolving the fundamental measures are each macroscopic.

With this foundation, consider time a convenient measure with which to better understand length and mass. Given a unit of time equal to one second what is the maximum count of length or mass measures that may be observed in a second? As example, we might consider a count of \( l_f \) relative to a count of \( t_f \) that light may traverse with respect to an inertial frame. We may consider the same experiment for mass. As such, the maximum rates, also referred to as the upper bounds, in SI units would be

\[
l_f / t_f = 2.99792458 \times 10^8 \text{ m/s},
\]

\[
m_f / t_f = 4.0371111 \times 10^{35} \text{ kg/s}.
\]

The first expression reflects the number of meters traversed by light in a second. The next expression reflects the number of kilograms that may be traversed in a second. By implication, each describes the maximum rate of change that may be observed relatively between two measures.

As such, we may interpret \( l_f / t_f \) as an upper bound to speed. While such an interpretation is valid, the focus of the expression is that this is an upper bound to observation. In a system with a fixed rate of change \( l / t_f \) and an upper bound to the observation of units of \( l_f \) per \( t_f \), once that boundary is reached the observer can no longer distinguish a greater number of events. To do so would violate our understanding of the fundamental measures of length and time implying that we were able to observe measures smaller than the fundamental measures. As well, it isn’t that physical phenomena cannot overlap in space-time, but an observer has a specific upper bound to the number of events that may be distinguished.

To express a count of distinguishable events for \( l_f, m_f \) or \( t_f \), we would take a rate and divide by the respective fundamental measure.
\[ n_L = 2.99792458 \times 10^8 / l_f = 1.85492 \times 10^{43} \text{units/s} \] (6)
\[ n_M = 4.0371111 \times 10^{45} / m_f = 1.85492 \times 10^{43} \text{units/s} \] (7)
\[ n_T = 1 / t_f = 1.85492 \times 10^{43} \text{units/s}. \] (8)

With this, we observe that

\[ O_1: \quad \text{A count of each of the fundamental measures with respect to any shared measure is the same.} \]

\[ n_L = n_T = n_M. \] (9)

Each count is best understood as an upper bound constrained by a count of a given fundamental measure relative to the fundamental measure of time from an inertial frame. Where one may say in a colloquial manner that matter is constrained to traverse no more than \( 2.99792458 \times 10^8 \) meters per second (i.e. the speed of light), we should say that matter is constrained to traverse no more than \( 1.85492 \times 10^{43} \) units of \( l_f \) per second. The presentation brings to our attention that change, whether that change is in length, mass or time, is constrained. This approach presents an upper bound to observation where an inertial frame may not observe any measure of a phenomenon in excess of:

- a maximum length frequency of \( l_f / t_f \)
- a maximum mass frequency of \( m_f / t_f \)
- a maximum count frequency of \( 1 / t_f \).

The observation is best understood in the first example, the speed of light. An observer cannot observe a phenomenon with a change in the count of \( l_f \) greater than a change in the count of \( t_f \). An upper bound to change applies to each measure in no lesser a way than our understanding of the speed of light.

While we are familiar with the upper bounds to length and time frequency, mass is a less familiar boundary. A maximum mass frequency does not constrain the mass of the universe, but it does provide an upper bound to the total mass that may be observed. Likewise, it follows that if an observer cannot observe mass beyond a count boundary, then that observer is also not subject to the associated gravitational effects. For all purposes, mass in excess of a measurement count boundary does not exist in the universe that is known to an observer. But, other parts of the universe that may be visible to other observers will be subject to the effects of mass within a bounded sphere of those inertial frames.

Currently there are no studies to support that the gravitational effects of mass outside of an observer’s bounded measurement sphere are not experienced. Rather, the conjecture rests on our initial argument that measurement (which includes gravity) is subject to an upper bound. This may further understood by breaking down the gravitational constant into its base frequencies.

Where the expression may be presented in terms from Informativity [1] (see equations 4-8), and where \( 2 \theta_o / l_f = m_f / t_f \) may be resolved from Eq. (3) then
The expression exposes two maximums, the maximum rate of $l_f$ with respect to $t_f$ (Eq. 4) divided by the maximum rate of $m_f$ with respect to $t_f$ (Eq. 5). It follows that the phenomenon of gravity is a composite of two measurement frequencies each constrained to an upper bound. Any effects of gravitation in excess of these bounds cannot be observed as doing so would violate our understanding of the fundamental measures.

To further our understanding of gravitation, also consider a cube with sides measured in $l_f$ equal to the distance that light can travel per second. We find that this cube contains a count of $c^3$ units of cubes each cube with sides equal to $l_f$. The parent cube provides a grid-like understanding of an inertial frame describing the maximum frequency of $l_f$ relative to $t_f$.

Next, consider $m_f/t_f$ as a scalar quantity defining a count of mass units that may exist along the edge of the parent cube, the maximum mass frequency. Thus, dividing the cubic length frequency $(l_f/t_f)^3$ by the mass frequency $m_f/t_f$ gives us the fixed relation of mass relative to an observer in space; in other words the most suited understanding of a gravitational field relative to an inertial frame.

We may expand on this approach by also noting, where $G$ expresses the phenomenon of gravity then the speed of gravity, $s_{\text{gravity}}$ may be resolved by a process of factoring out known components. First multiply $G$ by the mass frequency thus removing the mass component. Then reduce space-time by dividing the cubic length frequency in two of the three dimensions, $c^2$, such that the linear speed of gravity is

$$s_{\text{gravity}} = G \frac{m_f}{t_f} \frac{1}{c^2} = c \text{ m/s.}$$

(11)

Some might argue that this is merely a means of resolving the speed of light. In some sense this is true, but not in the context of expressions regarding the relative relation between length, mass and time. In that context, we are asking, what is the maximum rate of change with respect to space of mass. To succeed in this endeavor, we must first factor out the mass frequency and then reduce space to a measure in one dimension. In this context, we find the rate of change for the phenomenon of gravity relatively in one dimension is $c$. This could already be seen from Eq. (9) where a count of length measures $l_f$ will equal a count of mass measures $m_f$ such that $n_L = n_M$.

