Quantum Political Economics

---the characterization of productive forces and production relations

Erman ZENG[1]

Abstract: The mathematical characterization of “the Productive Forces” of a macro economic system is based upon the analogy between political economy and Newtonian mechanics, which is expressed as the product of the growth rate of the profit rate (p) and the surplus value (M), showing several quantum qualities like a photon quanta. The one-dimensional linear harmonic oscillator model can correlate the angular frequency with the change rate of the rate of profit thus with the economic growth rate, resulting the quantum-like interpretation of various business cycles. The matrix operator analysis of the Leontief’s input-output table, similar to the matrix mechanics of quantum physics, gives the Schrodinger function like value-price transformation eigen function ($\hat{g}\psi = \beta\psi$), with the reduced organic composite of capital β as the eigenvalue of the price wave function, namely the relations of production, leading to the "two Cambridge controversy" resolved. The statistic physical entropy increase theory combined with the Marx labour value function leads to the quantitative formulation of the relations of production.

Key words: quantum mechanics, harmonic oscillator, econophysics, labor theory of value, matrix operator, eigen-vector, eigen-function, eigenvalue, general equilibrium, wave function, transformation problem, input-output，Two Cambridge Controversy, Needham question, entropy theory, AK model, spontaneous order

JEL：A12 E11 E25 C67 C68 D24 D46 D57 D58 E25 O47

1、Classic mechanics model

Similar to the fractal space-time theory[2], the natural logarithm of the price of a commodity (lnP) is defined as the "displacement" (q) of the classical Newtonian physics, the speed (v=dq/dt) corresponding to the rate of the profit (p’):

$$p' = \frac{d\ln P}{dt} = \frac{dP}{Pdt}$$

the momentum (mv) as the surplus value:M =p’Cv，Cv=C+V, C=nK:constant capital，n:turnover rate of capital K; V=wL:variable capital，w：wage rate，L：labour; thus the “productive forces[3]”(Φ) can be expressed as：

---

[1] Amoy Institute of Technovation (XiaMen Productivity Promotion Center),1300 JiMei Blvd., XiaMen, FuJian 361024, PRChina, Email: zengerman@163.com

[2] Nottale, L. (2011)Scale Relativity and Fractal Space-Time: a New Approach to Unifying Relativity and Quantum Mechanics, Imperial College Press.

[3] 曾尔曼，《厦门科技》4（2016）50-56.

$$\Phi = Cv\frac{dp'}{dt} = p'Cv\frac{dp'}{p'dt} = Mp = FY = (1-\alpha)\dot{g}*Y = (1-\alpha)yL\dot{g}* = mY$$

$$Cv = C + V = (n + \frac{V}{K})K = (n+1)K, when: \mathrm{n} = 0,\ \mathrm{C} = \mathrm{nK} = 0,\ K = V = Cv, \mathrm{M}_0 = K\mathrm{p}',$$

$$\Phi = (n+1)\mathrm{pM}_0 = (n+1)\Phi_0;$$

$$M = C\frac{p'}{\beta} = nK\frac{p'}{\beta} = n\frac{\mathrm{M}_0}{\beta} \Rightarrow \Phi = \frac{n}{\beta}\Phi_0, \beta \equiv \frac{C}{Cv} = \frac{g}{g+1} = \frac{n}{n + Lw/K}$$

Formula $\Phi=\Phi_0(n+1)$ reflecting the photon-like quantum character, the productivity forces(Φ) includes all the elements described by Marx[4] such as the level of division of labor ($1-\alpha$), labor productivity y, number of workers L showing quantum character too, the organic composition of capital g (OCC, *: general equilibrium), the productivity coefficient F, and the Solow residual m (technical progress rate), under the condition of the Cobb-Douglas production function[5]: m=F, other analogies are as follows (Table 1)：

Table 1.	Classic Mechanics	Political Economics[6]	
mass	m	Cv=C+V	Capital input
displacement	q	lnP	P:price
velocity	v=dq/dt	p'=dlnP/dt	Profit rate
acceleration	a=dv/dt	dp'/dt	Growing rate of p’
momentum	p=mv	M=p’Cv	Surplus value
potential	T=p^2/2m	Mp’/2=Ψ	potential
force	F=ma	Φ=Mp	Productive forces

From the definition of annual profit rate：

$$p' = \frac{d\ln P}{dt} = p'_0 e^{pt} \Rightarrow$$

$$\ln P = \frac{p'_0}{p}(e^{pt}-1) = \frac{p'_0}{p}\sum_i(\frac{p^i}{i!}t^i - 1) = \frac{p'_0}{p}(pt + \frac{p^2}{2}t^2 + \ldots + \frac{p^i}{i!}t^i) \approx p'_0 t(1+\frac{p}{2}t)$$

the prices intend to increase, especially when p is positive; and the quantum character is displayed obviously。

The least action principle can be used to determine the economic equilibrium g*：

4 Karl Marx, The German Ideology, 1845

5 Cobb C.W., Douglas P.H. “A Theory of Production”, Amer. Econ. Rev. 1928,8(1), Spp1.139-165.

6 曾尔曼，（2016）《马克思生产力经济学导引》，厦门大学出版社. (Erman ZENG,“Introducing Marxian Economics of Productive Forces”,Xiamen Univ. Press,2016)

$$F=-\frac{\partial V}{\partial q}=ma=m\dot{v}\Leftrightarrow\Phi=-\frac{\partial\Psi}{\partial(\ln P)}=Mp$$

$$\Phi=Cv\frac{dp'}{dt}=Cv\frac{p'dp'}{p'dt}=M\frac{dp'}{p'dt}\Rightarrow\Psi=-Mp'=-Cv(p')^2$$

$$L=T-V=\frac{p^2}{2v}-V=\frac{M^2}{2Cv}-Mp'=-\frac{Mp'}{2}=-\frac{Cv}{2}p'^2$$

$$\frac{\partial L}{\partial\ln P}=-\frac{Cv}{2}\frac{2p'\partial p'}{p'\partial t}=-Cv\frac{dp'}{dt}=-Mp;\ \ \Psi=-Mp',\frac{d\ln P}{dt}=\frac{dP}{Pdt}=p'$$

$$\frac{d}{dt}(\frac{\partial L}{\partial p'})=\frac{d}{dt}[\frac{\partial}{\partial p'}(-\frac{Cv}{2}p'^2)]=-\frac{dM}{dt}=-Mp$$

$$\Rightarrow p=\dot{M}=p+\beta\dot{g}+\dot{V}=\dot{Y}+\dot{\alpha}-\dot{\beta}\approx\dot{C}+\dot{\alpha}-\dot{\beta}$$

$$\Rightarrow\dot{C}v=0=\beta\dot{g}+\dot{V}=\dot{C}-(1-\beta)\dot{g}=\dot{C}-\dot{\beta}$$

$$\Rightarrow dCv=0;\dot{C}=\dot{\beta},\dot{M}=p\approx\dot{\alpha}$$

$$\Rightarrow\dot{g}^*=-\dot{V}/\beta$$

According to the Lagrange equation, the "quality"of a system thus the capital input and output (Cv=C+V) is conserved. The least action principle reveals that the growth rate of constant capital C equals to the growth rate of the reduced organic composition of capital β, the growth rate of surplus value M is equal to the growth rate of profit rate P, thus the change rate of the capital output coefficient α of the CD production function .

