

# OPRA Technique for M-QAM over Nakagami-m Fading Channel with Imperfect CSI

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## Abstract

Analysis of an Optimum Power and Rate Adaptation (OPRA) technique has been carried out for Multilevel-Quadrature Amplitude Modulation (M-QAM) over Nakagami-m fading channels considering an imperfect channel estimation at the receiver side. The optimal solution has been derived for a continuous adaptation, which is a specific bound function and not possible to express in close mathematical form. Therefore, a sub-optimal solution is derived for the continuous adaptation and it has been observed that it tends to the optimum solution as the correlation coefficient between the true channel gain and its estimation tends to one. It has been observed that the receiver performance degrades with an increase in estimation error.

**Keywords:** Adaptive Transmission Technique, MQAM, Power and Rate Adaptation, PSAM, Imperfect Channel Estimation, Spectral Efficiency.

## 1 Introduction

The Adaptive Transmission Techniques is a solution to enhance the spectral efficiency, particularly in wireless communication systems. In an Adaptive Transmission Technique, a channel estimator is applied at the receiver to estimate the channel condition and the channel information is feedback to the transmitter through a lossless path. Depending upon the channel condition the transmitter varies the power and rate of transmission adaptively in different techniques. However, the channel estimation in a real time is challenging and there is a possibility of an imperfect estimation. Researchers have studied this topic in various communication models for more than a decade and presented in the literature [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], available now for different fading channels.

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A capacity analysis has been borne out in [1] for Optimal and Sub Optimal Power and Rate Adaptation Techniques considering an Imperfect Channel Estimation for Multilevel-Quadrature Amplitude Modulation (M-QAM) over a Rayleigh fading channel. In [2] the issues of ISI, time-varying, and multiple access in the context of an error about the channel measurement available at the receiver have been considered. In the same work it has also been shown that the time variation of the channel and the error on the estimate of the channel are tightly linked. In [3] the impact of imperfect channel estimation on the Variable-Rate Variable-Power QAM (Quadrature Amplitude Modulation) performance is contemplated for a flat fading environment. Here, a set of new analytical expression is derived that shows the high sensitivity of the BER (Bit Error Rate) to both the estimated MSE (Mean Square Error) and the system adaptation delay. Similarly, a general approach to calculate the exact BER of M-QAM with the PSAM (Pilot-Symbol Assisted Modulation) in flat Rayleigh fading channels is given in [4] when there are some channel estimation errors. In [5] the authors proposes an adaptive multi-mode transmission strategy to improve the spectral efficiency achieved in the multiple-input multiple-output (MIMO) broadcast channel with an imperfect channel state information. The adaptive strategy adjusts the number of active users, denoted as the transmission mode, to balance, the transmit array gain, the spatial division multiplexing gain, and the residual inter-user interference. The optimum power profile and the ergodic capacity have been derived for Rayleigh fading channels with respect to an average or a peak transmit power, along with more realistic interference outage constraints in [6]. Also, the impact of channel estimation quality on the ergodic capacity has been highlighted. The performance analysis of a space-time coded MIMO system with the Variable-Rate-Adaptive Modulation over flat Rayleigh fading channels for both perfect and imperfect channel state information (CSI) has been presented in [7]. In [8] the ergodic capacity of bidirectional amplify-and-forward relay selection network has been analyzed. Also, the imperfect CSI takes into account, which includes outdated CSI and channel estimation error, caused by the time-variation of the channel and the imperfect channel estimation. A system where the receiver should harvest energy from the transmitter by wireless energy transfer to bear out its wireless information transmission has been studied in [9]. In [10] an optimal precoding method for a multiple antenna relay node is investigated in order to maximize the achievable rate of the cooperative communication system. It is assumed that only the channel covariance matrices of the relays receive and transmit channels are available for the relay and that the antennas of the relay are correlated. The optimization of an amplify and forward (AF) relay network with time delay and estimation error in the channel state information (CSI) has been modeled by the channel time variation and the stochastic error, respectively in [11]. The effect of Rayleigh and Rician fading distribution for various transmit diversity and MIMO system has been analyzed in [15]-[29]. In [30] the Nakagami-m fading parameters are estimated for the transmit diversity. However, in the literature, the capacity analysis of adaptive transmission scheme is not available for Nakagami-m fading channels with an imperfect channel estimation. The Nakagami-m fading channels [12] receive attention of researchers due to its flexibility, simple analytical form and also because it fits best into the practically obtained data. This gets a motive to know the result of the imperfect channel estimation on the channel capacity of adaptive transmission scheme over the Nakagami-m fading channels. In this paper, a capacity analysis is done for an optimal and a sub-optimal power and rate adaptation techniques for an imperfect channel knowledge considering