3. RESULTS

3.1. Hubble’s Constant

To determine Hubble’s constant using the principles of Informativity consider the fundamental expression, the simplest relation describing length, mass and time,

$$l_f m_f = 2\theta_m t_f \quad [1] \text{ (see equation 51).}$$

(12)
We may expand the expression to include counts of the fundamental measures, \( n_L \) units of length, \( n_M \) units of mass and \( n_T \) units of time such that

\[
n_L l_f n_M m_f = 2 \theta , n_T t_f .
\]  

(13)

Where the system is the observable universe we may propose that the elapsed time (an increasing count in \( n_T \)) must correspond to an equivalent count increase in either length or mass. It should be noted that these are system properties and are not necessarily applied in a scalar fashion from the point of view of an inertial frame. System properties are applicable only with respect to the system as defined in the above expression. We will go over the process of applying system properties to derive values in the sections that follow. But for now, with respect to the fundamental expression, note that:

1. **The values of \( l_f, m_f \) and \( t_f \) are invariant.** Where each of the component measures as resolved in Eqs. (1-3) are known to be invariant we find support for invariance in the fundamental measures. Where \( c = l_f / t_f \) is invariant, it follows that \( l_f \) and \( t_f \) must also be invariant, each constrained to one another.

2. **The measure \( l_f \) is physically significant.** Support for the physical significance of \( l_f \) may be found in the example of momentum and velocity as applied to the uncertainty principle. Using Informativity the product may be reduced such that \( n_M l_f n_T t_f \geq l_f [1] \) (see equations 67-70) thus demonstrating the significance of \( l_f \).

3. **Any count of \( l_f \) must equal a count of \( t_f \).** Where \( c = n_L l_f / n_T t_f \), support for an invariant value for the speed of light \( c \) cannot be maintained unless \( n_L = n_T \).

4. **The count \( n_M \) must equal an invariant count of \( 2 \theta \).** (in this case \( n_M = 1 \)). Any variation in the value of \( n_M \) will be in conflict with support for the conservation of momentum.

Where these constraints are strongly supported, we conclude that elapsed time (an increasing count in \( t_f \)) must correspond with a universe that experiences increasing length (an increasing count of \( l_f \)). A better description of space is not a process of stretching, but a geometric relation that corresponds to an increasing count of length measures equal to the same count in time measures. New units of \( l_f \) are being added to the reference system uniformly and in a discrete manner. The process is best understood as a reference system that increases in volume in proportion to an increase in time, the two measures being defined against one another where the count of \( m_f \) is fixed.

Let us take this moment to reaffirm our understanding of space. Specifically, any inertial frame that presents no force of acceleration to the observer defines an origin of reference for that observer invariant in motion with respect to the measure of space. When we say that space expands, we say that static points of reference in relation to the inertial frame experience increasing relative distance without experiencing a force of acceleration.

With this, let us also take this moment to differentiate the expansion of space (universal expansion) from the expansion of matter within space (stellar expansion). There is no specific correlation between the two. At this point, we can only interpret the expressions above in such a way that space expands and that matter rests within space moving relatively by whatever means respective of its initial conditions.

It follows that a background independent system that has aged by a time \( A_U = n_T t_f \) must correspondingly expand by an equal count of \( l_f \) such that \( l_f / t_f = c \) for all inertial frames within the
system. This may occur specifically when $n_L = n_T$. Thus, with respect to an inertial frame, expansion must occur at the rate $H = 1/A_U$. To place $H$ in the proper form we multiply the inverse of the age of the universe in seconds by the unit conversion $I = km/Mpc$. 

Where the age of the universe is $13.799 \times 10^9$ years [3], where there are $3.15576 \times 10^7$ seconds in a Julian year and where there are $3.0857 \times 10^{19}$ kilometers per megaparsec [4], then space will expand at a rate of

$$H = \frac{km/Mpc}{A_U} = \frac{3.0857 \times 10^{19} \text{ km/Mpc}}{13.799 \times 10^9 \text{ y} \times 3.15576 \times 10^7 \text{ s/y}} = 70.860 \text{ km s}^{-1}\text{Mpc}^{-1}. \quad (14)$$

We will denote the expansion of space, universal expansion, with $H$ to distinguish the value from Hubble’s descriptor $H_o$ which describes the expansion of galaxies from one-another in space, stellar expansion. Converting this to SI units, we may also present universal expansion as a frequency where

$$H_f = \frac{70.860 \text{ km/s Mpc}}{3.0857 \times 10^{19} \text{ km/Mpc}} = 2.2964 \times 10^{-18} \text{ s}^{-1}. \quad (15)$$

Where the general expression as applied to stellar expansion is typically calculated using Hubble’s law as expressed with $v = H_0 D$, we find that Hubble’s expression may be understood in terms of Informativity as a factoring of the fundamental expression presented in Eq. (12). As such, the value for $H$ may be resolved for any moment in time as a necessary outcome of the preservation of the relation between length and time such that $l/f = c$.

We also find that the value for $H$ (but not necessarily $H_o$) decreases as the universe ages. Although each of the prior Informativity expressions describe the expansion of space and not the expansion of matter, both measures $H$ and $H_o$ demonstrate significant correlation as supported in several studies. Analysis of WMAP data over a seven-year period combined with other cosmological data using the simplest version of the $\Lambda$CDM model produced a complimentary value of $H_o = 70.4^{+1.3/-1.4} \text{ km s}^{-1}\text{ Mpc}^{-1}$ [5]. Another study using time delays between multiple Hubble space telescope images of distant variable sources produced by strong gravitational lensing resolved a value of $H_o = 71.9^{+2.4/-3.0} \text{ km s}^{-1}\text{ Mpc}^{-1}$ [6].

While the rate at which galaxies are moving away from one another may move faster or less than the expansion of space, it is interesting that most studies show a stellar expansion that nearly coincides with the Informativity calculation of $H$, universal expansion. This correlation provides support for a model where matter has had almost no relative motion to space since the big bang and has been carried apart by the expansion, stationary with respect to space, minus the effects of gravitational attraction.

