Isomorphism of Single Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce the homomorphism, weak isomorphism, co-weak isomorphism, and isomorphism of single valued neutrosophic hypergraphs. The properties of order, size and degree of vertices, along with isomorphism, are included. The isomorphism of single valued neutrosophic hypergraphs equivalence relation and of weak isomorphism of single valued neutrosophic hypergraphs partial order relation is also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism of single valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic set (NS) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories, and it is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in the real world. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^-0, 1^+[$. To conveniently use NS in the real-life applications, Wang et al. [9] introduced the single-valued neutrosophic set (SVNS), as a subclass of the neutrosophic sets. The same authors [10] introduced the interval valued neutrosophic set (IVNS), which is even more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent, and their values belong to the unit interval [0, 1]. The hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyse architecture structures and to represent system partitions. In this paper, we extend the concept into isomorphism of single valued neutrosophic hypergraphs, and some of their properties are introduced.

2 Preliminaries

Definition 2.1

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ a finite set of vertices;

(2) $E = \{E_1, E_2, \dots, E_m\}$ a family of subsets of X;

(3) E_j are not-empty for $j = 1,2,3, \dots, m$ and $\cup_j(E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyper-edges).

Definition 2.2

A fuzzy hypergraph H = (X, E) is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

Remark 2.3

The collection $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of edge sets of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form $H = (X, E, R)$, where $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of fuzzy subsets of X, that is $E_j : X \to [0, 1]$, $j = 1, 2, 3, \dots, m$ and $R : E \to [0, 1]$ is a fuzzy relation on fuzzy subsets E_j, such that:

$$R(x_1, x_2, \dots, x_r) \leq \min(E_j(x_1), \dots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$. For each point x ∈X; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.6

A single valued neutrosophic hypergraph (SVNHG) is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \dots, m$ and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyper-edges).

Proposition 2.7

The SVNHG is the generalization of the fuzzy hypergraphs and of the intuitionistic fuzzy hypergraphs.

Let be given a SVNHG $H = (X, E, R)$, with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of SVN subsets of X, and R being SVN's relation on SVN subsets E_j such that:

$$R_T(x_1, x_2, \dots, x_r) \leq \min([T_{E_j}(x_1)], \dots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \dots, x_r) \geq \max([I_{E_j}(x_1)], \dots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \dots, x_r) \geq \max([F_{E_j}(x_1)], \dots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Example 2.8

Consider the SVNHG $H = (X, E, R)$ with underlying set $X = \{a, b, c\}$, where $E = \{A, B\}$ and R, which is defined in the *Tables* given below.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

By routine calculations, $H = (X, E, R)$ is a SVNHG.

3 Isomorphism of SVNHGs

Definition 3.1

A homomorphism $f: H \to K$ between two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a mapping $f: X \to Y$, which satisfies:

$$\min[T_{E_j}(x)] \leq \min[T_{F_j}(f(x))], \tag{5}$$

$$\max[I_{E_j}(x)] \geq \max[I_{F_j}(f(x))], \tag{6}$$

$$\max[F_{E_j}(x)] \geq \max[F_{F_j}(f(x))], \tag{7}$$

for all $x \in X$, and

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{8}$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{9}$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{10}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Example 3.2

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, and $f: X \to Y$ defined by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.3,0.2,0.2)	(0.2,0.1,0.3)
y	(0.2,0.4,0.2)	(0.3,0.2,0.1)
z	(0.5,0.8,0.2)	(0.9, 0.7, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.3
D	0.1	0.7	0.3

By routine calculations, $f: H \rightarrow K$ is a homomorphism between H and K.

Definition 3.3

A weak isomorphism $f: H \rightarrow K$ between two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f : X \rightarrow Y$, which satisfies f is homomorphism, such that:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \quad (11)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \quad (12)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \quad (13)$$

for all $x \in X$.

Note

The weak isomorphism between two SVNHGs preserves the weights of vertices.

Example 3.4

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, and f: $X \rightarrow Y$ defined by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.2,0.3,0.2)	(0.2,0.1,0.8)
y	(0.2,0.4,0.2)	(0.1,0.6,0.5)
z	(0.5,0.8,0.9)	(0.9,0.9,0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.9

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.1	0.9	0.8

By routine calculations, f: $H \rightarrow K$ is a weak isomorphism between H and K.

Definition 3.5

A co-weak isomorphism f: $H \rightarrow K$ between two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping f: $X \rightarrow Y$ which satisfies f is homomorphism, i.e.:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{14}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{15}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{16}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Note

The co-weak isomorphism between two SVNHGs preserves the weights of edges.

Example 3.6

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}* ,*F = {C, D}*, *R* and *S* are defined in the *Tables* given below, and *f: X → Y* defined by *f(a)=x* , *f(b)=y* and *f(c)=z*.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.3,0.2,0.2)	(0.2,0.1,0.3)
y	(0.2,0.4,0.2)	(0.3,0.2,0.1)
z	(0.5,0.8,0.2)	(0.9, 0.7, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.1	0.9	0.8

By routine calculations, *f: H → K* is a co-weak isomorphism between *H* and *K*.

