

Primes obtained concatenating to the left a prime having an odd prime digit sum s with a multiple of $s-1$

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Abstract. In a previous paper, "Primes obtained concatenating to the left a prime having an odd prime digit sum s with a divisor of $s - 1$ ", I observed that for many primes p having an odd prime digit sum s there exist a prime obtained concatenating p to the left with a divisor of $s - 1$. In this paper I conjecture that for any prime p , $p \neq 5$, having an odd prime digit sum s there exist an infinity of primes obtained concatenating to the left p with multiples of $s - 1$. Yet I conjecture that there exist at least a prime obtained concatenating $n*(s - 1)$ with p such that $n < \sqrt{s}$.

Conjecture 1:

For any prime p , $p \neq 5$, having an odd prime digit sum s there exist an infinity of primes obtained concatenating to the left p with multiples of $s - 1$.

Note: see the sequence A046704 in OEIS for the primes having a prime digit sum.

Examples:

Such primes obtained for $p = 29$ ($s - 1 = 10$):

: 2029, 6029, 9029, 14029, 17020, 20029, 23029, 24029
(...)

Such primes obtained for $p = 89$ ($s - 1 = 16$):

: 4889, 8089, 9689, 12889, 14489, 19289, 32089, 46489
(...)

Conjecture 2:

For any prime p , $p \neq 5$, having an odd prime digit sum s there exist at least a prime obtained concatenating to the left p with the number $n*(s - 1)$ such that $n < \sqrt{p}$.

The sequence of the least primes obtained concatenating each prime p , $p \neq 5$, p having an odd prime digit sum s , to the left with numbers $n \cdot (s - 1)$:

: 23, 67, 823, 2029, 2441, 643, 6047, 661, 2467, 2083,
4889, 12113, 36131, 30137, 60139, 6151, 12157, 20173,
48179, 80191, 48179, 80191, 48193, 48197, 18199,
18223, 50227, 24229, 12241, 50263 (...)

obtained for the following values of $[p, s, n]$:

[3, 3, 1], [7, 7, 1], [23, 5, 2], [29, 11, 2], [41, 5, 6],
[43, 7, 1], [47, 11, 6], [61, 7, 1], [67, 13, 2], [83, 11,
2], [89, 17, 3], [113, 5, 3], [131, 5, 9], [137, 11, 3],
[139, 13, 5], [151, 7, 1], [157, 13, 1], [173, 11, 2],
[179, 17, 3], [191, 11, 8], [193, 13, 4], [197, 17, 3],
[199, 19, 1], [223, 7, 3], [227, 11, 5], [229, 13, 2],
[241, 7, 2], [263, 11, 5].

The conjecture was verified for the first 30 primes p with the defined property. The values of n closest by the values of $\text{sqr } p$ were:

$n = 1$ for $\text{sqr } 3 = 1.732$;
 $n = 6$ for $\text{sqr } 41 = 6.403$;
 $n = 9$ for $\text{sqr } 131 = 11.445$.

The least such primes obtained for 5 consecutive primes p with 6 digits, i.e. 287491, 287501, 287671, 287789, 287813:

: 120287491 obtained for $n = 4$ ($s = 30$);
: 110287501 obtained for $n = 5$ ($s = 22$);
: 90287671 obtained for $n = 3$ ($s = 30$);
: 440287789 obtained for $n = 11$ ($s = 40$);
: 56287813 obtained for $n = 2$ ($s = 28$).

Note for what low values of n were obtained the primes above!