Question 1760 : Euler-Mascheroni Constant

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Abstract

This note presents some double integrals for Euler-Mascheroni constant and related fractals.

Introduction

$$\gamma = \lim_{k\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{k}-\ln k\right)=0.577215... \qquad (1)$$

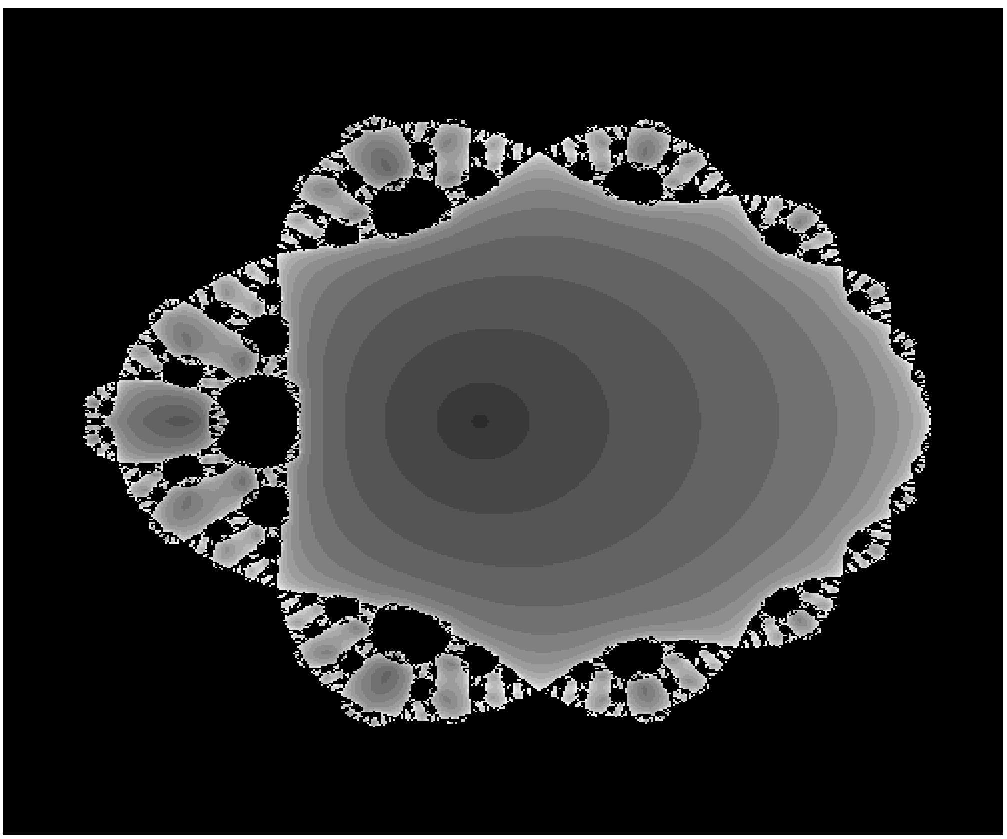

Figure 1.

Double Integrals

$$\gamma = \int_0^1 \int_0^1 \frac{\ln\left(-\ln\left(xy\right)\right)}{\ln\left(xy\right)} dxdy \tag{2}$$

$$\gamma = -\int_0^1 \int_0^1 \frac{\ln\left(x+y\right)}{x+y} e^{-x-y} dxdy \tag{3}$$

$$\gamma = 1 + \int_0^1 \int_0^1 \frac{\ln\left(-e\ln\left(xy\right)\right)}{\ln\left(xy\right)} dxdy \tag{4}$$

$$\gamma = \frac{1}{2} + \int_0^1 \int_0^1 \frac{\ln\left(-\ln\left(\left(xy\right)^{e^{xy}}\right)\right)}{\ln\left(xy\right)} dxdy \tag{5}$$

$$\gamma = \frac{3}{5} + 3\int_0^1 \int_0^1 \frac{\left(xy\right)^2 \ln\left(-\ln\left(\left(xy\right)^{3e^{\left(xy\right)^2}}\right)\right)}{\ln\left(xy\right)} dxdy \tag{6}$$

$$\gamma = \frac{4}{7} + 4\int_0^1 \int_0^1 \frac{\left(xy\right)^3 \ln\left(-\ln\left(\left(xy\right)^{4e^{\left(xy\right)^3}}\right)\right)}{\ln\left(xy\right)} dxdy \tag{7}$$