### 3.2. Self-Referencing and Self-Defining Measures

The universe as a self-defining system of measures is an important frame of reference when developing expressions that describe the universe. Our current model of measurement is premised on a framework of self-referencing measures. That is, we define each measure as an understanding of other measures. When the frame of reference is the universe, that methodology presents a problem. The universe has no framework with which to define measure. Specifically,
the universe is that which has no relation to any other thing. The issue pushes us to consider measurement of the universe with measures defined against the universe.

In this section, we will consider the idea that a framework of self-defining measures can describe characteristics of the universe. Phenomena will then consist of both self-referencing and self-defining terms respective of two frames of reference. To provide a grounded understanding of these differences, we will present measures for both, starting with the self-referencing expressions [1] (see equation 51):

\[
\begin{align*}
l_f &= \frac{2\theta_{\text{u}}t_f}{m_f}, \\
t_f &= \frac{m_f}{2\theta_{\text{u}}}, \\
m_f &= \frac{2\theta_{\text{u}}t_f}{l_f}.
\end{align*}
\]

To resolve self-defining expressions, we then expand these expressions, set the target measure equal to a value of one (that is, a measure defined against itself) and solve for counts of the remaining two measures.

To avoid confusion, we will denote self-defining measures as well as their counts with the subscript \( p \). As example, for length \( l_p = 1 \), for time \( t_p = 1 \) and for mass \( m_p = 1 \). This approach provides physically significant expressions that describe properties of our universe with respect to the universe. The procedure is appropriate for any measure that is conserved with respect to the universe. When a measure is not conserved, the reference varies and as such does not provide a fixed frame against which to resolve the corresponding measurement counts. For this reason, we will not work through the derivations of length and time, neither of which are conserved (i.e. elapsed time and an expanding universe). But, their derivations are available in Appendix B.

Conversely, mass is conserved and as such provides a physically significant frame of reference with which to understand the counts of length \( n_{L_p} \) and time \( n_{T_p} \). Starting with the self-referencing expression for mass in Eq. (18), then

\[
m_f = \frac{2\theta_{\text{u}}t_f}{l_f} = \frac{2\theta_{\text{u}}n_{T_p}}{c n_{L_p}}.
\]

Where \( c = n_{T_p}/n_{L_p} \) then \( n_{L_p} = n_T \). To resolve the self-defining expression for mass, we set the value of \( m_f \) equal to one. Substituting the representative term, \( m_p = 1 \) for \( m_p \), reducing and then substituting the self-referencing term for mass back into the expression (i.e. \( m_f = 2\theta_{\text{u}}/c \)), then

\[
\begin{align*}
m_p &= \frac{2\theta_{\text{u}}}{c n_{L_p}} = 1, \\
\frac{2\theta_{\text{u}}}{c} &= \frac{n_{T_p}}{n_{L_p}}.
\end{align*}
\]
\[ m_f = \frac{n_{lp}}{n_{tp}}. \] (22)

The approach presents an expression that is no longer self-referencing, but a self-defining count ratio. Keep in mind that \( n_{lp}/n_{tp} \) is also nondimensionalized. Thus, while the ratio is equivalent in value to \( m_f \), the expression has no units. Nondimensionalization is a physically significant feature of Informativity that is a product of the self-defining properties of a system.

It follows from this ratio multiplied by the speed of light that the value for Hubble’s constant \( H_U \) using self-defining terms is

\[ H_U = \frac{n_{lp}}{n_{tp}} c \ m/s \text{ per universe}. \] (23)

To distinguish the value from \( H \) which is measured in megaparsecs, here we use the subscript \( U \) to indicate that we are using the self-defining reference, the universe. Likewise, where \( m_f c = 2\theta_u \) from Eq. (18), we may write the value-equivalent expression

\[ H_U = \frac{n_{lp}}{n_{tp}} c = 2\theta_u. \] (24)

The expression although equivalent in value, differs in units. Expressing the Hubble constant in this way may be convenient, but presents a confusing mix of terms that are both self-referencing and self-defining. When expressions mix terms that derive from different frames of reference unit analysis will fail. The observation differs dramatically from an error in calculation. Errors are associated with a difference in value and units. Informativity expressions that mix differing frames of reference differ only in units. This will be further explored in Section 3.5.

### 3.3. Universal Expansion

Having resolved that time frequency must correspond to an expanding space as described in Eq. (14), we may now substitute and group counts of the fundamental units to resolve expressions for the diameter and age of the universe. First, multiply both sides of the fundamental expression in Eq. (12) by \((n_{tp} n_{lf})\). Then regroup the terms and substitute where the universe’s diameter is \( D_U = n_{lf} \) (billion light-years) and age is \( A_U = n_{tf} \) (billion years).

\[
\begin{align*}
(n_{tf} n_{lf}) l_f m_f &= 2\theta_{sf} (n_{tf} n_{lf}) \\
(n_{lf}) m_f \left( \frac{n_{tf} c}{n_{lf}} \right) &= 2\theta_s (n_{tf}) \\
D_U m_f \left( \frac{A_U c}{D_U} \right) &= 2\theta_s A_U
\end{align*}
\] (25-27)
Next, we move $D_U$ to the right and break down the right portion to determine its value. Where $A_U = n_T t_f$, $D_U = n_L l_f$ and where $m_f = 2\theta_s/c$ from Eq. (18) then

$$\frac{2\theta_s A_U}{D_U} = \frac{2\theta_s t_f}{l_f} \frac{n_T}{n_L} = \frac{2\theta_s}{c} \frac{n_T}{n_L} = m_f \frac{n_T}{n_L}$$

(28)

The result is a self-referencing expression. In turn, the self-defining expressions are $A_U = n_T t_f$ and $D_U = n_L l_f$. We may formally recognize the frame of reference as the system by replacing the count terms with their respective system terms, $n_L$ and $n_T$. The expression may then be reduced with the self-defining expression $m_f = n_L p/n_T p$ from Eq. (22).

$$m_f \frac{n_T}{n_L} = m_f \frac{1}{m_f} = 1$$

(29)

Thus, where $2\theta_s A_U/D_U = 1$, and $m_f c = 2\theta_s l_f$ from Eq. (18), then Eq. (27) may be reduced to

$$D_U = 2\theta_s A_U = 2 \times 3.26239 \times 13.799 = 90.035 \text{ bly}$$

(30)

$$m_f = \frac{D_U}{A_U c} = \frac{90.035}{13.799 \times 299792458} = 2.1764 \times 10^{-8} \text{ kg}$$

(31)

The second expression follows implicitly where Eq. (30) is established, then $2\theta_s A_U/D_U = 1$ may be substituted into Eq. (27) and reduced to produce the later. Equation (30) also confirms that the system constant between diameter and age is precisely $2\theta_s l_f$, as resolved in Eq. (24).