In addition, according to the Noether theorem, it is shown that under the condition of no external force, the conservation of energy is the conservation of the value of the commodity as well as the profit rate, unless the division of labor is changed:

$$H=p\dot{q}-L=Mp'-(-Mp'/2)=3Mp'/2=3p'^2\,Cv/2$$

$$\frac{dH}{dt}=0\Leftrightarrow\frac{dp'}{dt}=0=\frac{d}{dt}(\frac{M}{Cv})=\frac{d}{dt}(\frac{Q}{Cv})$$

$$\Leftrightarrow CvdQ-QdCv=0;\because\dot{C}v=0\Leftrightarrow dCv=0\therefore dQ=0$$

$$C+Y+Q\Rightarrow 1+\frac{p'}{\alpha}=\frac{1+p'}{\beta}=\frac{P'}{\beta}\Rightarrow p'=\frac{1-\beta}{\beta-\alpha}\alpha\Leftrightarrow$$

$$p'=\frac{(1-\beta)[1-(1-\alpha)]}{1-\alpha-(1-\beta)}=\frac{d_R(1-d_L)}{d_L-d_R},$$

$$d_R:=1-\beta,d_L:=1-\alpha$$

d_L：division of labor，d_R：roundabout production.

2、Harmonic Oscillator Model

One dimensional linear harmonic oscillator[7] model can be used to approximate an equilibrium state of a system, the potential energy and kinetic energy are equal：

[7] 曾谨严，（2016,第VI版）《量子力学》，科学出版社.

$$f = m_0 a = m_0 \ddot{x} = -kx, k = m_0 \omega^2 = Cv\omega^2$$

$$E = \frac{1}{2}kX^2 = \frac{1}{2}m_0(\omega X)^2, H = \frac{1}{2}Mp' = \frac{Cv}{2}p'^2 \Rightarrow$$

$$p' \Leftrightarrow \omega X = \omega \ln P, Cv \Leftrightarrow m_0, \ln P \Leftrightarrow X, \omega = \frac{p'}{\ln P} = \sqrt{\frac{k}{Cv}}$$

$$\ln P = W \sin \omega t, p' = W\omega \cos \omega t$$

$$\Rightarrow \mathrm{tg}\omega t = 1, \omega t = \frac{\pi}{4} + N\pi;$$

$$\Phi = Mp = k \ln P \Rightarrow p = \frac{k \ln P}{M} = \frac{\omega^2 \ln P}{p'} = \omega;$$

$$p > 0 \Leftrightarrow \frac{d \ln(\ln P)}{dt} > 0,\ \text{or:}\ P > 1(\text{when}: p' > 0);$$

$$\omega = \dot{M} = \dot{Y} + \dot{\alpha} - \dot{\beta} \cong \dot{Y};$$

$$T = \frac{1}{\omega} = \frac{1}{p} \cong \frac{1}{\dot{Y}},\ k = \omega^2 Cv = p^2 Cv$$

$$\omega = \sqrt{\frac{k}{Cv}} = \sqrt{\frac{k/V}{g+1}} = \sqrt{\frac{k}{V}(1-\beta)} = \sqrt{\frac{k\beta}{C}} = \sqrt{\frac{kp'}{M}} = \frac{\pi(1+4N)}{4T};$$

$$\ln P = e^{\omega t} \ln P_0 = \frac{Mp}{k} = \frac{M\omega}{k} = \frac{Cp'}{\beta k}\omega = n\frac{Kp'}{\beta k}\omega = \sqrt{\frac{Mp'}{k}} = \frac{M}{\sqrt{kCv}}$$

$$p = \frac{d \ln p'}{dt} = \frac{1-\alpha}{\alpha}\beta \dot{g}^* = \omega \Rightarrow \dot{g}^* = \frac{\alpha\omega}{(1-\alpha)\beta}, \ln p' = e^{\omega t} \ln p'_0;$$

$$m = F = \frac{\alpha}{\beta}p = \frac{\alpha}{\beta}\omega;$$

$$\beta = \frac{C}{Cv} = \frac{nK}{nK + wL} = \frac{n}{n + Lw/K}$$

$$1 - \beta = \frac{p'}{m'} = d_R, \dot{m}' = p + \beta\dot{g} = \omega + \beta\dot{g} = \omega - \dot{d}_R, \dot{d}_R = \omega - \dot{m}';$$

$$\dot{d}_L = \dot{d}_R + (\alpha - \beta)(\dot{g}^* - \dot{g})$$

$$\Phi = Mp = M\omega = C\frac{p'}{\beta}\omega = nK\frac{p'}{\beta}\omega = \frac{n}{\beta}M_0\omega = \frac{n}{1-d_R}\Phi_0;$$

$$\dot{\Phi} = \dot{M} = p = \dot{n} - \dot{\beta} = \dot{n} - \dot{C} = \dot{n} + \frac{1-\beta}{\beta}\dot{d}_R = \dot{n} + \frac{d_R}{1-d_R}\dot{d}_R$$

N is a natural number, therefore, the productive forces Φ, "price coefficient"lnP,the rate of technological progress m, various types of capitals (C, V, M) and the organic composition of capital g, the growth rate of profit rate (p')p , the labor division coefficient d_L and the production roundaboutness d_R, all the parameters of the economic system show the quantum characters . Similar to the quantum mechanics of the photon quantized energy E=hν and the one-dimensional harmonic oscillator energy E_n=(n+1/2)hϖ/2π, the economic system directly reflected its quantum-like behavior in the analytic expression of the "productive forces" Φ=(n+1)M_0ω； every intermediate inputs (C) and output surplus value (M) are right the contribution to the productive forces' growth, not only in a micro commodity, but also a meso industry, and even the whole macro economy.

The development of productive forcesΦ mainly resulted from the increased rates of both the capital turnover (n) and the production roundaboutness d_R , which can reduce the capital investment (C), avoid the falling rate of profit; in other words, the acceleration of the capital turnover can offset the insufficient capital investment and lower profit rate.

