Singlet Higgs Spontaneity in considered action: the Lie-dependent Masses

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Abstract

We derived the Lie-dependent masses of certain particles gauged as TeVeS in considered Lie groups raised from gauge couplings with constant global sections of singlet Higgs under the algorithm on mass terms which comes out naturally from the kinetic part of our considered TaLie action, and also available on the gauge fields as connections in formed Y-M actions. With the only parameters, *scaled mass* $M(H^D) \in \mathbb{R}^+$ of each Higgs section introduced in this mechanism, we concretely computed the masses $m_{W^\pm}$, m_{Z^0}, m_X and m_H under the gauge selection $E_{8(-24)}$ in *Lie Group Cosmology* (LGC), figuring out how the masses of every different singlet Higgs bosons all equal one real number $\sqrt{2} \cdot M(H^\Sigma)$. When comparing the results with recent experiments at LHC, we find the singlet Higgs spontaneity with algorithms derived from our considered action under the gauge selection of LGC is consistent with current data including the diphoton excess at 750 GeV, as well as stating some important implications from the derived Lie-dependent masses and our constructions on the mechanism.

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1 Introduction

Renormalized perturbation theory with Feynman rules provide the effective way to analyse the behaviour and interactions of gauged particles in SM. However, perturbative computations on higher orders, especially on fermions described by Dirac Lagrangian, are extremely complicated. In the light of simplification, we made attempts on improving [1] the basic structures in our considerations. For example, an idea first introduced by R. P. Feynman to gauge the Dirac spinors in a Lagrangian with second-ordered kinetic term was developed and performed [2, 3] in 2012, which leads to significant simplifications on the perturbative derivations.

We are considering non-perturbative valid methods on representing certain properties in the evolution of Lie-valued sections and gauged particles along the proper time [4]. The first step here is to introduce the mechanism developed from SSB of singlet Higgs and its coupling with Lie-valued gauged field under specific gauge conditions via the kinetic terms (written in a shortened form)

$$\mathcal{L}_H \sim (d + A)H(d - A)H \tag{1}$$

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$$\mathcal{L}_A \sim -H^2 AA \tag{2}$$

in our considered TaLie action, legible on TeVeS fields gauged in classical Y-M actions as well. Herein we also present the computational results and important implications under the gauge selection of Lie Group Cosmology [5] with the sections of frame-Higgs $\underline{e}\phi$ [6] from MacDowell-Mansouri gravity [7].

2 Constructed TaLie action

TaLie Lagrangian is constructed as second-ordered gauge-invariant Lagrangian with the variable inner product $n_{B(i)C(j)}$ introduced from the background of gauged simple Lie algebra and with our many designs on projected particles of sourced gauge fields from Lie-valued generators of considered simple Lie groups T_B, onto the curved spacetime $(\mathcal{M}, g_{\mu\nu})$ which is conformal to a regular $(Cl^1(1,3), \eta_{\alpha\beta})$ manifold where $\eta_{\alpha\beta} = (+,-,-,-)$, summing up all i and $j \neq i$ kinetic terms of the addressed 4-vectors notated $A_{\mu(i)}(x)$ and scalars $\phi_{(i)}(x)$ as gauged particles to raise its action.

Firstly we write down two pseudo- covariant derivatives on real potential $A_{\mu(i)}$ there $D_\mu \equiv \partial_\mu + A_{\mu(j)}$ and $\bar{D}_\mu \equiv \partial_\mu - A_{\mu(j)}$ for all $j \neq i$ with $A_{\mu(j)} \equiv A_{\mu(1)} + A_{\mu(2)} + ... + A_{\mu(n=j)}$. Observations around $\mathcal{M}$ are dependent locally on the metric $g_{\mu\nu}$ that from $\partial^\mu \equiv g^{\mu\rho}\partial_\rho$ and $A^\mu \equiv g^{\mu\rho}A_\rho$ we can define $\bar{D}^\mu \equiv \partial^\mu - A^{\mu(j)}$ for all $j \neq i$ leading to these pseudo- field tensors $F_{\mu\nu} = 2D_{[\mu}A_{\nu]}$, $\bar{F}_{\mu\nu} = 2\bar{D}_{[\mu}A_{\nu]}$, and $F^{\mu\nu} = 2D^{[\mu}A^{\nu]}$ with $\bar{F}^{\mu\nu} = g^{\mu\rho}g^{\nu\sigma}\bar{F}_{\rho\sigma}$ for

$$g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} = g^{\mu\rho}g^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho) = g^{\mu\rho}\partial_\rho\, g^{\nu\sigma}A_\sigma - g^{\nu\sigma}\partial_\sigma\, g^{\mu\rho}A_\rho = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{3}$$

an example of Abelian fields, where the repeated indices sum up automatically to leave the others as resulting components. Or one may use an invertible tetrad to convert $F_{\alpha\beta} = F_{\mu\nu}e^\mu_\alpha e^\nu_\beta$ that

$$F^{\alpha\beta} = \eta^{\alpha\gamma}\eta^{\beta\delta}F_{\gamma\delta} = \eta^{\alpha\gamma}\eta^{\beta\delta}F_{\mu\nu}e^\mu_\alpha e^\nu_\beta \tag{4}$$

results in $F_{\mu\nu}\bar{F}^{\mu\nu} = F_{\alpha\beta}\bar{F}^{\alpha\beta}$, suitable for e.g. de Sitter spaces.