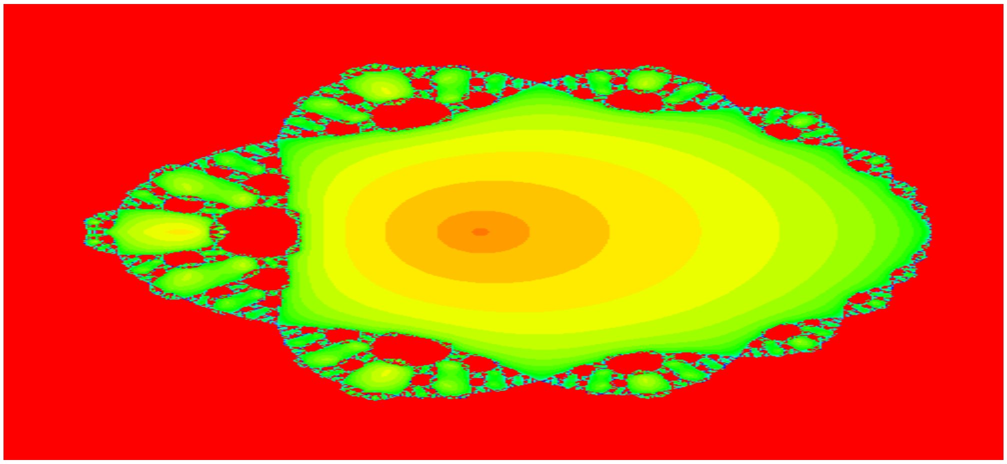

Figure 2.

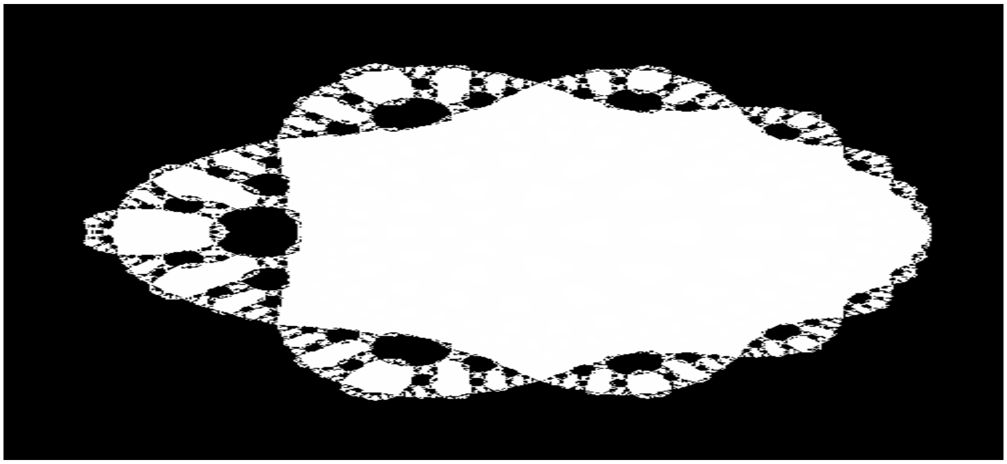

Figure 3.

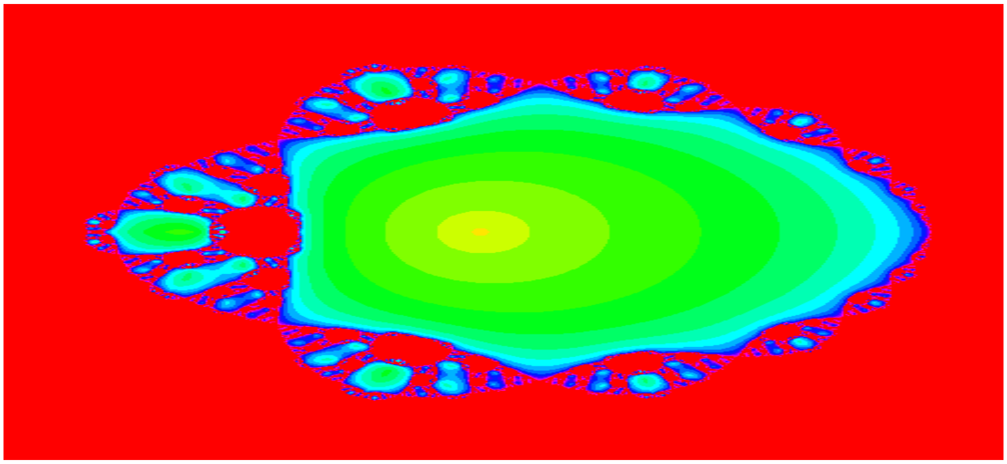

Figure 4.

Remark: For fractals we use the function: $f(x,y) = \dfrac{\ln(-\ln(xy))}{\ln(xy)}$.

References

1. Guillera, J. and Sondow, J. : Double integrals and infinite products for some classical constants via analytic continuations of lerch's transcendent. arXiv:math/0506319v3[math.NT] 5 Aug 2006.