Table 2 shows the calculation of the economic business cycles based on various growth rates of GDP ($\approx$p)：

Table2	dlnY/dt=	p	=	ω	=	π	/	4	T	*	(1+4N)
t=T=1	3	7	10	15	20	25	45	60	80	110	
0.785	**0.262**	**0.112**	**0.079**	0.052	0.039	0.031	0.017	0.013	0.010	0.007	1
3.927	1.309	0.561	0.393	**0.262**	0.196	**0.157**	**0.087**	0.065	0.049	0.036	5
7.069	2.356	1.010	1.414	0.471	0.353	**0.283**	0.157	**0.118**	**0.088**	0.064	9
10.21	3.403	1.459	2.042	0.681	0.511	0.408	**0.227**	0.170	**0.128**	**0.093**	13

After a comprehensive analysis of different economic cycles based upon his innovation theory, in 1936 Schumpeter[8] proposed that each long cycle (the Kondratiev wave or long technological cycle of 45 to 60 years) includes 6 medium cycles (the Juglar fixed-investment cycle of 7 to 11 years and the Kuznets infrastructural investment cycle of 15 to 25 years), each cycle includes three short cycle (the Kitchin inventory cycle of 3 to 5 years).

Since the mid eighteenth Century, the industrial revolution has occurred three times. The first created the steam machine era (1760-1840, 80years), marking the transition from agricultural civilization to industrial civilization; the second industrial revolution moved the human being into the electric chemical age (1840-1950, 110years); after the World War II, the third industrial revolution began and created an electronic-information age (1950-2011, 61years). Since 2011, the era of artificial intelligence stared . Each economic cycle with different length of the year can be explained properly.

3. Economic Operator--Matrices Mechanics

According to the function system of Marx's theory of value[9], all the input-output relationships are as follows：

[8] Schumpeter, J. A. (1954). History of Economic Analysis. London: George Allen & Unwin.

[9] 曾尔曼，（2016）《马克思生产力经济学导引》，p122, 厦门大学出版社. (Erman ZENG,"Introducing Marxian Economics of Productive Forces", Xiamen Univ. Press,2016)

$$LaborValue\,Theory:\ Q = C + V + M = C + Y = B_0 e^{ft} C^{\beta} V^{1-\beta};$$

$$CapitalOrganicComposite:\ g \equiv \frac{C}{V} = \frac{\beta}{1-\beta}, reducedOCC:\ \beta = \frac{g}{1+g} = \frac{C}{C+V};$$

$$Marx\,Production Function:\ Y = M + V = a_0 e^{Ft} C^{\alpha} V^{1-\alpha} = A_0 e^{mt} K^{\alpha} L^{1-\alpha},$$

$$"Okishio"\,Theorem: m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta} p \cong \frac{\alpha}{\beta} p = F;$$

$$Surplus\,Value Function:\ M = b_0 e^{pt} C^{\beta} V^{1-\beta}, \gamma = \frac{Q}{M} = \frac{P'}{p'}, \frac{Y}{M} = \frac{\beta}{\alpha}$$

$$\frac{C}{Q} = \frac{C}{C+V+M} = \frac{C}{(C+V)(1+p')} = \frac{\beta}{1+p'} = \frac{\beta}{P'},$$

$$\frac{C}{Y} = \frac{C}{Q}\frac{Q}{M}\frac{M}{Y} = \frac{\beta}{P'}\gamma\frac{\alpha}{\beta} = \frac{\alpha}{p'},$$

$$\frac{C}{M} = \frac{C}{Q}\frac{Q}{M} = \frac{\beta}{P'}\gamma = \frac{\beta}{p'}$$

For Marx's two Departments of production system, the Leontief intermediate input nXn matrix is non-negative, according to the Perron-Frobenius theorem[10] there is always a positive eigenvalue λ, and {Pi} standing for the Marx price eigenvector:

$$C_1 + V_1 + M_1 = C_1 + Y_1 = Q_1 = C_1 + C_2 = C, (Y_1 = C_2 : Simple\,Reproduction)$$

$$C_2 + V_2 + M_2 = C_2 + Y_2 = Q_2 = Y_1 + Y_2 = Y;\quad Q_1 + Q_2 = Q$$

$$p_1 C'_1 + p_2 Y'_1 = p_1 Q'_1$$

$$p_1 C'_2 + p_2 Y'_2 = p_2 Q'_2$$

$$\begin{pmatrix} C'_1 & Y'_1 \\ C'_2 & Y'_2 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} Q'_1 & 0 \\ 0 & Q'_2 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, p_i \equiv \frac{1}{P_i};$$

$$C'_1 + Y'_2 = Q'_1 + Q'_2 \Rightarrow Y'_1 = -C'_2$$

$$or: \begin{pmatrix} \frac{C'_1}{Q'_1} & \frac{Y'_1}{Q'_1} \\ \frac{C'_2}{Q'_2} & \frac{Y'_2}{Q'_2} \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \lambda\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \sum_i p_i^2 = 1;$$

$$\lambda_1 = 1, \lambda_2 = \frac{C_1}{Q_1} + \frac{Y_2}{Q_2} - 1 = \frac{C_1}{Q_1}\frac{Y_2}{Q_2} - \frac{C_2}{Q_2}\frac{Y_1}{Q_1}$$

[10] Seneta, E. (1973) Non-negative Matrices – An Introduction to Theory and Applications. London: George Allen and Unwin.

$$\begin{pmatrix} \frac{C'_1}{Q'_1} & \frac{Y'_1}{Q'_1} \\ \frac{C'_2}{Q'_2} & \frac{Y'_2}{Q'_2} \end{pmatrix} \begin{pmatrix} \frac{1}{P_1} \\ \frac{1}{P_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{P_1} \\ \frac{1}{P_2} \end{pmatrix} = \lambda \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, p_i \equiv \frac{1}{P_i}$$

$$1)\lambda_1 = 1 : p_2 = 1, p_1 = \frac{Q'_2 - Y'_2}{C'_2}; 2)\lambda_2 = \frac{C'_1}{Q'_1} + \frac{Y'_2}{Q'_2} - 1 : p_2 = 1, p_1 = \frac{C'_1 - Q'_1}{C'_2} \frac{Q'_2}{Q'_1};$$

$$\frac{dC'_1}{dt} p_1 + C'_1 \frac{dp_1}{dt} + \frac{dY'_1}{dt} p_2 + Y'_1 \frac{dp_2}{dt} = \frac{dQ'_1}{dt} p_1 + Q'_1 \frac{dp_1}{dt};$$

$$\frac{dC'_2}{dt} p_1 + C'_2 \frac{dp_1}{dt} + \frac{dY'_2}{dt} p_2 + Y'_2 \frac{dp_2}{dt} = \frac{dQ'_2}{dt} p_2 + Q'_2 \frac{dp_2}{dt} \Rightarrow$$

$$\frac{d(C'_1 - Q'_1)}{dt} p_1 + \frac{dY'_1}{dt} p_2 + Y'_1 \frac{dp_2}{dt} = (Q'_1 - C'_1) \frac{dp_1}{dt} \Rightarrow Y'_1 \left(\frac{dp_1}{dt} - \frac{dp_2}{dt} \right) = \frac{dY'_1}{dt} (p_2 - p_1)$$

