A Brief Approach to the Riemann Hypothesis Over the Lagarias Transformation

Mesut KAVAK*

Over the paper of Lagarias [1], for a positive integer n, let $\sigma(n)$ denote the sum of the positive integers that divide n. Let H_n denote the nth harmonic number by

$$H_n = \sum_{n=1}^{n} \frac{1}{n}$$

Does the following inequality hold for all $n \geq 1$ where $\sigma(n)$ is the sum of divisors function?

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$$

1 Definition for the solutions

Theorem: *First of all, let's define an imaginary function as $\rho(n)$, and know that this function is the sum of the elements which are not dividable being the result is an integer in a function as nH_n; so according to this definition, it becomes as the following.*

$$H_n = \frac{\sigma(n) + \rho(n)}{n}$$

By using the equation, $H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$ inequality turns into (1).

$$H_n + \ln(H_n)e^{H_n} \geq nH_n - \rho(n) \tag{1}$$

If it is edited, it becomes (2) over (2a).

$$\frac{\ln(H_n)e^{H_n} + \rho(n)}{n-1} \geq H_n \tag{2}$$

$$\ln(H_n)e^{H_n} \geq nH_n - H_n - \rho(n) \tag{2a}$$

Condition: *Right this point assume, that the actual inequality is not (2) but is (3).*

$$\frac{e^{H_n}}{n} \geq H_n \tag{3}$$

On (2), actually the numerator is always bigger than e^{H_n}, and also if the divisor was $n-1$, this would increase the possibility of to be greater than H_n of the division; so for the worst possibility, let's use this as (3).

2 Conclusion

The final inequality (3) is true for any $n \geq 1$ integer, and so as it is for the worst possibility, it means that for greater n values, accuracy of the main inequality increases; but how we can prove it?

Acknowledgment

I have been working about some unknown problems for a time [2] that Riemann Hypothesis is included as well, and a short time ago I supposed that I found a solution out to the Riemann Hypothesis; but I noticed that there is a stupid mistake; so, I want to only publish a very simple approach.

References

1. Jeffrey C. Lagarias. 2002 *An Elementary Problem Equivalent to the Riemann Hypothesis*, The American Mathematical Monthly. Vol. 109, No. 6, pp. 534-543
2. Kavak M. 2018, *Complement Inferences on Theoretical Physics and Mathematics*, OSF Preprints, Available online: https://osf.io/tw52w/

*kavakmesut@outlook.com.tr