Now we gauge the generators of considered Lie groups correctly as sources addressed in TaLie Lagrangian. Rescale $A_{\mu(j)} \rightarrow A^{C(j)}_{\mu(j)}$ and $A_{\mu(i)} \rightarrow A^{B(i)}_{\mu(i)}$ as the vector-like particles addressed on manifold $\mathcal{M}$ which are valued by the generators T_C or T_B correspondingly, of considered Lie algebra in their representational Killing inner spaces. Vectors $A_{\mu(i)}$ are different from the rescaled vectors $A^B_{\mu(i)}$ up to a Lie-dependent real factor, which will be discussed next and presented in the action. When considering the gauged field not have been addressed with label i (no sum) but as classical C^∞-field, an $A^B_\mu(x)$ over $\mathcal{M}$ is considered as $\underline{A} = d\underline{x}^\mu A^B_\mu(x)T_B$ a Lie-valued *connection*, also placed with a rescaling $A_\mu \rightarrow A^B_\mu$. We always introduce a Lie bracket in the *adjoint representation* to conserve Leibniz law on the wedge product between graded Lie-valued forms $A_1 \wedge A_2$. But for meeting the gauge-invariances of the addressed i-labelled generators, one should here introduce the Killing inner product $n_{BC} = (T_B, T_C)$ as Lie-dependent parameters derived from the measures of a linearized Lie group in the Lagrangian that

$$F_{\mu\nu}\bar{F}^{\mu\nu} = F^B_{\mu\nu}\bar{F}^{\mu\nu C}n_{BC} = g^{\mu\rho}g^{\nu\sigma}n_{BC}F^B_{\mu\nu}\bar{F}^C_{\rho\sigma} \tag{5}$$

consistent with the conditions on meeting the gauge-invariance $tr(F,F)$ when we applied trivially the Killing form n_{BC} in the Lagrangian. With

$$\mathcal{L} = \frac{1}{2}F_{\mu\nu}\bar{F}^{\mu\nu} = 2D_{[\mu}A_{\nu]}\bar{D}^{[\mu}A^{\nu]}$$

set and variable $n_{B(i)C(j)}$ for our arrangement [36] applied, we arrive at

$$\begin{aligned}\mathcal{L}_{(i)} &= \frac{1}{2}\{(\partial_\mu A^{B(i)}_{\nu(i)} - \partial_\nu A^{B(i)}_{\mu(i)})(\partial^\mu A^{\nu(i)B(i)} - \partial^\nu A^{\mu(i)B(i)})\|T_B\|^2\ [\mathbf{C}, T_B] \\ &- \sum_{j\neq i}(A^{C(j)}_{\mu(j)} A^{B(i)}_{\nu(i)} - A^{C(j)}_{\nu(j)} A^{B(i)}_{\mu(i)})(A^{\mu(j)C(j)} A^{\nu(i)B(i)} - A^{\nu(j)C(j)} A^{\mu(i)B(i)}) n_{B(i)C(j)}\ [T_C, T_B]\} \\ &= (\partial_\mu A^{B(i)}_{\nu(i)} \partial^\mu A^{\nu(i)B(i)} - \partial_\mu A^{B(i)}_{\nu(i)} \partial^\nu A^{\mu(i)B(i)})\|T_B\|^2\ [\mathbf{C}, T_B] \\ &- \sum_{j\neq i}(A^{C(j)}_{\mu(j)} A^{B(i)}_{\nu(i)} A^{\mu(j)C(j)} A^{\nu(i)B(i)} - A^{C(j)}_{\mu(j)} A^{B(i)}_{\nu(i)} A^{\nu(j)C(j)} A^{\mu(i)B(i)}) n_{B(i)C(j)}\ [T_C, T_B]\end{aligned} \tag{6}$$

a complete expression of the constructed *TaLie Lagrangian* where $\mathbf{C}$ inside Lie bracket $[\mathbf{C}, T_B]$ is the generator selected from a *Cartan subalgebra* of the considered simple Lie group or else a semi-simple Lie group (e.g. $Spin(4,4)$), leaving the indices (μ, ν and B, C) all right for every addressed i-labelled particles.