$$\Rightarrow \dot{Y}'_1 = \frac{d(p_1 - p_2)}{(p_2 - p_1)dt} = -\frac{d}{dt} \ln(p_1 - p_2);$$

$$\frac{d(Q'_2 - Y'_2)}{dt} p_2 + (Q'_2 - Y'_2) \frac{dp_2}{dt} = \frac{dC'_2}{dt} p_1 + C'_2 \frac{dp_1}{dt} \Rightarrow \frac{dC'_2}{dt} (p_2 - p_1) = C'_2 \left(\frac{dp_1}{dt} - \frac{dp_2}{dt} \right)$$

$$\Rightarrow \dot{C}'_2 = \frac{d(p_1 - p_2)}{(p_2 - p_1)dt} = \dot{Y}'_1$$

Based on the conservation of the value of input-outputs, the static/dynamic equilibrium conditions of Marx's simple reproduction/complex reproduction are obtained emergently. Or in a three-sector economic system--production material, living material, capital goods-- as proposed by J. Winternitz[11], we also have:

$$(1): P_1C_1 + P_2V_1 + P_3M_1 = P_1Q_1 = P_1C = P_1(C_1 + C_2 + C_3)$$

$$(2): P_1C_2 + P_2V_2 + P_3M_2 = P_2Q_2 = P_2V = P_2(V_1 + V_2 + V_3)$$

$$(3): P_1C_3 + P_2V_3 + P_3M_3 = P_3Q_3 = P_3M = P_3(M_1 + M_2 + M_3)$$

$$\begin{pmatrix} C_1 & V_1 & M_1 \\ C_2 & V_2 & M_2 \\ C_3 & V_3 & M_3 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix},$$

$$or : C_1 + V_2 + M_3 = Q_1 + Q_2 + Q_3, Y_1 = V_1 + M_1 = -(C_2 + C_3), M_2 = -V_3$$

$$\begin{pmatrix} \frac{C'_1}{Q'_1} & \frac{V'_1}{Q'_1} & \frac{M'_1}{Q'_1} \\ \frac{C'_2}{Q'_2} & \frac{V'_2}{Q'_2} & \frac{M'_2}{Q'_2} \\ \frac{C'_3}{Q'_3} & \frac{V'_3}{Q'_3} & \frac{M'_3}{Q'_3} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \lambda \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}, p_i \equiv \frac{1}{P_i}$$

All Marx-Leontief[12] input and output functions have the same form, not matter in the value form or in the price form:

---

[11] Winternitz，J. 'Values and Prices: a solution to the so-called transformation problem', Econ. Jour. 1948, 58, 276-280.

[12] Leontief，W. "Input-Output Economics". 2nd ed., New York: Oxford University Press, 1986.

$$\frac{Q'}{P_Q} \equiv \frac{\sum_{i=1}^{3} Q'_i}{P_Q} = \sum_{i=1}^{3} \frac{Q'_i}{P_i}, p_Q = \frac{1}{P_Q} = \sum_{i=1}^{3} \frac{Q'_i}{Q' P_i};$$

$$\frac{Y'}{P_Y} \equiv \frac{\sum_{i=1}^{3} V'_i + M'_i}{P_Y} = \sum_{i=1}^{3} (\frac{V'_i}{P_2} + \frac{M'_i}{P_3}), p_Y = \frac{1}{P_Y} = \sum_{i=1}^{3} \frac{p_2 V'_i + p_3 M'_i}{Y'};$$

$$\frac{Q'}{P_Q} = Q = B_0 e^{ft} (\frac{C'}{P_C})^{\beta} \frac{V'}{P_V})^{1-\beta} \Rightarrow Q' = \frac{P_Q}{P_C^{\beta} P_V^{1-\beta}} B_0 e^{ft} C'^{\beta} V'^{1-\beta} = B'_0 e^{ft} C'^{\beta} V'^{1-\beta}, B'_0 = \frac{B_0}{Ip_Q};$$

$$Y' = a'_0 e^{Ft} C'^{\alpha} V'^{1-\alpha}, B'_0 = \frac{a_0}{Ip_Y}; M' = b'_0 e^{pt} C'^{\beta} V'^{1-\beta}, b'_0 = \frac{b_0}{Ip_M};$$

$$C'v = \frac{I}{P_C^{\beta} P_V^{1-\beta}} c_0 C'^{\beta} V'^{1-\beta} = c_0 C'^{\beta} V'^{1-\beta}, \frac{dCv}{Cv} = \delta I = \ln(1+\delta I) = I = P_C^{\beta} P_V^{1-\beta}$$

$$r' = \frac{M'}{C' + V'} = \frac{M}{Ip_M Cv} = p' \frac{P_M}{I}$$

Since the mechanics quantities can be characterized as operators, the economic input-output eigen-equations are:

$$\frac{C}{Q} = \frac{\beta}{P'} \to \left| \sum_j \frac{C'_{ij}}{P_j} \right| = \frac{\beta_i}{P'_i} \left| \frac{Q'_i}{P_i} \right| = \lambda \left| \frac{Q'_i}{P_i} \right| \Leftrightarrow \sum_j \left| \frac{C'_{ij}}{Q'_i} \right| \left| \frac{1}{P_j} \right| = \frac{\beta_i}{1+p'_i} \left| \frac{1}{P_i} \right| = \lambda \left| \frac{1}{P_i} \right| \Rightarrow$$

$$\sum_j \frac{C'_{ij}}{C'_i} (\frac{C'_i}{Q'_i}) \left| \frac{1}{P_j} \right| = \frac{\beta_i}{P'_i} \left(\frac{1}{P_i} \right) \Leftrightarrow \sum_j \left| \frac{C'_{ij}}{C'_i} \right| \left| \frac{1}{P_j} \right| = \left| \frac{1}{P_i} \right| (C'_i = \sum_j C'_{ij}) : \mathrm{Pr}\, iceEigenfunction$$

$$\frac{C}{Y} = \frac{\alpha}{p'} \to \left| \sum_j \frac{C'_{ij}}{P_j} \right| = \frac{\alpha_i}{p'_i} \left| \frac{Q'_i - \sum_j C'_{ij}}{P_i} \right| = \frac{\alpha_i}{p'_i} \left| \frac{V'_i + M'_i}{P_i} \right| = \xi \left| \frac{Y'_i}{P_i} \right|$$

$$\sum_j \left| \frac{C'_{ij}}{Y'_i} \right| \left| \frac{1}{P_j} \right| = \frac{\alpha_i}{p'_i} \left| \frac{1}{P_i} \right| = \xi \left| \frac{1}{P_i} \right| \Leftrightarrow \sum_j \left| \frac{C'_{ij}}{C'_i} \left(\frac{C'_i}{Y'_i} \right) \right| \left| \frac{1}{P_j} \right| = \frac{\alpha_i}{p'_i} \left| \frac{1}{P_i} \right| \Rightarrow \sum_j \left| \frac{C'_{ij}}{C'_i} \right| \left| \frac{1}{P_j} \right| = \xi \left| \frac{1}{P_i} \right|$$