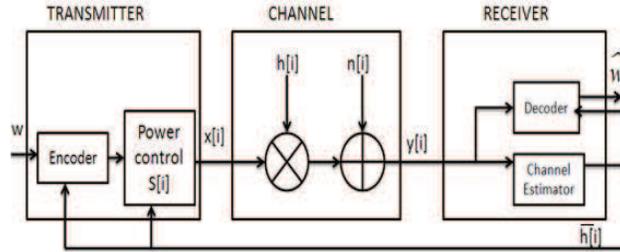


Figure 1: System Model for Power and Rate Adaptation

MQAM over the Nakagami- $m$  fading channels. A Minimum Mean Square Error (MMSE) based channelestimation is considered which uses a PSAM under average power and an instantaneous BER constrains using an M-QAM. We compare the optimal solution with sub-optimal derivations also. The paper is organized as follows: In section 2 the system model and problem formulation is presented. In section 3 the optimum power and rate adaptation is given. Accordingly in section 4 the sub-optimal power and rate adaptation is considered. In section 5 the numerical results and analysis is done. Conclusions are given in section 6.

## 2 System Model

An Adaptive Transmission Scheme as described in [4] has been seen here for the purpose of analysis, which is presented in Fig.1. The system model sends an input message  $w$  from the transmitter. The message is encoded into the codeword  $x$ , which is transmitted over the time-varying channel as  $x[i]$  at time  $i$ . Since the channel is changing with the time, then the channel gain  $h[i]$  changes over the transmission of the code as well. The power spectral density of additive noise  $n(i)$  has been assumed as  $\frac{N_0}{2}$ . Since the channel estimation is not complete, then the gain  $h[i]$  is not perfectly known to the receiver at time  $i$ . Then, just an imperfect estimation of  $h[i]$  which is considered to be  $\hat{h}(i)$  is sent back to the transmitter through a feedback path. The average power to be transmitted is denoted by  $\bar{S}$ . For a constant transmitted power  $\bar{S}$  the instantaneous SNR at the receiver will be  $\gamma(i) = \bar{S}|h(i)|^2/(N_0B)$ , where  $B$  is the received signal bandwidth. Since, the receiver adaptively adjusts its power based on feedback, to instantaneous transmit power at time  $i$  is the function of  $\hat{h}(i)$ . Accordingly an instantaneous estimated SNR at the receiver will be given by  $\hat{\gamma}(i) = \bar{S}|\hat{h}(i)|^2/(N_0B)$ . The instantaneous BER for the model, shown in Fig. 1, considering the BER bounds and flat-fading channels with the MQAM is given by [1],

$$BER(\gamma, \hat{\gamma}) \leq C_1 \exp\left(\frac{-C_2 \gamma}{M-1} \frac{S(\hat{\gamma})}{\bar{S}}\right) \quad (1)$$

where  $C_1$  and  $C_2$  are two positive constants and  $M$  is the size of constellation in the QAM.

