Correcting for Relativity in GPS makes no sense

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Abstract – Showing that the Special Theory of Relativity is an untenable theory, many times leads to the reaction that the GPS is so accurate thanks to the STR corrections. This article shows that the supposed relativity errors are by far negligible relative to the errors caused by atmospheric circumstances.

1. Introduction

In [1] the impact of the Special Theory of Relativity, by means of the supposed so-called time dilation, on the accuracy of a Global Positioning System is presented.
For more background information a reference herein is made to [2], from now on also referred to by means of "Ashby".
This article shows a phenomenon that has a much larger impact on the accuracy of the GPS than the one claimed by the concept "time dilation". It also shows that this phenomenon leads to errors of the same order of magnitude as shown in [3].

2. GPS configuration

In [2] the GPS configuration is described as follows.

> "The orbiting component of the GPS consists of 24 satellites (plus spares): four satellites in each of six different planes inclined 55^0 from Earth's equatorial plane. The satellites are positioned within their planes so that, from almost any place on Earth, at least four are above the horizon at any time. Orbiting about 20.000 km above Earth's surface, all satellites have periods of 11 hours and 58 minutes."

So the angular velocity of the satellites is twice as high as the one of the earth.

The basic equations are:

$|\mathbf{r} - \mathbf{r}_i| = c(t - t_i)$ with i representing satellite number 1 up to and including 4

$\mathbf{r}$	position of the receiver (on or near the earth's surface)
$\mathbf{r}_i$	position of the i^{th} satellite
t_i	time of emitting a signal from the i^{th} satellite
t	time of the clock of the receiver when it receives the 4 signals simultaneously

The "variable" c is described in [2] as:

> "The fundamental principle on which GPS navigation works is an apparently simple application of the second postulate of special relativity – namely, the constancy of c, the speed of light."

3. Error budgets

If the above shown definitions of the variables are correct then the system is seemingly be able to detect the simultaneousness of the receiving of the 4 signals. To prevent discussions about this it is also possible to look at the equations in another, more easier, way: t is the time that at position **r** a signal is simultaneously transmitted to 4 satellites at position $\mathbf{r}_i$, where they are received at time t_i respectively.

Ashby's argumentation, regarding the impact of the STR on the accuracy of the value of **r**, is that, given c as a physical constant, the values of t_i are influenced by time dilation due to the velocity of the satellites. A velocity of 4km/s is mentioned. That velocity indeed corresponds to an angular velocity twice as high as the one of the earth.

The most important mistake in [2] is that c is considered as constant on the trajectory $|\mathbf{r} - \mathbf{r}_i|$. The satellites orbit at a distance of about 20.000km above earth's surface. In between is the atmosphere. Its mean thickness is about 1000 km. As generally known the velocity of light in air is certainly not ***c***.
N.B. ***c*** is the symbol for the velocity of light in vacuum, leaving out for the moment the reference with respect to which this velocity is defined. Therefor this symbol has, properly, not been used up to now.

Propagating through air, the velocity ***c*** *decreases with the factor 1/n, with n the so-called refractive index. Due the fact that the refractive index depends on for example temperature, density and relative humidity, the 4 paths along which the 4 signals propagate in general are mutual different and thus is the mean propagation velocity of the light along these 4 paths.*

Suppose the distance between the receiver and satellite i is D_i, the mean refractive index of the atmosphere along the path of signal i to satellite i is n_i and the thickness of the atmosphere, along which this mean refractive index is applicable, is d_i. Then the mean velocity of signal i is:

$$c_i = \{\boldsymbol{c}(D_i - d_i) + (\boldsymbol{c}/n_i)*d_i\}/D_i = \boldsymbol{c}\,\{1 - (d_i/D_i)\,(1 - 1/n_i)\}$$

N.B. $c_i = \boldsymbol{c}$ for either $d_i = 0$ or $n_i = 1$.