Without the introduction of self-defining measures, the expressions are mere extrapolations of measure in the local frame. Only by setting our frame of reference to the universe can we produce valid descriptions of the universe from our point of view.

Current measurements for $D_U$ and $A_U$ are respectively 91 billion light-years and 13.799 +/- 0.021 billion years [3]. Thus, as the universe ages its diameter must expand by a factor of $2\theta_s l_f$. In 2011, formulations by Barrow and Douglas [7] comparing the cosmological constant and the age of the universe had been worked out predicting a constant relationship. In 2015, analysis of WMAP data by Gasanalizade and Hasanalizade [8] furthered our understanding confirming a constant correlation between the age of the universe and its expansion.

It should be noted, while each expression describes a property of the universe, the terms and their associated units can be misleading. As noted previously, system expressions are appropriately expressed in self-defining terms. Mass has already been resolved in Eq. (22) as $m_f = n_L p/n_T p$. Where Eq. (24) is $(n_L p/n_T p)c = 2\theta_s l_f$ with $D_U$ and $A_U$ in meters and seconds, then the self-defining presentation of Eq. (30) is

$$D_U = \frac{n_L p}{n_T p} c A_U .$$

(32)

The expression confirms itself and our understanding of self-defined measures. Where $D_U = n_L p l_f$, $A_U = n_T p t_f$ and $c = l_f / t_f$, the expression simplifies to $1 = 1$. Like self-referencing measures, self-defining measures are also measured against themselves. Note as well, removing the ratio...
\(n_{Lp}/n_{Tp}\) will give you the scalar expression for an expanding volume with respect to an inertial frame. And finally, the expression in Eq. (24) for Hubble’s constant (\(\text{m/s per universe}\)) combined with Eq. (32) are value equivalent in each of the following three forms depending on the frame of reference,

\[
H_U = \frac{n_{Lp}}{n_{Tp}} c = 2\theta_{si} = \frac{D_U}{A_U} c.
\]  

(33)

Most notably, we can now see that the repeated appearance of \(2\theta_{si}\) is in fact also a form of Hubble’s constant defined with respect to the universe. The term shows up not only in Eq. (30) as a \textit{system constant} that incorporates the co-moving element of universal expansion, but in many other expressions such as the fundamental expression \(l_m \equiv 2\theta_{si} t_f\) which relates length, mass and time in their most simple form. The term shows up in the definition of Planck’s reduced constant \(\hbar = 2\theta_{si} l_f\) [1] (see equation 35) and Newton’s Gravitational constant \(G = l_f c^3 / 2\theta_{si}\) [1] (see equation 16). In short, the \textit{system constant} is less a constant of the universe and more a descriptive count ratio that serves as a conversion metric between the local frame and the universe. With this broader understanding, the physical constants may be understood as a collection of measurement ratios each a variant of the \textit{system constant}.

### 3.4. Properties of the Universe

The \textit{fundamental expression} \(l_m \equiv 2\theta_{si} t_f\) is a system definition that not only provides the foundation for how length, mass and time interrelate within the system, but also defines both upper bounds and properties for the system. In this section, we will present expressions that allow us to calculate some of those properties at this point in time.

In the same way that length frequency is an upper bound, we may resolve system properties by multiplying the age of the universe \(A_U\) and the corresponding radial \textit{system constant} \(\theta_{si}\) (where \(A_U\) corresponds to half of \(2\theta_{si}\)) by the two system frequencies and corresponding system count. Where \(A_U = 4.3546 \times 10^{17} \text{ seconds}\) [3] at this point in time then

\[
n_f = A_U \frac{1}{t_f} = 4.3546 \times 10^{17} \frac{1}{5.39106 \times 10^{-44}} = 8.0775 \times 10^{60} \text{ units } t_f,
\]

(34)

\[
R_U = A_U \theta_{si} \frac{l_f}{t_f} = 4.3546 \times 10^{17} \times 3.26239 \frac{1.61620 \times 10^{-35}}{5.39106 \times 10^{-44}} = 4.2590 \times 10^{26} \text{ m},
\]

(35)

\[
M_f = A_U \theta_{si} \frac{m_f}{t_f} = 4.3546 \times 10^{17} \times 3.26239 \frac{2.17643 \times 10^{-8}}{5.39106 \times 10^{-44}} = 5.7353 \times 10^{51} \text{ kg}.
\]

(36)

Each expression is best understood as an observational boundary, but may also be understood as a physical property where \(\theta_{si}\) is the radial \textit{system constant} of a co-moving reference in the expansion. Also note, in the case of \(M_f\), the fundamental mass of the universe is not a straight-forward scalar bound at \(5.7353 \times 10^{53}\) kilograms. Where \(M_f\) is a function of mass frequency, \(m_f\) differs from the other measures as it is not the smallest measurable mass. Rather, \(m_f\) is a composite of our understanding of \(l_f\) and \(t_f\) and thus an important countable measure relative to the other measures. The ability to measure phenomenon smaller than \(m_f\) does not
violate the mass frequency boundary, but understanding how it constrains visible mass requires additional steps discussed in Section 3.6.

Conversely, the values \( n_T \) and \( R_U \) are scalar observational bounds. \( n_T \) , for instance, is a count of time units elapsed. \( R_U \) is the co-moving radius that corresponds to that count. The radius \( R_U = A_U \theta_{si} c \) and the dimensionless nature of the system constant \( \theta_{si} \) can be verified. Starting with Eq. (32) where \( D_U = (n_L / n_T) c A_U \), substitute \( (n_L / n_T) c = 2 \theta_{si} \) from Eq. (24), divide by two for radius \( R_U \) and then multiply \( A_U \) by \( c \) to convert to SI units.