C'_{ij}/C'_i is the Leontief intermediate input coefficient matrix, all the eigen equations, eigenvalues and eigenvector of the price coefficient are obtained; similarly：

$$\frac{C}{M} = \frac{\beta}{p'} \to \left| \sum_j \frac{C'_{ij}}{P_j} \right| = \frac{\beta_i}{p'_i} \left| \frac{M'_i}{P_i} \right| \Leftrightarrow \sum_j \left| \frac{C'_{ij}}{M'_i} \right| \left| \frac{1}{P_j} \right| = \frac{\beta_i}{p'_i} \left| \frac{1}{P_i} \right| = \mu \left| \frac{1}{P_i} \right|$$

$$\frac{C}{V} = g = \frac{\beta}{1-\beta} \to \left| \sum_j \frac{C'_{ij}}{P_j} \right| = \frac{\beta_i}{1-\beta_i} \left| \frac{V'_i}{P_i} \right| \Leftrightarrow \sum_j \left| \frac{C'_{ij}}{V'_i} \right| \left| \frac{1}{P_j} \right| = \frac{\beta_i}{1-\beta_i} \left| \frac{1}{P_i} \right| = \upsilon \left| \frac{1}{P_i} \right|;$$

$$or: \frac{C}{C+V} = \frac{C}{Q-M} = \beta, \sum_j \left| \frac{C'_{ij}}{Q'_i - M'_i} \right| \left| \frac{1}{P_j} \right| = \beta_i \left| \frac{1}{P_i} \right|$$

The eigenvalue (β/P’) of the eigen-equation of the intermediate input C and the total output of Q demonstrated that there existed in the economic system a ratio of the reduced organic composition of capital (ROCC) and the rate of profit of P' (P '=p' +1) , rather than the "uniform profit rate p’"[13]; Marx's "uniform profit rate" in the "Das Kapital" should be ROCC (β), which is the eigenvalue of the eigen-equation of the intermediate input C and the total input cost Cv=C+V。

The ratio (β/p’) is the eigenvalue of the eigen-equation of the input--constant capital C and the output--surplus value M;the ratio (α/p’) is the eigenvalue of the eigen-quation of the intermediate input C and the end use Y.Details are as following:

$$\begin{pmatrix} C'_{11} & C'_{12} & \cdots & C'_{1n} \\ C'_{21} & C'_{22} & \cdots & C'_{2n} \\ \vdots & & & \\ C'_{n1} & C'_{n2} & \cdots & C'_{nn} \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} = \frac{\beta}{1+p'}\begin{pmatrix} Q'_1 & & & \\ & Q'_2 & & \\ & & \ddots & \\ & & & Q'_n \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \Leftrightarrow \frac{C'_{ij}}{Q'_i}\left|p_j\right\rangle = \frac{\beta}{P'}\left|p_j\right\rangle;$$

$$= \frac{\beta}{p'}\begin{pmatrix} M'_1 & & & \\ & M'_2 & & \\ & & \ddots & \\ & & & M'_n \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \Leftrightarrow \frac{C'_{ij}}{M'_i}\left|p_j\right\rangle = \frac{\beta}{p'}\left|p_j\right\rangle;$$

$$= \frac{\alpha}{p'}\begin{pmatrix} Y'_1 & & & \\ & Y'_2 & & \\ & & \ddots & \\ & & & Y'_n \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \Leftrightarrow \frac{C'_{ij}}{Y'_i}\left|p_j\right\rangle = \frac{\alpha}{p'}\left|p_j\right\rangle;$$

$$= \frac{\beta}{1-\beta}\begin{pmatrix} V'_1 & & & \\ & V'_2 & & \\ & & \ddots & \\ & & & V'_n \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \Leftrightarrow \frac{C'_{ij}}{V'_i}\left|p_j\right\rangle = \frac{\beta}{1-\beta}\left|p_j\right\rangle;$$

$$= \beta\begin{pmatrix} C'_1+V'_1 & & & \\ & C'_2+V'_2 & & \\ & & \ddots & \\ & & & C'_n+V'_n \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \Leftrightarrow \frac{C'_{ij}}{C'_i+V'_i}\left|p_j\right\rangle = \beta\left|p_j\right\rangle;$$

$$= \beta\begin{pmatrix} Q'_1-M'_1 & & & \\ & Q'_2-M'_2 & & \\ & & \ddots & \\ & & & Q'_n-M'_n \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \Leftrightarrow \frac{C'_{ij}}{Q'_i-M'_i}\left|p_j\right\rangle = \beta\left|p_j\right\rangle$$

Wave function ψ is the price eigen-vector：

$$\psi_i = \left|p_i^{(j)}\right\rangle,$$

$$\psi^*\psi = \left\langle p_i^{(j)} \middle| p_i^{(j)}\right\rangle = \sum_i p_i^2 = 1$$

determined by price eigen-function：

---

13 王璐，柳欣，（2006）《马克思经济学与古典一般均衡理论》，人民出版社.

$$\begin{pmatrix} C'_{11} & C'_{12} & \cdots & C'_{1n} \\ C'_{21} & C'_{22} & \cdots & C'_{2n} \\ \vdots & & & \\ C'_{n1} & C'_{n2} & \cdots & C'_{nn} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} C'_1 & & & \\ & C'_2 & & \\ & & \ddots & \\ & & & C'_n \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix}, C'_i = \sum_k C'_{ik}$$

Similar to Schrodinger function, the price wave function can be defined as：

$$\hat{g}\psi = \beta\psi;$$

$$ROCCoperator: \hat{g} \equiv \left[\frac{C'_{ij}}{C'_i + V'_i}\right] = \left[\frac{C'_{ij}}{Q'_i - M'_i}\right], C'_i = \sum_i C'_{ij}$$

Similar to the intermediate input C=nK, each output--the added value Y, the surplus value of M, the variable capital/human capital V--displayed the quantum nature too. According to the USA 1997-2014 input-output data[14], the eigenvalue λ(=C'/Q') of intermediate input C-total output Q is about 0.479 (Table 3)，however in the year of 1997-2002、2004、2007、2008, the price eigenvectors are negative：