Definition 3.7

An isomorphism *f: H → K* between two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X → Y*, which satisfies:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{17}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{18}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{19}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{20}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{21}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Note

The isomorphism between two SVNHGs preserves both the weights of vertices and the weights of edges.

Example 3.8

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, and *f: X → Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*.

H	A	B
a	(0.2,0.3,0.7)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.2,0.3,0.2)	(0.2,0.1,0.8)
y	(0.2,0.4,0.2)	(0.1,0.6,0.5)
z	(0.5,0.8,0.7)	(0.9, 0.9, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.0	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.0	0.9	0.8

By routine calculations, $f: H \to K$ is an isomorphism between H and K.

Definition 3.9

Let $H = (X, E, R)$ be a SVNHG; then, the *order* of H is denoted and defined by:

$$O(H) = (\sum \min T_{E_j}(x), \sum \max I_{E_j}(x), \qquad (23)$$

and the *size* of H is denoted and defined by:

$$S(H) = (\sum R_T(E_j), \sum R_I(E_j), \sum R_F(E_j)). \qquad (24)$$

Theorem 3.10

Let $H = (X, E, R)$ and $K = (Y, F, S)$ be two SVNHGs, such that H is isomorphic to K.

Then:

(1) O(H) = O(K);

(2) S(H) = S(K).

Proof.

Let $f: H \to K$ be an isomorphism between H and K with underlying sets X and Y respectively.

Then, by definition, we have:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \qquad (25)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \qquad (26)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \qquad (27)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (28)$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (29)$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (30)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Consider:

$$O_T(H) = \sum \min T_{E_j}(x) = \sum \min T_{F_j}(f(x)) = O_T(K) \quad (31)$$

Similarly, $O_I(H) = O_I(K)$ and $O_F(H) = O_F(K)$, hence *O(H) = O(K).*

Next,

$$S_T(H) = \sum R_T(x_1, x_2, \dots, x_r) = \sum S_T(f(x_1), f(x_2), \dots, f(x_r)) = S_T(K) \quad (32)$$

Similarly, $S_I(H) = S_I(K)$, $S_F(H) = S_F(K)$, hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem need not to be true in general.

Example 3.12

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

H	A	B
a	(0.2, 0.5, 0.33)	(0.16,0.5,0.33)
b	(0.0,0.0,0.0)	(0.2,0.5,0.33)
c	(0.33,0.5,0.33)	(0.2,0.5,0.33)
d	(0.5,0.5,0.33)	(0.0,0.0,0.0)

K	C	D
w	(0.2,0.5,0.33)	(0.2,0.5,0.33)
x	(0.16,0.5,0.33)	(0.33,0.5,0.33)
y	(0.33,0.5,0.33)	(0.2,0.5,0.33)
z	(0.5,0.5,0.33)	(0.0,0.0,0.0)

R	R_T	R_I	R_F
A	0.2	0.5	0.33
B	0.16	0.5	0.33

S	S_T	S_I	S_F
C	0.16	0.5	0.33
D	0.2	0.5	0.33

Here, *O(H) = (1.06, 2.0, 1.32) = O(K)* and *S(H) = (0.36, 1.0, 0.66) = S(K)*, but, by routine calculations, *H* is not isomorphism to *K*.

Corollary 3.13

The weak isomorphism between any two SVNHGs preserves the orders.

Remark 3.14

The converse of above corollary need not to be true in general.

Example 3.15

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

H	A	B
a	(0.2,0.5,0.3)	(0.14,0.5,0.3)
b	(0.0,0.0,0.0)	(0.2,0.5,0.3)
c	(0.33,0.5,0.3)	(0.16,0.5,0.3)
d	(0.5,0.5,0.3)	(0.0,0.0,0.0)

K	C	D
w	(0.14,0.5,0.3)	(0.16,0.5,0.3)
x	(0.0,0.0,0.0)	(0.16,0.5,0.3)
y	(0.25,0.5,0.3)	(0.2,0.5,0.3)
z	(0.5,0.5,0.3)	(0.0,0.0,0.0)

Here, *O(H)= (1.0, 2.0, 1.2) = O(K),* but, by routine calculations, *H* is not weak isomorphism to *K*.

Corollary 3.16

The co-weak isomorphism between any two SVNHGs preserves sizes.

Remark 3.17

The converse of above corollary need not to be true in general.