One may also note what seems like the arbitrary introduction of \( \theta_{si} \), the system constant. This is an important system parameter that applies to most Informativity expressions and may be resolved from Eq. (30). Without the system constant, the expression would represent a calculation applicable to a volume expanding at the speed of light, but would not express the expansion we see in the universe which requires a self-defining frame of reference. One might also consider applying the system constant to the expression \( n_T = A_U \theta_{si} l_f \). Where \( 1/l_f \) is a count and not a relation, the system constant is not applicable.

Interpreting the units for \( \theta_{si} \) can be challenging as well. Units will be discussed in Section 3.5, but for now this consideration may be factored out. Where \( A_U = R_U l_f \theta_{si} \) from Eq. (35), then the fundamental mass is

\[
M_f = A_U \theta_{si} \frac{m_f}{l_f} = \frac{R_U l_f^{\theta_{si}} m_f}{l_f} = \frac{R_U}{l_f} m_f. \tag{37}
\]

You may have also noticed that this is the last unaccounted for frequency boundary, \( m_f / l_f \). We may as well reduce the fundamental mass expression into more familiar physical measures. Where the age of the universe is \( A_U = n_T l_f \) in seconds and where \( m_f / l_f = c^3 / G \) from Eq. (10), then

\[
M_f = A_U \theta_{si} \frac{m_f}{l_f} = n_T l_f \theta_{si} \frac{m_f}{l_f} = n_T m_f \theta_{si}, \tag{38}
\]

\[
M_f = A_U \frac{\theta_{si} c^3}{G}. \tag{39}
\]

The four ratios \( l / l_f, l / l_f, m_f / l_f \) and \( m_f / l_f \) each describe an important property of our universe. It should also be noted that Eq. (39) is valid only because the measure of \( G \) is made macroscopically and as such corresponds to the proper value reflective of the Informativity differential \([1]\) (see equation 28).

We may next resolve the volume of the universe \( V_U \) using its radius \( R_U \) from Eq. (35).

\[
V_U = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R_U^3 = \frac{4}{3} \pi (4.2590 \times 10^{26})^3 = 3.2360 \times 10^{80} \text{ m}^3. \tag{40}
\]

To resolve the corresponding matter density of the universe using the expression \( \rho_m = M_f / V_U \), we substitute in the expression for volume and reduce where \( R_U = A_U \theta_{si} c \) from Eq. (35), where \( M_f = R_U m_f / l_f \) from Eq. (37), where \( h = 4 \pi \theta_{si} l_f \) \([1]\) (see equation 14) and where \( 2 \theta_{si} / m_f = l_f / l_f = c \) from Eq. (12).
\[
\rho_m = \frac{M_f}{V_U} = \left( R_U \frac{m_f}{l_f} \right) \left( \frac{3}{4 \pi R_f^3} \right) = \frac{3m_f}{4 \pi R_f^3 l_f} = \frac{3m_f}{4 \pi (A_f \theta_f c)^2 l_f} . \tag{41}
\]

\[
\rho_m = \frac{3m_f}{A_f^2 \theta_f c^3 (4 \pi \theta_f l_f)} = \frac{3m_f}{A_f^2 \theta_f c^3 h} = \frac{6}{A_f^2 c^3 h} 2\theta_f = \frac{6}{A_f^2 c^3 h} . \tag{42}
\]

Where Planck’s constant is adjusted for the Informativity differential, \[1\] (see equation 35) the corresponding matter density is

\[
\rho_m = \frac{6}{A_f^2 c^3 h} = \frac{6}{4.3546 \times 10^{17} \times 299792458 \times 6.62584 \times 10^{-34}} = 1.7723 \times 10^{-27} \text{ kg/m}^3 . \tag{43}
\]

### 3.5. Unit Analysis as a Function of the Frame of Reference

As with many Informativity expressions, unit analysis is challenged because self-referencing and self-defining terms are mixed. This happens wherever we present an expression that incorporates a system characteristic, such as the system constant. The issue differs significantly from a calculation error where both the value and units of an expression are incorrect. Properly resolved Informativity expressions will have the correct value. Rather, unit issues arise as a result of expressions that mix two frames of reference.

In this section, we will use Eq. (43) as an example to demonstrate the context and methods involved in resolving units for Informativity expressions. We begin by first demonstrating that the resulting value is the same value as would be resolved if we had solved the initial expression. Where \( m_f = 5.7353 \times 10^{53} \text{ kg} \) from Eq. (36) and \( V_U = 3.2360 \times 10^{80} \text{ m}^3 \) from Eq. (40), then

\[
\rho_m = \frac{5.7353 \times 10^{53}}{3.2360 \times 10^{80}} = 1.772 \times 10^{-27} \text{ kg/m}^3 . \tag{44}
\]

The values are the same. Note, there is variation in the fifth significant digit arising from a difference between Planck’s constant and the respective value adjusted for the Informativity differential \[1\] (see equation 35). Conversely, the unit issue began with the introduction of the system constant \( \theta_f \).

The self-defining dimensionless nature of \( \theta_f \) is important when we make substitutions like \( m_f = 2\theta_f / c \) as we did in the final reduction. In consideration of this variation of the fundamental expression where \( m_f = 2\theta_f / c = (n_{Lp} / n_{Tp}) \) from Eqs. (21-22) has the units of kilograms and such that \( H_U = (n_{Lp} / n_{Tp}) c = 2\theta_f \) m/s from Eq. (24) where \( (n_{Lp} / n_{Tp}) = 2\theta_f / c \) has the units seconds/meter, then kg = s/m is the ‘conversion metric’ between the self-defining and self-referencing value \( 2\theta_f / c \).

Secondly, where the frame of reference is a circle with an angle of \( \theta_f \) then \( h = 4\pi l_f \theta_f \) \[1\] (see equation 14) has a similar ‘conversion metric’; its units are meters. Making the substitution \( s/m = kg \) and meters for \( h \) into Eq. (43) resolves the unit conflict.

\[
\rho_m = \frac{6}{A_f^2 c^3 h} \left( \frac{1}{s^2 (m^3 / s^3) m} \right) = \frac{1}{m^3 m} = \frac{kg}{m^3} . \tag{45}
\]
The practice of mixing self-defining and self-referencing terms is difficult, but can be carried out safely so long as one is aware of the frame of reference under consideration. Units may always be verified by agreement on the calculated value and associated units at a point earlier in the derivation.