Notice: when considering the action in a gauged simple Lie group with Cartan-Weyl basis that $\|T_B\|$ are usually fixed at $\sqrt{2}$ or 1 in the representation space of their Lie algebra, values of the Lie brackets in *fundamental representation* drop to $\mathbf{1}$ because for each $B(i)$ and different T_C, the adjoint $ad_{\mathfrak{h}} = [\mathbf{C}, T_B]$ and $ad_{\Sigma\subset\mathfrak{g}} = [T_C, T_B]$ will result in different generators T_R of their 1-dimensional subspaces in the linearized Lie group extended as the linear space where Lagrangian Eq.(6) is staying in.

In this way the derived *TaLie action* for all particles addressed locally as $A_{(i)} \in \mathcal{M}$ reads

$$\mathcal{S}_{TaLie} = \sum_i \mathcal{L}_{(i)}. \tag{7}$$

summing up labels i and j (when doing this, the summing up of indices B and C have been considered. Thus as in Eq.(6) they should not be treated as summing-up indices where we have placed the labels i and j). The coupling constant for each section is absorbed into A^B_μ as the real physical fields, leaving A_μ just as an abstract functional.

With $F^{\mu\nu} = g^{\mu\rho} g^{\nu\sigma} \overline{F}_{\rho\sigma}$ and an applied $n_{B(i)C(j)}$ it is designed to be different from classical Y-M styled actions that A^3 ordered terms are here vanished. That we can construct the action as a sum of sourced i-labelled Lagrangian but not an integration of forms over the base manifold is because the gauge informations are now absorbed into $n_{B(i)C(j)}$ to provide effectively its computing on specific considered *particles*. The action leads to non-perturbative motions. Variable $n_{B(i)C(j)} = (T_{B(i)}, T_{C(j)})$ is derived from Killing product n_{BC} as the representation of $A^{B(i)}_{\mu(i)}$ or $A^{C(j)}_{\mu(j)}$ generators' projections from an entire deforming Lie group onto $\mathcal{M} \simeq Cl^1(1,3)$ the curved spacetime, dependent on the positional distinctions between these two projected generators on $\mathcal{M}$. For scalar-like generators $H_{(i)}(x) \in \mathcal{M}$ the Lagrangian should be

$$\mathcal{L}_{H(i)} = (\partial_\mu H^D_{(i)} \partial^\mu H^D_{(i)})\|T_D\|^2 - \sum_{j\neq i}(A^{C(j)}_{\mu(j)} H^D_{(i)} A^{\mu(j)C(j)} H^D_{(i)}) n_{D(i)C(j)} \tag{8}$$

raised basically in terms of the principles in SR, or if we consider TaLie Lagrangian for vector-like particles under the Lorentz gauge $\partial_\mu A^\mu = 0$ then

$$-\partial_\mu A^{B(i)}_{\nu(i)} \partial^\nu A^{\mu(i)B(i)} = -(0 - A^{B(i)}_{\nu(i)} \cdot \partial_\mu \partial^\nu A^{\mu(i)B(i)}) = A^{B(i)}_{\nu(i)} \cdot \partial^\nu \partial_\mu A^{\mu(i)B(i)} = 0 \tag{9}$$

which can also be derived from the equation of motion of A^ν, results in a kinetic part of Lorentz gauged TaLie Lagrangian as an analogy of Eq.(8) on spin-1 bosons,

$$\mathcal{L}_{(i)} \sim (\partial_\mu A^{B(i)}_{\nu(i)} \partial^\mu A^{\nu(i)B(i)}) \|T_B\|^2. \tag{10}$$

Moreover, if we add an analogized spin-1 mass term $-m^2 A_{\nu(i)} A^{\nu(i)} \equiv -m^2 A^{B(i)}_{\nu(i)} A^{\nu(i)B(i)} \|T_B\|^2$ with the pure kinetic ∂-terms of a vector-like boson (and here without the Lorentz gauge condition), it gives the equations of motion well considered in vacuum

$$-m^2 A^{\nu(i)} + \partial_\mu F^{\mu\nu(i)} = 0 \tag{11}$$

which will later be discussed in Section 4. In order to derive the algorithms on Lie-dependent masses we provisionally require little more preparations on the projector $n_{B(i)C(j)}$ which can also be used to generate the Swinger sources $J^{\mu(i)}$ in the motion, but further structures of the unpolarized Lie-valued generators in their representation spaces, because the spontaneous mechanism on particle masses from singlet Higgs we introduce next are also available for classical C^∞- Y-M fields. For scalar Higgs as sources of the masses, we start from reviewing the singlet Higgs.