For an instantaneous BER the conditional expectation of BER  $(\gamma, \hat{\gamma})$  when the  $\hat{\gamma}$  is known is given by [1],

$$BER \leq \int_0^{\infty} C_1 \exp\left(\frac{-C_2\gamma}{M(\hat{\gamma})-1} \frac{S(\hat{\gamma})}{\bar{S}}\right) \rho_{\gamma/\hat{\gamma}}\left(\frac{\gamma}{\hat{\gamma}}\right) d\gamma \quad (2)$$

where  $\rho_{\gamma/\hat{\gamma}}(\gamma/\hat{\gamma})$  denotes the conditional probability density function of  $\gamma$  and  $\hat{\gamma}$  is known. Accordingly  $M\hat{\gamma}$  indicates that the rate of adaptation is also adapted at the channel estimation.

In this paper we have considered the Continuous Power and Rate Adaptation. To maximize the spectral efficiency, we have to take the help of the following constrained optimization formulation:

$$\begin{aligned} & E_{\hat{\gamma}} [\text{Log}_2 (M(\hat{\gamma}))] \\ \text{s.t. } & \begin{cases} E_{\hat{\gamma}} [S(\hat{\gamma})] \leq \bar{S} \\ BER(\hat{\gamma}) < \varepsilon \end{cases} \end{aligned} \quad (3)$$

The instantaneous BER, given in (2), assumes the knowledge of the joint PDF  $\rho_{\gamma, \hat{\gamma}}(\gamma, \hat{\gamma})$ . It is directly related to the channel model and it's possible to use in the channel estimation. Here, we derived it for the Nakagami- $m$  flat-fading channel with a linear MMSE channel estimation using the PSAM technique.

The SNR PDF  $\gamma$  for the Nakagami- $m$  fading distribution for the perfect channel estimation is given by [15],

$$\rho_{\gamma}(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma} \Gamma(m)} \exp\left(\frac{-m\gamma}{\bar{\gamma}}\right) \quad (4)$$

where,  $m$  is the Nakagami- $m$  fading parameter. Accordingly the estimated SNR PDF  $\hat{\gamma}$  will be given by,

$$\rho_{\hat{\gamma}}(\hat{\gamma}) = \frac{m^m \hat{\gamma}^{m-1}}{\bar{\gamma} \Gamma(m)} \exp\left(\frac{-m\hat{\gamma}}{\bar{\gamma}}\right) \quad (5)$$

Now the conditional PDF of  $\gamma$  and  $\hat{\gamma}$  will be given by [5],

$$\begin{aligned} \rho_{\gamma/\hat{\gamma}}\left(\frac{\gamma}{\hat{\gamma}}\right) &= \frac{m}{(1-\rho)\Gamma} \left(\frac{\gamma}{\rho\hat{\gamma}}\right)^{(m-1)/2} I_{m-1}\left(\frac{2m\sqrt{\rho}}{(1-\rho)} \sqrt{\frac{\hat{\gamma}\gamma}{\Gamma\hat{\Gamma}}}\right) \\ &\times \exp\left(\frac{-m(\rho\hat{\gamma} + \gamma)}{(1-\rho)\Gamma\hat{\Gamma}}\right) \end{aligned} \quad (6)$$

where  $\Gamma = E[\gamma]$ ,  $\hat{\Gamma} = E[\hat{\gamma}]$ ,  $\rho = \text{cov}(\gamma, \hat{\gamma})/(\text{var}(\gamma) \text{var}(\hat{\gamma}))^{1/2}$  and  $I_{m-1}(\cdot)$  is the modified Bessel function of order  $m-1$ .  $\rho$  is known as a correlation coefficient and its values always lie between 0 and 1. This indicates the channel Doppler spread or pilot-symbol spacing. For the channel estimation by substituting (6) into (2) and using PSAM technique, the expression can be bounded as,

$$BER(\hat{\gamma}) \leq C_1 f(\hat{\gamma}) \exp\left(\frac{-m\rho}{(1-\rho)} \frac{\hat{\gamma}}{\hat{\Gamma}} (1 - f(\hat{\gamma}))\right) \quad (7)$$