表3	特征值	11	21	22	23	31G	42	44RT	48TW	51	FIRE	PROF	6	7	81	G
t	λ	Agric	Minin	Utili	Const	Manuf	Whole	Retai	Trans	Infor	Finan	Profe	Educa	Arts,	Other	Govern.
2014	**0.485**	0.4364	0.1773	0.1735	0.2956	**0.4964**	0.1413	0.1544	0.3348	0.2270	**0.1185**	0.1586	0.1909	0.2433	0.1786	0.2000
2013	**0.477**	0.4171	0.1636	0.1727	0.3065	**0.4977**	0.1414	0.1605	0.3389	0.2283	**0.1221**	0.1606	0.1946	0.2455	0.1791	0.2102
2012	**0.481**	0.4807	0.1558	0.1500	0.3017	**0.4942**	0.1327	0.1467	0.3235	0.2247	**0.1045**	0.1469	0.1834	0.2355	0.1662	0.2083
2011	**0.478**	0.4452	0.1551	0.1567	0.3110	**0.4967**	0.1417	0.1527	0.3291	0.2192	**0.1160**	0.1526	0.1927	0.2482	0.1723	0.2117
2010	**0.463**	0.4784	0.1548	0.1738	0.3136	**0.4851**	0.1306	0.1546	0.3007	0.2034	**0.1255**	0.1530	0.1926	0.2471	0.1717	0.2092
2009	**0.447**	0.5229	0.1260	0.1628	0.3206	**0.4783**	0.1000	0.1361	0.2798	0.2027	**0.1293**	0.1510	0.1890	0.2444	0.1622	0.2085
2008	**0.492**	-0.4683	-0.1664	-0.2360	-0.3257	**-0.482**	-0.1294	-0.1367	-0.3143	*-0.1738*	**-0.132**	*-0.1353*	-0.1836	-0.2335	-0.1672	-0.2044
2007	**0.49**	-0.4614	-0.1786	-0.2261	-0.3201	**-0.494**	-0.1262	-0.1378	-0.3116	-0.1845	**-0.131**	-0.1437	-0.1865	-0.2293	-0.1600	-0.2008
2006	**0.484**	0.4518	0.2100	0.2120	0.3321	**0.4931**	0.1296	0.1350	0.2896	0.2007	**0.1371**	0.1420	0.1913	0.2301	0.1579	0.2020
2005	**0.493**	0.4320	0.2351	0.2509	0.3280	**0.4815**	0.1367	0.1401	0.2848	0.1790	**0.1519**	0.1463	0.2016	0.2350	0.1591	0.1964
2004	**0.473**	-0.4090	-0.2505	-0.2055	-0.3496	**-0.501**	-0.1344	-0.1435	-0.2692	-0.1964	**-0.138**	-0.1435	-0.1982	-0.2410	-0.1633	-0.2003
2003	**0.466**	0.4398	0.2467	0.2125	0.3430	**0.4957**	0.1275	0.1304	0.2499	0.2176	**0.1209**	0.1381	0.1992	0.2404	0.1582	0.1946
2002	**0.467**	-0.4634	-0.2187	-0.1925	-0.3412	**-0.505**	-0.1324	-0.1277	-0.2439	-0.2236	**-0.109**	-0.1345	-0.1986	-0.2391	-0.1484	-0.1915
2001	**0.477**	-0.4479	-0.2341	-0.2544	-0.3252	**-0.493**	-0.1165	-0.1196	-0.2293	-0.2476	**-0.110**	-0.1391	-0.1973	-0.2406	-0.1553	-0.1909
2000	**0.489**	-0.4301	-0.2612	-0.2337	-0.3268	**-0.492**	-0.1226	-0.1312	-0.2481	-0.2556	**-0.123**	-0.1462	-0.1955	-0.2373	-0.1354	-0.1854
1999	**0.486**	-0.4651	-0.2341	-0.1781	-0.3439	**-0.510**	-0.1184	-0.1259	-0.2355	-0.2102	**-0.111**	-0.1455	-0.1949	-0.2532	-0.1409	-0.1811
1998	**0.487**	-0.4431	-0.2400	-0.1565	-0.3518	**-0.523**	-0.1064	-0.1147	-0.2250	-0.2066	**-0.110**	-0.1470	-0.2020	-0.2792	-0.1440	-0.1819
1997	**0.489**	-0.4310	-0.2342	-0.1379	-0.3607	**-0.523**	-0.1119	-0.1220	-0.2564	-0.2022	**-0.103**	-0.1345	-0.1971	-0.2869	-0.1404	-0.1818

In addition, the eigen-equations can also be expressed in Dirac ket-bra notation[15]：

[14] http://www.bea.gov/industry/io_annual.htm

[15] PAM Dirac (1939). "A new notation for quantum mechanics". *Mathematical Proceedings of the Cambridge Philosophical Society*. **35** (3): 416–418；狄拉克，（1979）《量子力学原理》，科学出版社.

$$P\hat{C}P^{-1} =< P\left|\hat{C}\right|P^{-1} >= C' = Q'\frac{\beta}{1+p'} = Y'\frac{\alpha}{p'} = M'\frac{\beta}{p'} = V'\frac{\beta}{1-\beta} = Cv'\beta = (Q'-M')\beta$$

$$P = Q'/Q, C = \hat{C}\left|P^{-1}\right> = \sum_i\sum_j C'_{ij}\,p_j = Q\frac{\beta}{1+p'}$$

$$\frac{V}{Y} = \frac{\alpha}{p'}\frac{1-\beta}{\beta} \Rightarrow w = \frac{\alpha}{p'}\frac{1-\beta}{\beta}y$$

$$\begin{pmatrix} P_1* & P_2* & \cdots & P_n* \end{pmatrix}\begin{pmatrix} C'_{11} & C'_{12} & \cdots & C'_{1n} \\ C'_{21} & C'_{22} & \cdots & C'_{2n} \\ \vdots & & & \\ C'_{n1} & C'_{n2} & \cdots & C'_{nn} \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix}$$

$$= \begin{pmatrix} P_1, & P_2, & \cdots & P_n \end{pmatrix}\begin{pmatrix} C'_1\,p_1 = C_1 \\ C'_2\,p_2 = C_2 \\ \vdots \\ C'_n\,p_n = C_n \end{pmatrix}$$

$$= P_1C_1 + P_2C_2 + \cdots P_nC_n = C'_1 + C'_2 + \cdots + C'_n = \sum_i C'_i = C'$$

Because the production and sale of each commodity is affected by other commodities, it is impossible to solve the "price wave function" independently like the schrodinger equation. Because all the commodities' price are known, using the Leontief intermediate input n x n matrix, the price eigenstate vector as the wave function are obtained. "Two Cambridge Controversy"[16] about the difficulty of the aggregation of heterogeneous capital goods in the CD production function, the "uniform profit rate" in the general equilibrium state problem, and the wage income distribution[17] issues were resolved. The wage (w) is proportional to the labor productivity (y) and inversely proportional to the rate of profit p'.