3 Singlet Higgs spontaneity

As first introduced [8–10] by P. W. Higgs in the 1950s, Higgs spontaneity always shows its youth [11–20] and vitality [21–30]. The type of singlet Higgs spontaneity considered here is from an spontaneous symmetry breaking (SSB) of rank-1 Lie group, but not to involve additional structures as "Little Higgs" [31, 32]. Rank-1 Higgs bosons break a dim-$(g_1 + h_1)$, rank-g_1 Lie group $\phi \in G_1$ down to its dim-(g_2+h_2), rank-g_2 Lie subgroup $\phi' \in G_2$ with $g_1-g_2 = 1$. While the $(g_1+h_1-g_2-h_2)$ NGBs are broken up, Higgs bosons as extra roots addressed with their extremal points r should conserve the equivalence between the sums of roots before and after the spontaneous symmetry breaking:

$$\begin{gathered} \#H + h_1 = h_2 + (g_1 + h_1 - g_2 - h_2) = h_1 + g_1 - g_2 \\ \#H = g_1 - g_2 = 1 \end{gathered} \tag{12}$$

With a gauge transform $\phi(x) \to \phi' = e^{i\theta(x)}\phi(x)$ stated, the approximation

$$\phi_0 = \frac{1}{\sqrt{2}} r e^{i\theta/r} \cong \frac{1}{\sqrt{2}}(r + i\theta) \tag{13}$$

is true iff $\lambda \to 0$. Set $V(\phi\phi^*)|_{min} = \dfrac{\mu^2}{2\lambda} \equiv \dfrac{r^2}{2}$ in the potential as $V(\phi\phi^*) = -\mu^2(\phi\phi^*) + \lambda(\phi\phi^*)^2$ then one obtain

$$\mathcal{L}_\theta = \frac{1}{2}\partial\theta\partial\theta + \frac{\mu^4}{4\lambda} - \frac{1}{4}\lambda\theta^4 \tag{14}$$

the Lagrangian from $\phi = \dfrac{1}{\sqrt{2}}(r + i\theta)$. The θ^4 term is compulsorily vanished by a phase selection $\theta'_{(j)} = \theta_{(j)} + (-\theta_{(j)}) = 0$ that $\mathcal{L}_\theta$ just contains purely a kinetic term of $\theta_{(j)}$. In this case the Higgs' Lagrangian from $\phi = \dfrac{1}{2}(r + H(x))$,

$$\mathcal{L}_H = \frac{1}{2}\partial H \partial H - \mu^2 H^2 - \mu\sqrt{\lambda} H^3 - \frac{\lambda}{4} H^4 \tag{15}$$

gives H^3, H^4 Higgs' self-coupling terms besides an $m_H = \sqrt{2}\mu$ mass.

4 Singlet Higgs mechanism in Talie action

Whenever a Lie-valued field $A_\mu^B(x)$ on the base manifold is addressed, sourced or degenerated, it couples with the sections of singlet Higgs H^D as global sections from the generators T_D. In terms of the scalarization of mass in singlet Higgs spontaneity, the value of a scalar-like Higgs boson is real and commutative with other scalars or vectors as

$$-HA_\mu HA^\mu = -HHA_\mu A^\mu = -H^D H^D A_\mu^B A^{\mu B} \|T_B\|^2. \tag{16}$$

When considering all the fields gauged in *simple Lie group*, we absorb other factors that may appear with the Higgs bosons into the constant H^D. This responsible arrangement will naturally leads to $\|T_D\| = \|T_B\|$, and here the constant global section of scalarized H^D becomes a real multiplier acting on the vector A_μ, leaving the norm of A_μ^B as $\|T_B\|^2$.

A gauge coupling of $H^D - A_\mu^B$ in the description of fundamental TaLie Lagrangian reads

$$\begin{aligned} \mathcal{L} \sim \mathcal{L}_{H-A_\mu} &= 2\partial_{[\mu} A_{\nu]} \partial^{[\mu} A^{\nu]} - HA_\nu HA^\nu \\ &= (\partial_\mu + H)A_\nu(\partial^\mu - H)A^\nu \\ &= \partial_\mu A_\nu^B \partial^\mu A^{\nu B} \|T_B\|^2 - H^D H^D A_\nu^B A^{\nu B} \|T_B\|^2 \end{aligned} \tag{17}$$

with the vector field A_μ under Lorentz gauge condition $\partial_\mu A^\mu = 0$, and

$$\begin{aligned} \mathcal{L} \sim \mathcal{L}_{A_\mu - H} &= (\partial_\mu + A_\nu)H(\partial^\mu - A^\nu)H \\ &= (\partial_\mu \partial^\mu \|\mathbf{C}\|^2 - A_\mu^B A^{\mu B} \|T_B\|^2) H^D H^D \end{aligned} \tag{18}$$

where the derivatives ∂_μ with respect to proper time τ on the scalarized constant section H become multipliers that are commutative with H along the differentiated $d\tau$, and $\|\mathbf{C}\|$ is the norm of Cartan subalgebra in the considered simple Lie algebra, i.e. in the representational inner product space with Killing form,

$$\|\mathbf{C}\|^2 = \sum_{orthonormal}^{rank(G)} \|T_B\|^2. \tag{19}$$

By each $H^D - A_\mu^B$ coupling, a global section of singlet Higgs H^D gives the mass raised from Eq.(18) to particle $A_{\mu(i)}$ via Eq.(17), leaving itself as massless, constant background field but *related* to the coupled particle $A_{\mu(i)}$.