Let us consider the most simple situation, meaning that all d_i's, respectively D_i's, are mutual equal. Then:

$$c_i - c_j = \boldsymbol{c}\,(d/D)\,(1/n_i - 1/n_j) = \boldsymbol{c}\,(d/D)\,(n_j - n_i)\,/\,n_i\,n_j$$

If n_i is rewritten as $1 + \Delta n_i$, then $c_i - c_j = \boldsymbol{c}\,(d/D)\,(\Delta n_j - \Delta n_i)$.

The factor $n_i\,n_j$ is omitted, because it only represents a multiplication factor, close to 1, for the accuracy $\Delta n_j - \Delta n_i$ to be investigated.

The absolute value of the relative difference between the two velocities, $(c_i - c_j)/\boldsymbol{c}$, is $(d/D)*|\Delta n_j - \Delta n_i|$, resulting in relative time errors of the same value.
According to Ashby, relative time errors, *due to "time delation",* are considered to be much too large to ignore. He claims relative time errors of about 10^{-10}.
Ashby writes:

"
$$t' = \sqrt{(1 - v^2/c^2)} \, . \, t \qquad (27)$$
Thus, a clock moving relative to a system of synchronized clocks in an inertial frame beats more slowly. The square root in Eq. (27) can be approximately expanded using the binomial theorem:
$$\sqrt{(1 - v^2/c^2)} \approx 1 - v^2/2c^2 \qquad (28)$$
In the GPS, satellite velocities are close to 4000 m/s, so the order of magnitude of the time dilation effect is $v^2/2c^2 \approx 10^{-10}$. This is also a huge effect."

The absolute error of a normal GPS is about 15 m [3]: "The standard accuracy of about 15 meters can be..............". Relatively speaking this is an error of $15/2{*}10^7 \sim 10^{-6}$.
So Ashby claims an error due to "time dilation" 10.000 times *smaller* than the real error.

The refractive index of air at atmospheric pressure is 1.0003. Above 100 km height the pressure is so low that we can assume a refractive index of 1.
Suppose the mean refractive index of this layer of 100 km is 1.0002, so the mean Δn_i is $2{*}10^{-4}$. Suppose 10% of this mean value causes the difference $|\Delta n_j - \Delta n_i|$, being $2{*}10^{-5}$.

Then $(d/D){*}|\Delta n_j - \Delta n_i| = (100/20000){*}\ 2{*}10^{-5} = 10^{-7}$.

Compared to the real relative accuracy of 10^{-6} it turns out that the assumed mutual differences in refractive indices are most likely too optimistic.
But besides that, there are several other sources for inaccuracies [3].
The relative error as a result of the relative velocity between satellite and earth surface ((4-2)km/s) is 2/300000000, being about 10^{-8}, so negligible. See [4], [5] and [6].

4. The misconception: time dilation

Einstein introduced this concept based on, among others, the nonsense hypothesis:

> *Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body.*

The result is that (atomic) clocks will run with a speed dependant on their state of *constant* velocity. But a clock in a state of constant velocity represents an inertial system. So, given the hypothesis: all physical laws are the same in any inertial system, a clock in a state of an arbitrary constant velocity will not show any deviation.

Satellites can, given their large distance relative to the centre of the earth, be considered as inertial systems!

Conclusion

Claiming that the GPS is so accurate due to relativity corrections is, for more than one reason, nonsense.

References

[1] Every Day Einstein, Philip Yam, Scientific American, Special Issue, September 2004
[2] Relativistic Effects in the Global Positioning System, Neil Ashby, July 2006 https://www.aapt.org/doorway/TGRU/articles/Ashbyarticle.pdf
[3] https://en.wikipedia.org/wiki/Global_Positioning_System
[4] http://vixra.org/abs/1505.0072 From Astronomical to Terrestrial Light
[5] http://vixra.org/abs/1505.0145 The Doppler shift and the Special Theory of Relativity
[6] http://vixra.org/abs/1611.0111 The Principle of Absolute Relativity