3.6. Dark Energy

To this point, several calculated values of the universe have been presented. The same may be accomplished to resolve the distributions of visible mass and dark energy. From a theoretical point of view, the calculation has not been possible as we do not know the total or visible mass nor what distinguishes the two.

With Eq. (36) we have introduced the idea of mass frequency $m_f/t_f$ as an important ratio that describes a measurement boundary beyond which mass is not visible. Unfortunately, unlike the speed of light $l/t_f$, the scalar interpretation of mass frequency does not define that boundary. The relationship is complicated by our point of view; a translation is needed between the self-referencing and self-defined measurement frameworks. The solution is also challenged as we do not know the total mass against which we can apply the distribution expression. This challenge is met by using critical mass as a placeholder reference. The ratio can then be applied to what we know about the density of the universe and the closeness of that value to critical density to resolve the distribution.

We begin by noting that the mass density $\rho_m$ of the universe is the product of Hubble’s expression for critical density $\rho_c$ and the mass distribution $M_{in}$ associated with visible matter. As noted, although mass density is contingent on the spatial curvature of the universe, based on observations of the cosmic microwave background (CMB) from the Wilkinson Microwave Anisotropy Probe, curvature is measured to be close to zero, supporting an approximation where the mass density of the universe is close to the product $\rho_c M_{in}$. Thus, visible mass $M_{vis}$ may be expressed as

$$\rho_m = \rho_c M_{in}, \quad (46)$$

$$M_{vis} = V_U \rho_m. \quad (47)$$

With this, we may then calculate the dark energy distribution $M_{out}$ by subtracting the fundamental mass $M_f = A_U \theta_s m_f/t_f$ presented in Eq. (36) from the visible mass $M_{vis} = V_U \rho_m$. But, we do not divide the difference by the self-referencing visible mass. This is a distribution expression that solves for the self-defining non-visible portion of the total. Thus, we want to divide by the fundamental mass giving us a ratio of mass in excess of the mass frequency divided by the mass associated with the mass frequency. Where the total mass $M_{tot}$ is the sum of visible and non-visible (dark energy) mass, then their relation may be expressed as

$$2M_{tot} M_f = M_{vis} (M_{tot} + M_f). \quad (48)$$

To solve for the relative distributions, where $V_U = (4/3)\pi R_U^3$, where $G = c^3 t_f/m_f$ from Eq. (10), where $R_U/A_U = \theta_s c$ from Eq. (35), and where the critical density of the universe $\rho_c = H_f^2/8\pi G$ [9] is a function of Hubble’s frequency as presented in Eq. (15), then
Next, where the sum of visible $M_{\text{in}}$ and non-visible $M_{\text{out}}$ mass percentages must equal one, 100% of the mass in the universe, then

\[ 2M_{\text{out}} + 2 = M_{\text{in}} \theta_{si}^2, \quad (54) \]
\[ M_{\text{out}} + M_{\text{in}} = 1. \quad (55) \]

The respective distributions are then

\[ M_{\text{out}} = \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} = 68.3624\%, \quad (56) \]
\[ M_{\text{in}} = \frac{4}{\theta_{si}^2 + 2} = 31.6376\%. \quad (57) \]

The most recent measurement data regarding mass/energy distributions [10] indicate 4.8% ordinary matter, 0.1% neutrinos, 26.8% dark matter and 68.3% dark energy. Also relevant, note that a study of the CMB published in 2015 [11] presents compelling data that dark matter is fine dust and can be measured by studying its gravitational effects on galaxies. In reflection of this research, combining dark matter with visible matter, we find 31.6% of the mass associated with the visible distribution matching our expectations.

Mass frequency plays an important role in what may be seen in our universe such that the principle precisely defines the 68.3% measure attributed to dark energy. Most interesting is that mass density appears to be close to Hubble’s expression for critical density. Considering the case where total mass $M_{\text{tot}}$ equals fundamental mass $M_{f}$, then Eq. (48) reduces to

\[ M_{\text{vis}}(M_{\text{vis}} - M_{f}) = 0. \quad (58) \]

Such that $M_{\text{vis}} \neq 0$, then the relation can be true only where $M_{\text{vis}} = M_{f}$ and there exists no dark energy component. The exploration is interesting when used to understand that the total mass $M_{\text{tot}}$ may take on any value greater than $M_{f}$ to produce the dark energy phenomenon. Finally, the expression brings to our attention that dark energy is also matter which cannot be
seen because it is in excess of the mass frequency constraint with respect to the self-defining framework, the universe.

With this, we may use Hubble’s expression for critical density

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3(2.296 \times 10^{-18})^2}{8\pi G} = 9.4316 \times 10^{-27} \text{ kg/m}^3$$

(59)

to estimate the mass density associated with each distribution such that

$$\rho_{m/in} = \rho_c M_{in} = 9.4316 \times 10^{-27} \times 316376 = 2.9838 \times 10^{-27} \text{ kg/m}^3,$$

(60)

$$\rho_{m/out} = \rho_c M_{out} = 9.4316 \times 10^{-27} \times 683624 = 6.4479 \times 10^{-27} \text{ kg/m}^3.$$

(61)

Solving for visible mass $M_{vis}$ and the mass associated with dark energy $M_{dke}$ we find that

$$M_{vis} = V_U \rho_{m/in} = 3.2360 \times 10^{30} \times 2.9838 \times 10^{-27} = 9.6556 \times 10^{53} \text{ kg},$$

(62)

$$M_{dke} = V_U \rho_{m/out} = 3.2360 \times 10^{30} \times 6.4479 \times 10^{-27} = 2.0865 \times 10^{54} \text{ kg}. $$

(63)

3.7. Reducing a Physical Expression Back to the Fundamental Expression

The Informativity conjecture is that every physical law is an expression of and thus can be reduced to the fundamental expression. We have used this principle as a basis to explain dark energy and to resolve several properties of our universe. In this example, we will demonstrate this by taking the fundamental mass which we may use to resolve the visible and non-visible mass distributions and reduce it back to the fundamental expression.

