

**Primes obtained concatenating to the left a prime having an odd prime digit sum  $s$  with a multiple of  $s-1$**

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**Abstract.** In a previous paper, "Primes obtained concatenating to the left a prime having an odd prime digit sum  $s$  with a divisor of  $s - 1$ ", I observed that for many primes  $p$  having an odd prime digit sum  $s$  there exist a prime obtained concatenating  $p$  to the left with a divisor of  $s - 1$ . In this paper I conjecture that for any prime  $p$ ,  $p \neq 5$ , having an odd prime digit sum  $s$  there exist an infinity of primes obtained concatenating to the left  $p$  with multiples of  $s - 1$ . Yet I conjecture that there exist at least a prime obtained concatenating  $n*(s - 1)$  with  $p$  such that  $n < \sqrt{s}$ .

**Conjecture 1:**

For any prime  $p$ ,  $p \neq 5$ , having an odd prime digit sum  $s$  there exist an infinity of primes obtained concatenating to the left  $p$  with multiples of  $s - 1$ .

Note: see the sequence A046704 in OEIS for the primes having a prime digit sum.

**Examples:**

Such primes obtained for  $p = 29$  ( $s - 1 = 10$ ):

: 2029, 6029, 9029, 14029, 17020, 20029, 23029, 24029  
(...)

Such primes obtained for  $p = 89$  ( $s - 1 = 16$ ):

: 4889, 8089, 9689, 12889, 14489, 19289, 32089, 46489  
(...)

**Conjecture 2:**

For any prime  $p$ ,  $p \neq 5$ , having an odd prime digit sum  $s$  there exist at least a prime obtained concatenating to the left  $p$  with the number  $n*(s - 1)$  such that  $n < \sqrt{p}$ .

The sequence of the least primes obtained concatenating each prime  $p$ ,  $p \neq 5$ ,  $p$  having an odd prime digit sum  $s$ , to the left with numbers  $n \cdot (s - 1)$ :

: 23, 67, 823, 2029, 2441, 643, 6047, 661, 2467, 2083,  
4889, 12113, 36131, 30137, 60139, 6151, 12157, 20173,  
48179, 80191, 48179, 80191, 48193, 48197, 18199,  
18223, 50227, 24229, 12241, 50263 (...)

obtained for the following values of  $[p, s, n]$ :

[3, 3, 1], [7, 7, 1], [23, 5, 2], [29, 11, 2], [41, 5, 6],  
[43, 7, 1], [47, 11, 6], [61, 7, 1], [67, 13, 2], [83, 11,  
2], [89, 17, 3], [113, 5, 3], [131, 5, 9], [137, 11, 3],  
[139, 13, 5], [151, 7, 1], [157, 13, 1], [173, 11, 2],  
[179, 17, 3], [191, 11, 8], [193, 13, 4], [197, 17, 3],  
[199, 19, 1], [223, 7, 3], [227, 11, 5], [229, 13, 2],  
[241, 7, 2], [263, 11, 5].

The conjecture was verified for the first 30 primes  $p$  with the defined property. The values of  $n$  closest by the values of  $\text{sqr } p$  were:

$n = 1$  for  $\text{sqr } 3 = 1.732$ ;  
 $n = 6$  for  $\text{sqr } 41 = 6.403$ ;  
 $n = 9$  for  $\text{sqr } 131 = 11.445$ .

The least such primes obtained for 5 consecutive primes  $p$  with 6 digits, i.e. 287491, 287501, 287671, 287789, 287813:

: 120287491 obtained for  $n = 4$  ( $s = 30$ );  
: 110287501 obtained for  $n = 5$  ( $s = 22$ );  
: 90287671 obtained for  $n = 3$  ( $s = 30$ );  
: 440287789 obtained for  $n = 11$  ( $s = 40$ );  
: 56287813 obtained for  $n = 2$  ( $s = 28$ ).

Note for what low values of  $n$  were obtained the primes above!