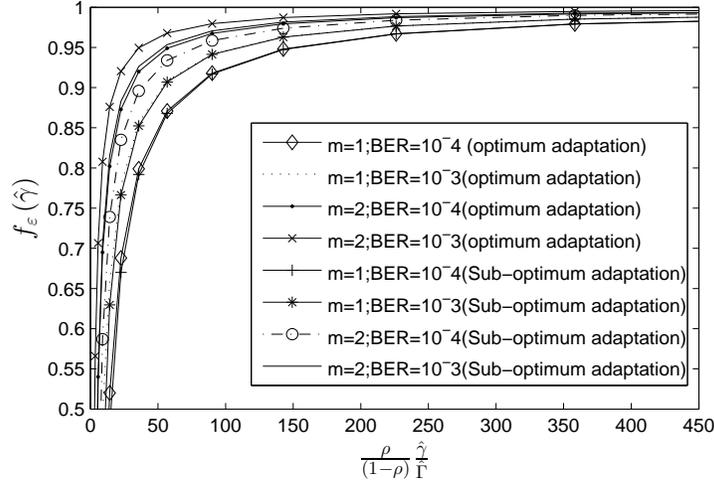


Figure 2: System Model for Power and Rate Adaptation

where  $f(\hat{\gamma}) = \left(1 + \frac{C_2(1-\rho)\Gamma}{m} \frac{S(\hat{\gamma})}{S(M(\hat{\gamma})-1)}\right)^{-1}$  assuming  $S(\hat{\gamma}) \geq 0$  and  $M(\hat{\gamma}) \geq 1$  it can be said that  $f(\hat{\gamma})$  always lies between 0 and 1 for all the values of  $\hat{\gamma}$ .

### 3 Optimum Power and Rate Adaptation

In this paper, we have considered the Continuous Rate Power and Rate Adaptation process. From eq. (7) we can calculate the upper bound of the BER for  $M(\hat{\gamma})$  (Optimum Rate) and  $S(\hat{\gamma})$  (Optimum Power) which is given in (3). For the maximum BER consideration let  $f(\hat{\gamma})$  be substituted as  $f_\varepsilon(\hat{\gamma})$ . Then (7) can be written as

$$C_1 f_\varepsilon(\hat{\gamma}) \exp\left(\frac{-m\rho}{(1-\rho)} \frac{\hat{\gamma}}{\Gamma} (1 - f_\varepsilon(\hat{\gamma}))\right) = \varepsilon \quad (8)$$

In fig: 2 where  $f_\varepsilon(\hat{\gamma})$  is plotted against  $\frac{\rho}{(1-\rho)} \frac{\hat{\gamma}}{\Gamma}$  for different values of  $m$  and BER where we can see that for each of these values,  $f_\varepsilon(\hat{\gamma}) \rightarrow 1$  when  $\hat{\gamma} \rightarrow +\infty$  substituting  $f(\hat{\gamma})$  as  $f_\varepsilon(\hat{\gamma})$ , we can find a relation between power and rate as given below:

$$\frac{S(\hat{\gamma})}{S} = \mu \left[ \left( \frac{1}{f_\varepsilon(\hat{\gamma})} \right) - 1 \right] [M(\hat{\gamma}) - 1] \quad (9)$$

where,  $\mu = m/c_2(1-\rho)\Gamma$ . Now applying the Lagrangian for the optimization problem (3) [1] and using (9) and taking the help of calculus [Setting  $(\{\partial L[M(\hat{\gamma})]\}/[\partial M(\hat{\gamma})]) = 0$ ] the optimum

continuous rate adaptation can be formulated as,

$$M(\hat{\gamma}) = \max \left\{ 1, -\frac{1}{\mu\lambda \ln 2\bar{S}} \left[ \left( \frac{1}{f_\varepsilon(\hat{\gamma})} \right) - 1 \right]^{-1} \right\} \quad (10)$$

By substituting (10) into (9) we can calculate the Optimum Power Adaptation as,

$$\frac{S(\hat{\gamma})}{\bar{S}} = \max \left\{ 0, -\frac{1}{\mu\lambda \ln 2\bar{S}} - \mu \left[ \left( \frac{1}{f_\varepsilon(\hat{\gamma})} \right) - 1 \right] \right\} \quad (11)$$