Where $M_f$ is the fundamental mass of the universe in kilograms, $V_U$ is the volume in cubic meters, $R_U$ is the radius in meters, $\rho_m$ is the mass density—an estimate based on the product of observed matter as a percentage and the critical density of the universe—then

$$M_f = V_U \rho_m,$$

(64)

We may then reduce the expression where $V_U=\frac{4}{3}\pi R_U^3$, where $\hbar=\hbar/2\pi$, where $\hbar=2l_f \theta_{si}$ [1] (see equation 35) and where expressions from Eqs. (35, 36 and 43) are

$$R_U = A_U \theta_{si} \frac{l_f}{t_f},$$

(65)

$$M_f = A_U \theta_{si} m_f \frac{R_U}{t_f},$$

(66)

$$\rho_m = \frac{6}{A_U^2 c^3 \hbar},$$

(67)

then
\[ M_f = V_U \rho_m = \frac{4}{3} \pi R_U^3 \frac{6}{A_U c^3 h} = \frac{8\pi}{A_U^2 c^3 h} R_U^3 = \frac{8\pi}{A_U^2 c^3 h} A_U^3 \theta_s^3 c^3, \quad (68) \]

and making a substitution for \( M_f \) and reductions on the right then

\[ A_U \theta_s \frac{m_f}{l_f} = \frac{2\pi}{h} 4A_U \theta_s^3 \frac{4A_U \theta_s^3}{h} = \frac{4A_U \theta_s^3}{2l_f \theta_s} = \frac{2A_U \theta_s^2}{l_f}, \quad (69) \]

\[ l_f m_f = 2\theta_s l_f. \quad (70) \]

Thus, the expression \( M_f = V_U \rho_m \) is a derivative representation of the fundamental expression. This is not to say that we can understand why the mass of the universe is what it is, only that the mass distribution associated with dark energy is an Informativist effect.

The lack of a formal expression which defines system laws and properties leads to questions that are in hindsight meaningless, such as what exists outside of the universe. In the context of a logical construct, such questions can now be more clearly defined. In the same way that the maximum speed is \( c = l/f \) and the system volume is \( V_U = (4/3) \pi R_U^3 \), when applying expressions that describe physical phenomena we find that each are an outcome of the fundamental expression. To ask what is outside of the reference system is meaningless because there is no means to define phenomena outside of a background independent system of relatively defined measures. Such limits, though, would not apply in consideration of a self-defining framework or the possibility that the fundamental measures are inherited from a multi-verse.

## 4. DISCUSSION

Informativity is a model drawn from fundamental units of measure: length, time and mass. The study of how the fundamental measures are related can reveal important expressions regarding the behavior of matter, a field of science where the use of Planck Units is a common tool. With this we have explored how certain properties of our universe are expressed and constrained by their relation, such as the speed of light (the length frequency), mass and time frequency, and how \( G \) is a space-time composite of these frequencies. We have explored how these relations prescribe that our understanding of space must expand in a precise and consistent fashion with elapsed time. That expansion not only requires that our universe expand outward, but in all directions.

Also noteworthy is the observation that physical constants are variations of the fundamental expression, \( l_m = 2\theta_s l_f \). Whether that is the Gravitational constant, Planck’s reduced constant or Hubble’s constant, there exists a foundation of understanding that these values are convenient arrangements of the fundamental measures as defined by the fundamental expression relative to the system constant \( 2\theta_s \). The observation is expanded to include more general expressions by taking the expression for the fundamental mass of the universe and reducing it back to the fundamental expression.

Perhaps one of the most significant discoveries is not that our expanding universe is a phenomenon that can be expressed as a measurement boundary, but that phenomena have measurement boundaries. Keep in mind, length, mass and time are related and relatively defined. To understand visible mass as a function of mass frequency is as valid as to say that the age of the universe is an increasing upper bound constrained by its relation to length frequency.
The conjecture is a mathematical equivalence that offers no frequency affinity. On such a foundation, it is possible that the apparent age of the universe is a frequency boundary, that there exists a multi-verse that extends indefinitely in length and time and that the phenomena we observe are the product of frequency boundaries of the multi-verse, not the universe. But I leave this conjecture for future investigation.

In light of the many expressions presented herein, we conclude with two experiments that not only allow for a test of expansion, but continue to provide support for the principles of Informativity.

4.1. Measuring Universal Expansion

A measure of expanding space has particular value as it can greatly assist in understanding the difference between the expansion of space and the expansion of galaxies away from one another with respect to space, two distinct effects that do not have a specific correlation. Secondly, such an experiment would confirm that expansion is a phenomenon that also occurs in the local frame, not a quality that appears only on a cosmological scale. Thirdly, where such measurements show no effects related to Special Relativity, the experiment supports the idea that this is a geometric property of space and not a property of the inertial frame.

Specifically, space is not a tangible, measurable phenomenon. Rather, the process of measurement is geometric in origin. Further, the reference system against which everything is defined, the *fundamental expression*, consists of measurement counts that change with elapsed time and thus change our understanding of length.

The expansion of the universe in the local frame is not as small as one might anticipate. Using Eq. (14) as a starting point we may resolve the expansion between the Earth and a satellite in the same orbit as the Earth on the other side of the sun. To begin with, we recognize the calculation where approximately \(3.0857 \times 10^{22}\) meters are in a megaparsec. The value is based on the IAU 2012 SI definition of the astronomical unit with example calculations in several texts.

Due to the expansion of space, the trip distance, two times the average distance \(d\) between the Earth and the Sun \(2 \times 1.496 \times 10^9 = 2.992 \times 10^9 \text{m} \) will increase at a rate of \(H_d\). The displacement \(D\) will then be

\[
H_d = \frac{\text{km/Mpc}}{1000m} \frac{1\text{Mpc}}{1\text{km}} \frac{2.992 \times 10^9 \text{m}}{d} = 6.871 \times 10^{-8} \text{m/s}, \quad (71)
\]

\[
D = H_d \frac{d}{c} = 6.871 \times 10^{-9} \frac{2.992 \times 10^9}{299792458} = 6.857 \times 10^{-8} \text{m}. \quad (72)
\]

In other words, excluding the effects of gravity, the distance between the Earth and the satellite will increase by 69 nanometers as a result of universal expansion during the trip.