4. Statistic thermodynamics--Entropy of an Economic system[18]

The second law of thermodynamics states that an isolated system's entropy never decreases. Such systems spontaneously evolve towards thermodynamic equilibrium, the state with maximum entropy.

For a two-sector economic system producing means of production (C) and living materials (V), total input cost Cv(=C+V） can be regarded as the invested currency with no inflation：

---

16 Joseph E. Stiglitz, "The Cambridge-Cambridge Controversy in the Theory of Capital; A View from New Haven: A Review Article," Journal of Political Economy, 82(4), Jul.-Aug. 1974: 893-903.

17 Erman Zeng,"From Labor Theory of Value to Price Eigen Function--Microfoundation of General Equilibrium", http://vixra.org/abs/1702.0174

18 Atkins, P.W., de Paula, J. (2006). *Atkins' Physical Chemistry*, eighth edition, W.H. Freeman, New York.

$$Q_1 = C_1 + V_1 + M_1 = C_1 + \underline{Y_1} = C_1 + C_2 = C$$

$$Q_2 = C_2 + V_2 + M_2 = \underline{C_2} + Y_2 = Y_1 + Y_2 = Y = V\frac{P'}{1-\alpha}$$

$$\text{since}: Y = V + M = V(1+m') = V\frac{P'}{1-\alpha}, 1-\alpha = \frac{p'+1}{m'+1} = \frac{P'}{m'+1}$$

dCv=0; Q_1=C, Q_2=Y, the state number of the economic system can be expressed by Stirling formula[19] according to the statistical physics theory:

$$\Omega_{input} = \frac{Cv!}{C!V!}, Cv \equiv C + V, \beta \equiv \frac{g}{1+g} = \frac{C}{C+V} = \frac{C}{Cv}$$

$$\ln\Omega_{input} = \ln Cv! - \ln C! - \ln V! = Cv\ln\frac{Cv}{e} - C\ln\frac{C}{e} - V\ln\frac{V}{e} = Cv\ln\frac{1}{\beta^{\beta}(1-\beta)^{1-\beta}}$$

$$S = k_B \ln\Omega_{input} = k_B Cv\ln C_0 = -k_B Cv[\beta\ln\beta + (1-\beta)\ln(1-\beta)], \frac{C+V}{C^{\beta}V^{1-\beta}} = C_0$$

$$\frac{\partial S}{\partial\beta} = -k_B Cv\ln\frac{\beta}{1-\beta} = -k_B Cv\ln g = 0 \Rightarrow$$

$$\beta = 0.5, g = 1, S_{MAX} = k_B Cv\ln 2;$$

$$when: S_{\min} \to 0 \Leftrightarrow \beta \to 0 or 1$$

S is the entropy of the economic system , the relative maximum value is when β = 0.5, which characterizes the relations of production.

$$for: \beta = \frac{1}{2}, g = \frac{\beta}{1-\beta} = 1 \Leftrightarrow C = V, \frac{C}{Q} = \frac{\beta}{P'}, \frac{C}{Y} = \frac{\alpha}{p'}$$

$$Q = B_0 e^{ft} C^{\beta} V^{1-\beta} = B_0\sqrt{CV} = B_0 C = P'C = B_0 V, f = (1-\beta)\dot{g}^* = 0;$$

$$Y = a_0 e^{Ft} C^{\alpha} V^{1-\alpha} = a_0 C = p'C = p'nK = AK, F = (1-\alpha)\dot{g}^* = 0;$$

$$\dot{Y} = \dot{C} = \dot{Q}$$

Therefore, the nature of the endogenous economic growth AK model[20] is completely free market competition achieving general equilibrium[21]。Similarly, the state number of the income distribution of the economic system is:

$$\Omega_{\text{income}} = \frac{Y!}{M!V!}, Y = M + V, m' = \frac{M}{V}$$

$$\ln\Omega_{\text{income}} = Y\ln Y - M\ln M - V\ln V = -k_B Y(\frac{M}{Y}\ln\frac{M}{Y} + \frac{V}{Y}\ln\frac{V}{Y})$$

$$dS = k_B d\ln\Omega_{\text{income}} = -k_B Y\ln m' d(M/Y) = 0 \Rightarrow m' = 1 = 100\%$$

$$\Leftrightarrow M = V = Y/2$$

The most equilibrated distribution of the income between the wages and the surplus

[19] 朗道理论物理教程(卷 05)-统计物理学 I(第 5 版)-[俄]朗道&栗弗席兹-束仁贵&束纯(译)， 高等教育出版社，2011.

[20] Romer, Paul M. (1986). "Increasing Returns and Long-Run Growth". *Journal of Political Economy*. **94** (5): 1002–1037.

[21] 曾尔曼，《厦门科技》，2016（2）27.

value is half-half, as well as the surplus value, the ratio of the tax(T) to the net profit(P) equals 1:

$$\Omega_{\text{tax}} = \frac{M!}{P!T!}, M = P + T, \frac{P}{T} \equiv \kappa$$

$$\ln\Omega_{\text{tax}} = M\ln M - P\ln P - T\ln T - = -k_B M(\frac{P}{M}\ln\frac{P}{M} + \frac{T}{M}\ln\frac{T}{M})$$

$$dS = k_B d\ln\Omega_{\text{tax}} = -k_B M\ln\kappa d(P/M) = 0 \Rightarrow \kappa = 1 \Leftrightarrow P = T = M/2$$

$$p'_{net} = \frac{P}{Cv} = \frac{P}{M}p' = \frac{P}{P+T}\frac{M}{C+V} = \frac{1}{2}\frac{1}{2} = \frac{1}{4} = 25\%$$

Obviously, the optimum profit rate after tax would be 25%, several countries among OECD have achieved that level, for example, USA,France,Norway, Hungry, etc.[22]

Based upon Debreu-Arrow's general equilibrium theory and statistic physical partition function, Tao's research[23] seemed solved the Needham puzzle[24]: the level of technology progress is proportional to the entropy of the economic system, therefore the social freedom.

The second law of the thermodynamics reveals that for an isolated single system, its entropy increase process is monotonic and spontaneous , the freedom of the particles of the system is increased as well as the equilibrium degree of the system[25].