4.1 The mass of singlet Higgs bosons

In our mechanism on the Lie-dependent masses raised from scalarized global Higgs sections, there can be more than one generators of considered Lie algebra which lead to different Higgs, and the mass terms raising their different masses are derived from their kinetic terms in TaLie action, dependent on the Lie-values of the generators to the particles that they are coupling with, respectively, in the representational space of considered Lie algebra. Let us turn to a free singlet Higgs boson,

$$\mathcal{L}_{H_{(i)}^{D(i)}} = \partial_\mu H_{(i)}^{D(i)} \partial^\mu H_{(i)}^{D(i)} \tag{20}$$

that couples with the *constant Higgs background*, i.e. all the sections of Higgs generators in our considered gauged Lie algebra, raising its mass term as

$$\mathcal{L}_{H_{(i)}^{D(i)}} = \partial_\mu H \partial^\mu H - M(H^\Sigma)^2 H^2 = \partial_\mu H_{(i)}^{D(i)} \partial^\mu H_{(i)}^{D(i)} - M(H^\Sigma)^2 H_{(i)}^{D(i)} H_{(i)}^{D(i)} \tag{21}$$

where ∂_μ acting on addressed Higgs boson but not a constant global section of Higgs is the usual 4-derivative, and here we introduced a real parameter $M(H^\Sigma) \in \mathbb{R}^+$ as the *scaled mass* of constant Higgs background, and for each Higgs generator T_D to Higgs section H^D, we introduce a real parameter $M(H^D) \in \mathbb{R}^+$ as the *scaled mass* of Higgs section H^D that

$$M(H^\Sigma) = \sum_D M(H^D). \tag{22}$$

Now compare the mass term in Eq.(21) with Eq.(15), and the consequence $m_H = \sqrt{2}\mu$ where μ is the coefficient of Higgs' *quadratic term*, leading the scaled mass $M(H^\Sigma)$ in Eq.(22) as the coefficient of Higgs' *quadratic term* to be as

$$M(H^\Sigma)^2 = \frac{m_H^2}{2} \tag{23}$$

when the pure kinetic term $\partial_\mu H \partial^\mu H$ from Eqs.(6), (8) has no factor $\frac{1}{2}$ that appeared in some classical expressions, the Lagrangian of a *Higgs boson* which is coupling with the *constant Higgs background* to obtain its mass should be

$$\mathcal{L}_{H_{(i)}^{D(i)}} = \partial_\mu H \partial^\mu H - \frac{m_H^2}{2} H^2 = \partial_\mu H_{(i)}^{D(i)} \partial^\mu H_{(i)}^{D(i)} - \frac{m_{H^D}^2}{2} H_{(i)}^{D(i)} H_{(i)}^{D(i)} \tag{24}$$

that result in the mass $m_{H^D} = \sqrt{2}M(H^\Sigma)$, consistent with the condition $m_H = \sqrt{2}\mu$ on quadratic coefficient $M(H^\Sigma)$ or μ. From Eq.(23) we see, there can actually be different *Higgs sections* with different scaled masses, but the detected mass of every *Higgs bosons* can only be one real number,

$$\langle m_{Higgs} \rangle \simeq m_H = \sqrt{2}M(H^\Sigma) = \sqrt{2}\sum_D M(H^D) \tag{25}$$

with each Higgs section H^D under a free kinetic term as

$$\mathcal{L}_H = \partial_\mu \partial^\mu \|\mathbf{C}\|^2 H^D H^D \tag{26}$$

in which ∂ become multipliers when acting on constant global section H^D along $d\tau$, and $\|\mathbf{C}\|$ is the norm in Eq.(19).

We denote $H_{(i)}^{D(i)}$ for every addressed Higgs bosons only when there is a non-zero mass term in its Lagrangian

$$\mathcal{L}_{H_{(i)}^{D(i)}} \sim -\frac{m^2_{H_{(i)}^{D(i)}}}{2} H_{(i)}^{D(i)} H_{(i)}^{D(i)}. \tag{27}$$

As gauged fields derived from generators T_D in the considered Lie algebra, Higgs bosons in this description of Eqs.(20) − (25) from TaLie action have non-zero rest mass thus it can be detected as addressed particles but can not be coupled with other gauge fields to deliver its mass. Constant global sections of Higgs couple with other gauge particles via Eqs.(17) − (19) with a real parameter, the *scaled mass* of each Higgs section, respectively raising the mass of the coupled (or related) particle, which is rescaled Lie-dependently from the scaled mass. In our i-labelled notations, Higgs bosons are addressed as $H_{(i)}^{D(i)}$ while constant Higgs sections are denoted as $H^{D(j)}$, where the label (j) distinguishes the sections from the *relation* of each to different particles $A_{\mu(i)}$, respectively.

