Primes obtained concatenating to the left a prime having an odd prime digit sum $s$ with a divisor of $s-1$

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Abstract. In a previous paper, “Primes obtained concatenating a Poulet number $P$ with $(s - 1)/n$ where $s$ digits sum of $P$ and $n$ is 2, 3 or 6”, I noticed that in almost all the cases that I considered if a prime was obtained through this concatenation than the digits sum of $P$ was a prime. That gave me the idea for this paper where I observe that for many primes $p$ having an odd prime digit sum $s$ there exist a prime obtained concatenating $p$ to the left with a divisor of $s - 1$ (including 1 and $s - 1$).

Observation:

For many primes $p$ having an odd prime digit sum $s$ there exist a prime obtained concatenating $p$ to the left with a divisor $d$ of $s - 1$ (including 1 and $s - 1$).

Note: see the sequence A046704 in OEIS for the primes having a prime digit sum.

Verifying the observation:
(true for 15 from the first 16 primes $\neq 5$ with an odd prime digit sum)

: $13$ is obtained from $P = 3$ with $s = 3$ for $d = 1$, also $23$ is obtained from $P = 3$ for $d = 2$;

: $17$ is obtained from $P = 7$ with $s = 7$ for $d = 1$, also $37$ is obtained from $P = 7$ for $d = 3$, also $67$ is obtained from $P = 7$ for $d = 6$;

: $223$ is obtained from $P = 23$ with $s = 5$ for $d = 2$;

: $229$ is obtained from $P = 29$ with $s = 11$ for $d = 2$;

: $241$ is obtained from $P = 41$ with $s = 5$ for $d = 2$;

: $643$ is obtained from $P = 43$ with $s = 7$ for $d = 6$;
547 is obtained from \( P = 47 \) with \( s = 11 \) for \( d = 5 \);

661 is obtained from \( P = 61 \) with \( s = 7 \) for \( d = 6 \);

167 is obtained from \( P = 67 \) with \( s = 13 \) for \( d = 1 \),
also 367 is obtained from \( P = 67 \) for \( d = 3 \), also 467 is obtained from \( P = 67 \) for \( d = 12 \);

for \( p = 89 \) with \( s = 17 \) is obtained no prime but a square of prime, 289 for \( d = 2 \); indeed the observation could include those as well: from the cases above 529 for \( p = 29 \) and \( d = 5 \); 361 for \( p = 61 \) and \( d = 3 \);

2113 is obtained from \( P = 113 \) with \( s = 5 \) for \( d = 2 \);

2131 is obtained from \( P = 131 \) with \( s = 5 \) for \( d = 2 \);

2137 is obtained from \( P = 137 \) with \( s = 11 \) for \( d = 2 \);

4139 is obtained from \( P = 139 \) with \( s = 13 \) for \( d = 4 \);

1151 is obtained from \( P = 151 \) with \( s = 7 \) for \( d = 1 \),
also 6151 is obtained from \( P = 151 \) for \( d = 6 \);

4157 is obtained from \( P = 157 \) with \( s = 13 \) for \( d = 4 \),
also 12157 is obtained from \( P = 157 \) for \( d = 12 \).

**Verifying the observation:**
(true for 4 from the first 6 primes having 5 digits with an odd prime digit sum)

210037 is obtained from \( P = 10037 \) with \( s = 11 \) for \( d = 2 \);

110039 is obtained from \( P = 10039 \) with \( s = 13 \) for \( d = 1 \), also 1210039 is obtained for \( d = 12 \);

810079 is obtained from \( P = 10079 \) with \( s = 17 \) for \( d = 8 \);

for \( p = 10091 \) and \( p = 10093 \) is not obtained a prime;

910099 is obtained from \( P = 10099 \) with \( s = 19 \) for \( d = 9 \).