The value of  $\lambda$  should be set so that the power constraint of the problem is met, which indicates that  $\lambda < 0$ . As given in [1] defining  $\bar{S}_T = \mu ((C_1/\varepsilon) - E_{\hat{\gamma}} \{ [1/f_\varepsilon(\hat{\gamma})] \})$  the Optimum Power Adaptation can be written as,

$$\frac{S(\hat{\gamma})}{\bar{S}} = \max \left\{ \begin{array}{l} \mu \left[ \left( \frac{1}{f_\varepsilon(\hat{\gamma}_0)} \right) - \left( \frac{1}{f_\varepsilon(\hat{\gamma})} \right) \right] \hat{\gamma} > \hat{\gamma}_0 \\ 0 \dots \dots \dots \text{Otherwise} \end{array} \right\} \quad (12)$$

$\hat{\gamma}_0$  is known as a threshold value so that,  $f_\varepsilon(\hat{\gamma}_0) = \left( 1 - \left\{ \frac{1}{\mu\lambda \ln 2\bar{S}} \right\} \right)^{-1}$ . Now for the Optimum Rate Adaptation the expression can be given by,

$$M(\hat{\gamma}) = \left\{ \begin{array}{l} \left( \frac{1}{f_\varepsilon(\hat{\gamma}_0)} \right) - 1 / \left( \frac{1}{f_\varepsilon(\hat{\gamma})} \right) - 1 \dots \dots \hat{\gamma} > \hat{\gamma}_0 \\ 0 \dots \dots \dots \hat{\gamma} < \hat{\gamma}_0 \end{array} \right\} \quad (13)$$

From (13) we can find an expression for the Optimum Power and Rate Adaptation which should always be satisfied by  $\hat{\gamma}_0$ . The condition is,

$$\int_{\hat{\gamma}_0}^{\infty} \mu \left( \left( \frac{1}{f_\varepsilon(\hat{\gamma}_0)} \right) - \left( \frac{1}{f_\varepsilon(\hat{\gamma})} \right) \right) \rho_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1 \quad (14)$$

Like [1] also for Nakagami- $m$  fading distribution, we can formulate the spectral efficiency, as

$$\begin{aligned} E_{\hat{\gamma}} [\log_2 M(\hat{\gamma})] &= \log_2 \left( \frac{1}{f_\varepsilon(\hat{\gamma}_0)} - 1 \right) - \int_{\hat{\gamma}_0}^{\infty} \log_2 \left( \frac{1}{f_\varepsilon(\hat{\gamma})} - 1 \right) \\ &\quad \times \rho_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}, \bar{S} < \bar{S}_T \end{aligned} \quad (15)$$

where  $\rho_{\hat{\gamma}}(\hat{\gamma})$  is given by,  $\rho_{\hat{\gamma}}(\hat{\gamma}) = \frac{m^m \hat{\gamma}^{m-1}}{\hat{\gamma} \Gamma(m)} \exp\left(-\frac{m\hat{\gamma}}{\hat{\gamma}}\right)$

## 4 Sub Optimum Power and Rate Adaptation

Since it is not possible to find a close form equation for the Optimum Power and Rate Adaptation as shown in the above section. Therefore, here, we try to find an approximate value for  $f_\varepsilon(\hat{\gamma})$  and

then it is possible to have a close form equation for the Optimum Power and Rate Adaptation. Let the approximate value of  $f_\varepsilon(\hat{\gamma})$  be  $\varphi_\varepsilon(\hat{\gamma})$ , for which (7) can be written as,

$$BER \leq C_1 \exp\left(\frac{-m\rho}{(1-\rho)} \frac{\hat{\gamma}}{\Gamma} (1 - f(\hat{\gamma}))\right) \quad (16)$$