Therefore, Hayek's Spontaneous Order[26] can be explained by the entropy theory too:

$$TdS = dE = d(\frac{Mp'}{2}) = p'dM = Mdp' = Mpp'dt = \Phi p'dt = \Phi d\ln P > 0 \Leftrightarrow \Phi > 0$$

$$S = k_B\ln\Omega = k_B Cv\ln C_0 = -k_B Cv[\beta\ln\beta + (1-\beta)\ln(1-\beta)],$$

$$dS = -k_B Cv\ln gd\beta = k_B Cv\ln gd(1\text{-}\beta) = \Phi p'dt/T > 0, (dCv = 0)$$

$$or: \dot{S} = \dot{C}v + \frac{d\ln C_0}{\ln C_0 dt} = \dot{C}v + \frac{\dot{C}_0}{\ln C_0} = \frac{\dot{C}_0}{\ln C_0}, \quad \dot{C}_0 = \ln g\frac{d(1\text{-}\beta)}{dt} = \ln g(1\text{-}\beta)\dot{d}_R,$$

$$\dot{S} = \frac{dS}{Sdt} = \dot{C}v - \frac{(1\text{-}\beta)\ln g\dot{d}_R}{\beta\ln\beta + (1-\beta)\ln(1-\beta)} = \frac{\ln g\dot{d}_R}{(1+g)\ln(1+g) - g\ln g} > 0$$

$$p' = \frac{M}{Cv} = b_0 e^{pt}\frac{C^\beta V^{1-\beta}}{C+V} = b_0 e^{pt - S/Cv/k_B} \Rightarrow \ln\frac{p'}{b_0} = pt - \frac{S}{k_B Cv},$$

$$\Leftrightarrow S = k_B Cv(pt + \ln\frac{b_0}{p'}), dS = tk_B Cvdp > 0$$

The entropy change of the macro-economy is proportional to the change of the degree

---

22 曾尔曼.《马克思生产力经济学导引》，厦门大学出版社，2016，p158.

23 Yong Tao, Competitive market for multiple firms and economic crisis, Physical Review E 82 (2010) 036118

24 *Joseph Needham (1969). The Grand Titration: Science and Society in East and West.*

25 F. 旦尼尔斯，《物理化学》，曾国洲等译，上海科学技术出版社，1983. p

26 Berry,Norman (1982)"The Tradition of Spontaneous Order",*Literature of Liberty*. Vol. v, no. 2, pp. 7-58. Arlington, VA: Institute for Humane Studies ; 哈耶克．自由秩序原理[M].邓正来，译．北京：生活•读书•新知三联书店，1997.

of the round-about production or the change of the profit rate, as well as the change rate of the technology:

$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta}p \cong \frac{\alpha}{\beta}p = F, \frac{\beta}{\alpha} = \frac{Y}{M}, \Phi = Mp = YF$$

$$\Rightarrow S \propto ptCv = \frac{\beta}{\alpha}mtCv = \frac{\beta}{\alpha}FtCv = \frac{\Phi}{p'}t$$

$$dS = tk_B Cvdp = -k_B Cv \ln gd\beta = tk_B \frac{\beta}{\alpha}Cvdm = tk_B Cv\frac{Y}{M}dF = \frac{k_B t}{p'}d\Phi > 0$$

$$\Leftrightarrow tk_B Cdm = tk_B nKdm = \alpha dS; p'dS = k_B td\Phi;$$

$$tdp = -\ln gd\beta = \ln gd(1-\beta) \Leftrightarrow d(1-\beta) = tdp/\ln g$$

$$-\ln g\frac{d\beta}{\beta} = t\frac{dF}{\alpha} = \dot{g}*t\frac{d(1-\alpha)}{\alpha} \Leftrightarrow$$

$$\dot{\alpha} = \frac{\ln g}{\dot{g}*t}\dot{\beta} \Leftrightarrow \dot{d}_L = \frac{\ln g}{\gamma\dot{g}*t}\dot{d}_R, \gamma = \frac{\beta}{1-\beta}\frac{1-\alpha}{\alpha}$$

$$\dot{p} = \frac{dp}{pdt} = \frac{\ln gd(1-\beta)}{tpdt} = \frac{(1-\beta)\ln g}{tp}\dot{d}_R = \frac{\ln g}{\gamma\dot{g}*t}\dot{d}_R = \dot{d}_L$$

$$p = c'_0 d_L = c'_0(1-\alpha), c'_0 = \frac{\beta}{\alpha}\dot{g}*$$

$$T = \frac{dE}{dS} = \frac{Mpp'dt}{k_B Cv\ln gd(1-\beta)} = \frac{(1-\beta)p}{k_B \ln g\dot{d}_R}p'^2, \dot{T} = 2p + \dot{d}_R - \frac{\dot{g}}{\ln g};$$

$$or: T = \frac{dE}{dS} = \frac{Mpp'dt}{tk_B Cvdp} = \frac{p'^2\, pdt}{k_B tdp} = \frac{p'^2}{k_B t\dot{p}}, \dot{T} = 2p - t^{-1}$$

$$r' = \frac{M'}{C'+V'} = b'_0 e^{pt}\frac{(CPc)^{\beta}(VPv)^{1-\beta}}{CPc+VPv} = b'_0 e^{pt}\beta^{\beta}(1-\beta)^{1-\beta}P^{\beta}\frac{g+1}{g'+1}, g' \equiv Pg, P \equiv \frac{Pc}{Pv},$$

$$r = \frac{d\ln r'}{dt} = p - \frac{dS}{Cvk_B dt} + (\beta-\beta')\dot{g}';$$

$$\frac{dS}{Cvk_B dt} = p - r - (\beta'-\beta)\dot{g}' > 0 : \text{SpontaneousOrder}$$

$$\Leftrightarrow p = \dot{M} = \dot{\Phi} > r + (\beta'-\beta)\dot{g}' = r + (P-1)\beta(1-\beta')(\dot{P}+\dot{g})$$

$$\beta' = \frac{g'}{1+g'} = \frac{P\beta}{1+(P-1)\beta}, 1-\beta' = \frac{1-\beta}{1+(P-1)\beta}$$

5. Discussion

The discreteness of physical goods in the market, and the price based currencies have the smallest unit (￥; $cent, penny, etc.) and the individual nature of the labor force, is the objective basis of the discussion of the quantum nature of macroscopic economic system。It is worth mentioning that the uncertainty principle in quantum mechanics discovered by Heisenberg[27] 90 years ago, reflects that the value and the price cannot be determined simultaneously：

$$[x,p]=i\hbar=[\ln P,M],\quad \Delta(\ln P)\Delta M\geq\hbar/2$$

$$\Delta M=\Delta Q/\gamma,\gamma=P'/p'=1+(p')^{-1}\Rightarrow\Delta(\ln P)\Delta Q\geq\hbar\gamma/2$$

The negative value of the price eigen-vectors, corresponding to USA's economic / financial crisis (2002/2008), need further empirical research, which may be carried out by the Feynman path integral[28] of quantum mechanics representation, to study the time evolution the price state vector in the macro-economic system. It can provide the great help for the decision-making.

---

[27] Heisenberg, W. The physical content of quantum kinematics and mechanics. In Wheeler, J. A. & Zurek, W. H. (eds.) Quantum Theory and Measurement, 62–84 (Princeton UP, Princeton, NJ, 1983). [Originally published: Z. Phys., 43, 172-98 (1927)].

[28] Feynman, R. P.; Hibbs, A. R. (1965). Quantum Mechanics and Path Integrals. New York: McGraw-Hill.