Now we head for the introduction of algorithms on other particles' gauge coupling with the global sections of singlet Higgs derived from TaLie action, dependent on Lie-values of the considered generators T_A in gauged Lie algebra correspondingly.

4.2 Gauge couplings with Higgs sections

The gauge coupling of $H^D - A_\mu^B$ is described by Eqs.(17) – (19), in which the Higgs section raises its scaled mass $M(H^D - A_\mu^B)$ from the norms of the kinetic terms in Eq.(18) of this gauge coupling

$$\partial_\mu \partial^\mu \|\mathbf{C}\|^2 - A_\mu^B A^{\mu B} \|T_B\|^2 \longrightarrow M(H^D - A_\mu^B)^2 \tag{28}$$

with $M(H^D - A_\mu^B)^2$ as the real constant coefficient, raising the mass term of A_μ^B

$$-H^D H^D A_\nu^B A^{\nu B} \|T_B\|^2 \longrightarrow -M(H^D - A_\mu^B)^2 A_\nu^B A^{\nu B} \|T_B\|^2. \tag{29}$$

Compared with a free Higgs section with the norm of its kinetic term as $\|\mathbf{C}\|^2$ in Eq.(26) which leads to the scaled mass $M(H^D)$ for the uncoupled Higgs, the mechanism here we derived on the Lie-dependent scaled mass in gauge coupling with particles A_μ^B of the root vector T_B is

$$\frac{\|\mathbf{C}\|^2 - \|T_B\|^2}{M(H^D - A_\mu^B)^2} = \frac{\|\mathbf{C}\|^2}{M(H^D)^2} \tag{30}$$

This theorem concretely defined the *Lie-dependent scaled mass* in each gauge coupling of $H^D - A_\mu^B$ from the uncoupled $M(H^D)$ and norms in the representational space with Killing form of the considered Lie algebra. In consequence the Lagrangian of free, massive field A_μ^B is determined as

$$\begin{aligned} \mathcal{L}_{A_\mu} &= (\partial_\mu A_\nu^B \partial^\mu A^{\nu B} - \sum_D M(H^D - A_\mu^B)^2 A_\nu^B A^{\nu B}) \|T_B\|^2 \\ &= \partial_\mu A_\nu \partial^\mu A^\nu - \sum_D M(H^D - A_\mu^B)^2 A_\nu A^\nu \\ &= \partial_\mu A_\nu \partial^\mu A^\nu - M(H^\Sigma)^2 \sqrt{\frac{\|\mathbf{C}\|^2 - \|T_B\|^2}{\|\mathbf{C}\|^2}} A_\nu A^\nu \end{aligned} \tag{31}$$

and for a free, massive scalar ϕ^E of generator T_E to couple with the section H^D,

$$\begin{aligned} \mathcal{L}_\phi &= (\partial_\mu \phi^E \partial^\mu \phi^E - \sum_D M(H^D - \phi^E)^2 \phi^E \phi^E) \|T_E\|^2 \\ &= \partial_\mu \phi \partial^\mu \phi - \sum_D M(H^D - \phi^E)^2 \phi\phi \\ &= \partial_\mu \phi \partial^\mu \phi - M(H^\Sigma)^2 \sqrt{\frac{\|\mathbf{C}\|^2 - \|T_E\|^2}{\|\mathbf{C}\|^2}} \phi\phi. \end{aligned} \tag{32}$$

From basic principles in SR, as observed $E^2 = \mathbf{p}^2 + m^2$ which leads to K-G Lagrangian for spin-0 scalars and the exactly analogized Lagrangian Eq.(10) – (11) for spin-1 vectors under Lorentz gauge, without an extra factor of $\frac{1}{2}$ in our arrangement, it should appear

$$\mathcal{L}_{\phi(K-G)} = \frac{1}{\not{2}}(\partial_\mu \phi \partial^\mu \phi - m_\phi^2 \phi\phi) \quad \Rightarrow \quad \sum_D M(H^D - \phi^E) = m_{\phi^E} = m_\phi \tag{33}$$

$$\mathcal{L}_{A_\mu(Lorentz\ gauged)} = \frac{1}{2}(\partial_\mu A_\nu \partial^\mu A^\nu - m_A^2 A_\nu A^\nu) \quad \Rightarrow \quad \sum_D M(H^D - A_\mu^B) = m_{A_\mu^B} = m_A \tag{34}$$

where m_ϕ is the rest mass of spin-0 scalar ϕ^E and m_A is the rest mass of spin-1 vector A_μ^B.