Considering a maximum BER ( $\varepsilon$ ) in (16) an expression of  $f_\varepsilon(\hat{\gamma})$  (maximum bit error rate function) can be given as,

$$f_\varepsilon(\hat{\gamma}) = 1 - \frac{(1-\rho)\bar{\gamma}}{m\rho\hat{\gamma}} \ln \frac{C_1}{\varepsilon}, \quad 0 < f_\varepsilon(\hat{\gamma}) < 1 \quad (17)$$

from (17) we can write the expression of  $f_\varepsilon(\hat{\gamma})$  in terms of  $\varphi_\varepsilon(\hat{\gamma})$  as

$$\varphi_\varepsilon(\hat{\gamma}) = \begin{cases} 1 - \frac{\hat{\gamma}_T}{\hat{\gamma}}, \hat{\gamma} \geq \hat{\gamma}_T \\ 0 \dots \dots \dots otherwise \end{cases} \quad (18)$$

where  $\hat{\gamma}_T = \frac{(1-\rho)\bar{\Gamma}}{m\rho} \ln \frac{C_1}{\varepsilon}$ . In fig:2  $f_\varepsilon(\hat{\gamma})$  and  $\varphi_\varepsilon(\hat{\gamma})$  are plotted for  $\varepsilon = 10^{-4}$  and  $\varepsilon = 10^{-3}$  for different values of  $m$ . From the plot it is clear that if  $\varphi_\varepsilon(\hat{\gamma}) \rightarrow f_\varepsilon(\hat{\gamma})$  then  $\rho \rightarrow 1$  for all values of  $m$ . As [1] for the Nakagami- $m$  also the Sub optimal solution for the power adaptation is given by,

$$\frac{S(\hat{\gamma})}{\bar{S}} = \max \left\{ \begin{array}{l} \mu(\hat{\gamma}_T) \left[ \left( \frac{1}{\hat{\gamma}_0 - \hat{\gamma}_T} \right) - \left( \frac{1}{\hat{\gamma} - \hat{\gamma}_T} \right) \right] \hat{\gamma} \geq \hat{\gamma}_0 \\ 0 \dots \dots \dots Otherwise \end{array} \right\} \quad (19)$$

Here,  $\gamma_0 > \gamma_T$  and must satisfy the condition, and must satisfy the condition,

$$\int_{\hat{\gamma}_0}^{\alpha} \mu(\hat{\gamma}_T) \left( \left( \frac{1}{\hat{\gamma}_0 - \hat{\gamma}_T} \right) - \left( \frac{1}{\hat{\gamma} - \hat{\gamma}_T} \right) \right) \rho_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = 1 \quad (20)$$

Accordingly, for the rate adaptation, the expression is given by,

$$M(\hat{\gamma}) = \begin{cases} \frac{(\hat{\gamma} - \hat{\gamma}_T)}{(\hat{\gamma}_0 - \hat{\gamma}_T)} \\ 1 \dots \dots \dots otherwise \end{cases} \quad \hat{\gamma} \geq \hat{\gamma}_0 \quad (21)$$

Now from (21) the spectral efficiency can be calculated as given bellow:

$$E_{\hat{\gamma}} [\log_2 M(\hat{\gamma})] = \int_{\hat{\gamma}_0}^{\alpha} \log_2 \frac{(\hat{\gamma} - \hat{\gamma}_T)}{(\hat{\gamma}_0 - \hat{\gamma}_T)} \rho_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \quad (22)$$

where  $\rho_{\hat{\gamma}}(\hat{\gamma})$  is given by,  $\rho_{\hat{\gamma}}(\hat{\gamma}) = \frac{m^m \hat{\gamma}^{m-1}}{\hat{\gamma} \Gamma(m)} \exp\left(\frac{-m\hat{\gamma}}{\bar{\gamma}}\right)$  If we put  $\rho = 1$  in eq.(22) and  $m = 1$  (Rayleigh Fading case) then the expression (22) will be converted to the close form equation available in literature.