4.2. The System Constant and its Effect on Mass Distribution

At the center of Informativity is the observation that \(\theta_{si} = 3.26239 \text{ radians} \) \[1\] (see equation 32), a measure correlated with the polarization of an electric field with respect to the scattering plane needed to create quantum entanglement of X-rays in specific Bell states. The measure may also be correlated with the momentum of half a fundamental measure of mass. While evidence is presented that shows \(\theta_{si}\) conforms to the Informativity interpretation, estimates
based on the standard model interpretation of Planck’s reduced constant can suggest a value for \( \theta_{si} = 3.26250 \, \text{kg m/s} \). \[1\] (see equation 33).

Fortunately, there are several tests that bring a greater understanding to \( \theta_{si} \). As resolved in Eqs. (56-57), the value of \( \theta_{si} \) has a significant effect on the calculation of mass distribution. A study of the distributions to two orders of magnitude greater than current measurements will decidedly favor only one of the interpretations.

### TABLE I. The distribution of mass in the universe respective of the mass frequency bound

<table>
<thead>
<tr>
<th>Mass Distribution</th>
<th>( \theta_{si}=3.26239 )</th>
<th>( \theta_{si}=3.26250 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{in} ): Visible Mass</td>
<td>31.6376%</td>
<td>31.6358%</td>
</tr>
<tr>
<td>( M_{out} ): Non-visible Mass (dark energy)</td>
<td>68.3624%</td>
<td>68.3642%</td>
</tr>
</tbody>
</table>

### 5. APPENDICES

**Appendix A: A short review of \( \theta_{si} \)**

The term \( \theta_{si} \) represents a fundamental constant of nature that has two applicable interpretations. On the one hand the term can accurately describe half the momentum of a fundamental unit of mass, \( m_f \). On the other hand, \( \theta_{si} \) can also describe the polarization of an electric field with respect to the scattering plane needed to create quantum entanglement of X-rays in specific Bell states.

More specifically, there are five pump angles representing two of the Bell states that can generate entangled photons and \( l_f c^3/2G \) is uniquely distinguished where \( \theta_p \) is at its maximum. Shwartz and Harris recognize these Bell states, where \(|H>\) is the polarization of the electric field of the X-ray in the scattering plane and \(|V>\) is the polarization orthogonal to the scattering plane which contains the incident \( k \) vector and the lattice \( k \) vector \( G \). Subscripts \( p, s, \) and \( i, \) respectively, denote the pump, signal and idler. Measurement data is presented in a paper by Shwartz and Harris \[12\] summarized in Table II aside each of the respective Informativity expressions that denote \( \theta_{si} \).

### TABLE II. Angle setting in radians of the \( k \) vectors of the pump, signal and idler for maximally entangled states at the degenerate frequency with corresponding Shwartz and Harris values (Ref. [12]).

<table>
<thead>
<tr>
<th>Bell’s State</th>
<th>( \theta_p )</th>
<th>( \theta_s )</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>H_s, V_i +</td>
<td>V_s, H_i&gt;)/\sqrt{2} )</td>
<td>( (l_f c^3/2G) - \pi \cdot (0.1208) )</td>
</tr>
<tr>
<td>( 2\pi - (l_f c^3/2G) \cdot (3.02079) )</td>
<td>( (l_f c^3/2G) \cdot (3.26239) )</td>
<td>( (l_f c^3/2G) \cdot (3.26239) )</td>
<td></td>
</tr>
</tbody>
</table>

Calculation of \( \theta_{si} \) may be presented in several forms, but most straight-forwardly as

\[
\theta_{si} = \frac{l_f c^3}{2G} = 1.616199 \times 10^{-35} \cdot (299792458)^3 = 3.26239 \, \text{radians}. \quad \text{(A1)}
\]

The term alludes to recognizing the angular measure of the signal and idler under some conditions and momentum under others. Although both interpretations are applicable, \( \theta_{si} \) is
retained emphasizing that we are not working with a theoretical value, but an invariant macroscopic measure. Additional research regarding the measure of $\theta_{si}$ has also been reported [13], where the error in angular measurement is estimated to be less than 2 micro-radians.

**Appendix B: Resolving System Counts of Self-Defining Measures**

We are unable to resolve meaningful self-defining measures for length and time because neither are conserved with respect to the universe. Nevertheless, we may resolve the respective counts by expanding each expression and setting the target measure equal to one. We will denote system measures as well as system counts with the subscript $p$. Thus, where $l_p=1$, we may reduce using the expression $l_pl_f=2\theta_{si}l_f$ to express the count ratio in its most simple form.

\[
l_f = \frac{2\theta_{si}l_f}{m_f} = \frac{2\theta_{si}n_pl_f}{n_{si}m_f}, \quad (B1)
\]

\[
l_p = \frac{2\theta_{si}n_pl_f}{n_{si}m_f} = 1. \quad (B2)
\]

\[
\frac{m_f}{2\theta_{si}l_f} = \frac{n_{Tp}}{n_{M_p}}. \quad (B3)
\]

\[
l_f = \frac{n_{Tp}}{n_{M_p}}. \quad (B4)
\]

The same approach may be taken with time:

\[
t_f = \frac{m_fl_f}{2\theta_{si}} = \frac{n_{si}m_ll_f}{2\theta_{si}}. \quad (B5)
\]

\[
t_p = \frac{n_{M_p}m_ll_{p}l_f}{2\theta_{si}} = 1. \quad (B6)
\]

\[
n_{M_p}n_{Tp} = \frac{2\theta_{si}}{m_fl_f}. \quad (B7)
\]

\[
t_l = \frac{n_{M_p}}{n_{M_p}}. \quad (B8)
\]

Unfortunately, the mass of the universe is conserved. And as such a count of fundamental units of mass $n_{M_p}$ is invariant. As the values for the fundamental measures of $l_f$ and $t_f$ are also invariant, we find a singular invariant value for the count of $n_{Tp}$ in the first case, and the count of $n_{Lp}$ in the second case. Distinguished from mass which is conserved, we find these expressions inappropriate.
REFERENCES


