Primes obtained concatenating a Poulet number \( P \) with \( (s-1)/n \) where \( s \) digits sum of \( P \) and \( n \) is 2, 3 or 6

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I conjecture that there exist an infinity of Poulet numbers \( P \) such that concatenating \( P \) to the left with the number \( (s(P) - 1)/2 \), where \( s \) is the sum of digits of \( P \), is obtained a prime; also I make the same conjecture for \( (s(P) - 1)/3 \) respectively for \( (s(P) - 1)/6 \).

Conjecture 1:

There exist an infinity of Poulet numbers \( P \) such that concatenating \( P \) to the left with the number \( (s(P) - 1)/2 \), where \( s \) is the sum of digits of \( P \), is obtained a prime.

The sequence of primes obtained concatenating a Poulet number \( P \) to the left with \( (s(P) - 1)/2 \):

: 91387 obtained from \( P = 1387 \) with \( s = 19 \);
: 62047 obtained from \( P = 2047 \) with \( s = 13 \);
: 66601 obtained from \( P = 6601 \) with \( s = 13 \);
: 98911 obtained from \( P = 8911 \) with \( s = 19 \);
: 914491 obtained from \( P = 14491 \) with \( s = 19 \);
: 1219951 obtained from \( P = 19951 \) with \( s = 25 \);
: 1549981 obtained from \( P = 49981 \) with \( s = 31 \);
: 12271951 obtained from \( P = 271951 \) with \( s = 25 \);
: 9314821 obtained from \( P = 314821 \) with \( s = 19 \).

Conjecture 2:

There exist an infinity of Poulet numbers \( P \) such that concatenating \( P \) to the left with the number \( (s(P) - 1)/3 \), where \( s \) is the sum of digits of \( P \), is obtained a prime.

The sequence of primes obtained concatenating a Poulet number \( P \) to the left with \( (s(P) - 1)/3 \):

: 61729 obtained from \( P = 1729 \) with \( s = 19 \);
: 42821 obtained from \( P = 2821 \) with \( s = 13 \);
: 63277 obtained from \( P = 3277 \) with \( s = 19 \);
: 46601 obtained from \( P = 6601 \) with \( s = 13 \);
Conjecture 3:

There exist an infinity of Poulet numbers $P$ such that concatenating $P$ to the left with the number $(s(P) - 1)/6$, where $s$ is the sum of digits of $P$, is obtained a prime.

The sequence of primes obtained concatenating a Poulet number $P$ to the left with $(s(P) - 1)/6$:

- $31387$ obtained from $P = 1387$ with $s = 19$;
- $31729$ obtained from $P = 1729$ with $s = 19$;
- $314491$ obtained from $P = 14491$ with $s = 19$;
- $130121$ obtained from $P = 30121$ with $s = 7$;
- $331609$ obtained from $P = 31609$ with $s = 19$;
- $352633$ obtained from $P = 52633$ with $s = 19$;
- $357421$ obtained from $P = 57421$ with $s = 19$;
- $465077$ obtained from $P = 65077$ with $s = 25$;
- $3115921$ obtained from $P = 115921$ with $s = 19$;
- $3196021$ obtained from $P = 196021$ with $s = 19$;
- $3228241$ obtained from $P = 228241$ with $s = 19$;
- $6275887$ obtained from $P = 275887$ with $s = 37$;
- $3334153$ obtained from $P = 334153$ with $s = 19$.

Observation:

Note that in all the 31 cases considered above (when a prime was obtained through the defined concatenation) the digits sum of the Poulet number was a prime (7, 13, 19, 31, 37 or a square of a prime, 25). This fact is not a characteristic of Poulet numbers, many of them having as a sum of digits an even or odd composite number.