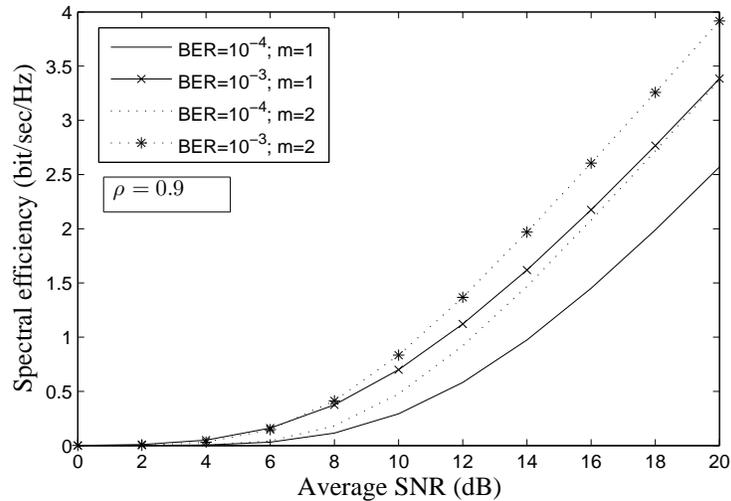


Figure 3: Spectral Efficiency for OPRA considering imperfect channel state estimation

## 5 Numerical Results and Analysis

The obtained results have been numerically evaluated and plotted for the purpose of analysis. The plots have been observed for different values of the Nakagami- $m$  fading parameter  $m$ , BER and  $\rho$ . The spectral efficiency is studied in fig: 3 considering the correlation coefficient=0.9 and the BER ( $10^{-4}$  and  $10^{-3}$ ). It has been observed that as expected the spectral efficiency increases exponentially with the increase of the average SNR. Merely, for different values of the Nakagami-fading parameter  $m$  the spectral efficiency is different. With the increase of  $m$  value the spectral efficiency has also increased. This is because of the fact that the channels improve with increase in  $m$ . In fig:4 the Spectral efficiency vs the Average SNR has been plotted for the Sub-Optimal and the Optimal Power and Rate Adaptation techniques for different values of  $\rho$ ,  $m$  and the BER. It has been observed that the difference between optimal and suboptimal is very narrow for the same value of  $m$ , BER and  $\rho$ . It can likewise be noted from the plot that the decrease in  $\rho$  for the constant  $m$  and the BER reduce the spectral efficiency. It is due to the increase in the estimation error, because of which the receiver performs poorly.

## 6 Conclusions

In this paper we have analyzed the Optimum Power and the Rate Adaptation considering the imperfect channel state information over the Nakagami- $m$  fading channels and the MQAM. Since, it is difficult to derive a closed form expression for an optimal technique using some assumptions,

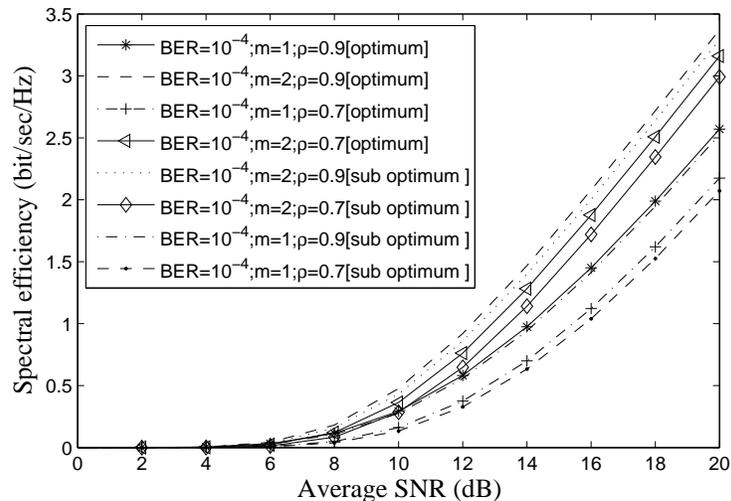


Figure 4: Spectral Efficiency for OPRA and Sub-OPRA considering imperfect channel state estimation

a closed form sub-optimal solution has been proposed. It has also been observed that the optimal and sub-optimal solution has a very narrow gap. The derived expressions are verified with the well-known expressions available in the literature.

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