Now we determine the masses of gauged particles coupling with singlet Higgs sectors H^D in this mechanism derived from TaLie action.

For $W^\pm$ bosons $\underline{W} = d\underline{x}^\mu W_\mu^W(x) T_W$ with $T_W \in su(2)$,

$$m_{W^\pm} = \sum_D M(H^D - W_\mu^W) = M(H^\Sigma)\sqrt{\frac{\|\mathbf{C}\|^2 - \|T_W\|^2}{\|\mathbf{C}\|^2}} \tag{35}$$

For Z^0 boson $\underline{Z} = d\underline{x}^\mu Z_\mu^Z(x) C_Z$ which lives in *Cartan subalgebra* as an Abelian field, Eq.(18) should be

$$\mathcal{L}_{Z_\mu - H} = (\partial_\mu \partial^\mu \|\mathbf{C}\|^2 - Z_\mu^Z Z^{\mu Z} \|T_Z\|^2) H^D H^D \tag{36}$$

where T_Z is the projected root of generator C_Z in Cartan subalgebra with T_N a non-zero root vector selected alternatively in the root system in considered Lie algebra

$$\|T_Z\| \equiv \frac{(C_Z, T_N)}{\|T_N\|} = 0 \tag{37}$$

that

$$m_{Z^0} = \sum_D M(H^D - Z_\mu^Z) = M(H^\Sigma)\sqrt{\frac{\|\mathbf{C}\|^2 - \|T_Z\|^2}{\|\mathbf{C}\|^2}} = M(H^\Sigma) \tag{38}$$

with Eq.(25),

$$m_{Z^0} = \frac{1}{\sqrt{2}} m_H. \tag{39}$$

Notice: in modern detections of the particle masses by measurements on the decayed final states, an unknown decaying channel to sections as *frame-Higgs* [5–7] $\underline{e}\phi \in sp\underline{i}n(4,4)$ will necessarily result in a lower detected mass than the authentic mass of some specific particles. In Lie Group Cosmology, H and $W^\pm$ are both $SU(2)$ particles but here H is scalarized to constant global sections while the $W^\pm$ boson has only one component, W_μ^W of root vector T_W that consistent with the K-G formed motion from Eq.(11). They both have the channels as

$$H_+ + \bar{\tau}^\wedge \to e_S^\vee \phi_0^* \qquad W^- + \bar{\tau}^\vee \to e_S^\wedge \phi_1^* \tag{40}$$

where the final states, spin-2 bosons $e\phi$, are the deformed de Sitter spacetime itself. But for Z^0, the channels of this sector can only be

$$Z \to e\phi(e\phi)^* \to \gamma\gamma \tag{41}$$

with the final states as photons γ, hence the $Z \to e\phi$ channels will not make the detected mass of a Z^0 boson lower than its authentic mass, i.e.

$$\langle m_{Z^0} \rangle \simeq m_{Z^0}. \tag{42}$$

When considering the undetected frame-Higgs that $\langle m_{W^\pm} \rangle < m_{W^\pm}$ and $\langle m_H \rangle < m_H$, we experimentally use $\langle m_{Z^0} \rangle \simeq 91.188$ GeV [34] to determine the only parameters appearing in our mechanism, the scaled mass of a uncoupled Higgs section

$$\langle m_{Z^0} \rangle \simeq M(H^\Sigma). \tag{43}$$

4.3 Determining the Lie-dependent masses under gauge selection of LGC

Here we compute the Lie-dependent masses of the gauge particles in *Lie Group Cosmology* (LGC) [5,6], the alive theory with a $spin(12,4)$ split of considered gauged Lie group $E_{8(-24)}$.

For Pati- X bosons, $\underline{X} = \underline{X}^Y(x)T_Y \in 3\times(3+\bar{3}) \subset spin(8)$ which would lead to proton decay, the norms of their root vectors are $\|T_Y\|$. Each of the X bosons has the mass

$$m_Y = \sum_D M(H^D - X^Y) = M(H^\Sigma)\sqrt{\frac{\|\mathbf{C}\|^2 - \|T_Y\|^2}{\|\mathbf{C}\|^2}}. \tag{44}$$

The X bosons are from 18# different root vectors of the Lie algebra $spin(8)$, and each pair of conjugate root vectors

$$XX^* \to \gamma\gamma \tag{45}$$

can not be combined in one section since they decay into photon γ in pairs. Thus in the section of $sp\underline{i}n(8)$, the combined mass of X bosons should be

$$m_X = 9\cdot m_Y = 9\cdot M(H^\Sigma)\sqrt{\frac{\|\mathbf{C}\|^2 - \|T_Y\|^2}{\|\mathbf{C}\|^2}}. \tag{46}$$

When considering the Lie-dependent masses raised from singlet Higgs in the mechanism of TaLie action under the selection of gauged Lie groups in Lie Group Cosmology, we have $\|T_W\|^2 = 2$, $\|T_Y\|^2 = 2$ and $\|\mathbf{C}\|^2 = \dfrac{34}{3}$ that result in

$$m_{Z^0} = M(H^\Sigma) \simeq 91.188 \text{ GeV} \tag{47}$$

$$m_{W^\pm} = M(H^\Sigma)\sqrt{\frac{\|\mathbf{C}\|^2 - \|T_W\|^2}{\|\mathbf{C}\|^2}} \simeq 91.188 \text{ GeV}\cdot\frac{\sqrt{238}}{17} = 82.752 \text{ GeV} \tag{48}$$

$$m_X = 9\cdot M(H^\Sigma)\sqrt{\frac{\|\mathbf{C}\|^2 - \|T_Y\|^2}{\|\mathbf{C}\|^2}} \simeq 9\cdot 91.188 \text{ GeV}\cdot\frac{\sqrt{238}}{17} = 744.766 \text{ GeV} \tag{49}$$

$$m_H = \sqrt{2}\cdot M(H^\Sigma) \simeq \sqrt{2}\cdot 91.188 \text{ GeV} = 128.960 \text{ GeV} \tag{50}$$

where $m_{W^\pm} = \langle m_{W^\pm}\rangle + m_{W\to e\phi}$ and $m_H = \langle m_H\rangle + m_{H\to e\phi}$, the combined mass m_X [33] be detected via Eq.(45) the annihilation $XX^* \to \gamma\gamma$ as the 750 GeV diphoton excess [34,35] at CERN.

For Dirac fermions and Majorana fermions, their components e.g. $\psi^\wedge_{uL} \in 16S_+$ satisfying K-G motion are coupled with constant Higgs sections in a non-trivial way, which leads to hard task on determining the masses of diverse hadron, an open question, in need of more attention.

That a gauged particle can only couple with at most one section $H^{D(j)}$ of H^D is because, for $T_D \in su(2) \subset 8_v \times 8_V$ or $T_D \in su(2) \subset 16S_+$ in Lie Group Cosmology, their anti-symmetric wave functions are restricted by *Pauli exclusion principle*.

5 Conclusion

In this paper we studied the Lie-dependent rest masses of test particles considered in gauged Lie groups, raised by the mass terms derived from their gauge couplings with the constant global sections of singlet Higgs via under the algorithm Eq.(30) on kinetic terms determining the Lie-dependent $M(H^D - A_\mu^B)$ of gauge coupling $H^D - A_\mu^B$ presented in TaLie action with the only parameters, scaled mass $M(H^D)$ of each Higgs section, introduced in this mechanism. A brief summary of the main results is given in Section 1; herein we conclude with a few additional remarks.

The scalarization of mass on constant Higgs sections result in H^D as the real constant multipliers in Eqs.(29), (31) and (32) on vectors A_μ^B or scalars ϕ^E, leaving the norms of their mass terms as $\|T_B\|^2$ or $\|T_E\|^2$ but not an inner product derived from Killing form, which indicates that the mechanism on gauge couplings with singlet Higgs we discussed here is also suitable for gauged fields or connection 1-forms in classical Y-M actions, that are only dependent on the norms of root vectors for the fields in considered Lie algebras.

There can be more than one root vectors T_D in the gauged Lie algebra to produce different Higgs bosons $H_{(i)}^{D(i)}$, but every Higgs boson as spin-0 scalars can only have one real constant rest mass m_H, i.e. Eq.(20) $-$ (25), since they couple with the constant Higgs background for $M(H^\Sigma)$, and the factor $\sqrt{2}$ in $m_H = \sqrt{2} \cdot M(H^\Sigma)$ is from the extremal value in Higgs potential Eq.(15) of spontaneous symmetry breaking, which leads to the restriction on $M(H^\Sigma)$ as the quadratic term of Higgs' Lagrangian $\mathcal{L}_H$.

As the forward-looking direction, we find the mechanism applied under the gauge selection of Lie Group Cosmology could give the sensible Lie-dependent rest masses of $W^\pm$, Z^0, H and X bosons with scaled mass $M(H^\Sigma) = \sum_D M(H^D)$ as the only real parameters introduced. There are $(2 + 3 \times 2)\#$ different Higgs sections H^D in the gauged Lie group $E_{8(-24)}$ of LGC, and we suggest the masses of fermions are dependent on the inner product $n_{B(i)C(j)}$ derived from Killing form, relevant to their vector components in the infinite momentum frame, which need to be investigated more in the future.

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