Example 3.18

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*,*R* and *S* are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

H	A	B
a	(0.2,0.5,0.3)	(0.14,0.5,0.3)
b	(0.0,0.0,0.0)	(0.16,0.5,0.3)
c	(0.3,0.5,0.3)	(0.2,0.5,0.3)
d	(0.5,0.5,0.3)	(0.0,0.0,0.0)

K	C	D
w	(0.0,0.0,0.0)	(0.2,0.5,0.3)
x	(0.14,0.5,0.3)	(0.25,0.5,0.3)
y	(0.5,0.5,0.3)	(0.2,0.5,0.3)
z	(0.3,0.5,0.3)	(0.0,0.0,0.0)

R	R_T	R_I	R_F
A	0.2	0.5	0.3
B	0.14	0.5	0.3
S	S_T	S_I	S_F
C	0.14	0.5	0.3
D	0.2	0.5	0.3

Here, *S(H) = (0.34,1.0,0.6) = S(K)*, but, by routine calculations, *H* is not co-weak isomorphism to *K*.

Definition 3.19

Let *H = (X, E, R)* be a SVNHG; then the degree of vertex x_i is denoted and defined by:

$$\deg(x_i) = (deg_T(x_i), deg_I(x_i), deg_F(x_i)), \quad (33)$$

where

$$deg_T(x_i) = \sum R_T(x_1, x_2, \dots, x_r), \quad (34)$$

$$deg_I(x_i) = \sum R_I(x_1, x_2, \dots, x_r), \quad (35)$$

$$deg_F(x_i) = \sum R_F(x_1, x_2, \dots, x_r), \quad (36)$$

for $x_i \neq x_r$.

Theorem 3.20

If *H* and *K* are two isomorphic SVNHGs, then the degree of their vertices is preserved.

Proof.

Let *f: H* → *K* be an isomorphism between *H* and *K* with underlying sets *X* and *Y* respectively; then, by definition, we have

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \quad (37)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \quad (38)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \quad (39)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \ldots, x_r) = S_T(f(x_1), f(x_2), \ldots, f(x_r)), \tag{40}$$

$$R_I(x_1, x_2, \ldots, x_r) = S_I(f(x_1), f(x_2), \ldots, f(x_r)), \tag{41}$$

$$R_F(x_1, x_2, \ldots, x_r) = S_F(f(x_1), f(x_2), \ldots, f(x_r)), \tag{42}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of *X.*

Consider:

$$deg_T(x_i) = \sum R_T(x_1, x_2, \ldots, x_r) = \sum S_T(f(x_1), f(x_2), \ldots, f(x_r)) = deg_T(f(x_i)). \tag{43}$$

Similarly:

$$deg_I(x_i) = deg_I(f(x_i)), deg_F(x_i) = deg_F(f(x_i)) \tag{44}$$

Hence:

$$deg(x_i) = deg(f(x_i)). \tag{45}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b}* and *Y = {x, y}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in the *Tables* given below, where *f* is defined by, *f(a)=x, f(b)=y,* here *deg(a) = (0.8, 1.0, 0.6) = deg(x)* and *deg(b) = (0.45, 1.0, 0.6) = deg(y).*

H	A	B
a	(0.5,0.5,0.3)	(0.3,0.5,0.3)
b	(0.25,0.5,0.3)	(0.2,0.5,0.3)

K	C	D
x	(0.3,0.5,0.3)	(0.5,0.5,0.3)
y	(0.2,0.5,0.3)	(0.25,0.5,0.3)

S	S_T	S_I	S_F
C	0.2	0.5	0.3
D	0.25	0.5	0.3

R	R_T	R_I	R_F
A	0.25	0.5	0.3
B	0.2	0.5	0.3

But H is not isomorphic to K, i.e. H is neither weak isomorphic nor co-weak isomorphic to K.

Theorem 3.23

The isomorphism between SVNHGs is an equivalence relation.

Proof.

Let $H = (X, E, R)$, $K = (Y, F, S)$ and $M = (Z, G, W)$ be SVNHGs with underlying sets X, Y and Z, respectively:

- Reflexive.

Consider the map (identity map) $f: X \to X$ defined as follows: $f(x) = x$ *for all* $x \in X$, since identity map is always bijective and satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{E_j}(f(x))], \quad (46)$$

$$\max[I_{E_j}(x)] = \max[I_{E_j}(f(x))], \quad (47)$$

$$\max[F_{E_j}(x)] = \max[F_{E_j}(f(x))], \quad (48)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = R_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (49)$$

$$R_I(x_1, x_2, \dots, x_r) = R_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (50)$$

$$R_F(x_1, x_2, \dots, x_r) = R_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (51)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Hence f is an isomorphism of SVNHG H to itself.

- Symmetric.

Let $f: X \to Y$ be an isomorphism of H and K, then f is bijective mapping, defined as $f(x) = y$ for all $x \in X$.

Then, by definition:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \quad (52)$$

$$\max[I_{E_j}(x)] \ = \max[I_{F_j}\big(f(x)\big)], \tag{53}$$

$$\max[F_{E_j}(x)] \ = \max[F_{F_j}\big(f(x)\big)], \tag{54}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \ = S_T(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{55}$$

$$R_I(x_1, x_2, \dots, x_r) \ = S_I(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{56}$$

$$R_F(x_1, x_2, \dots, x_r) \ = \ S_F(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{57}$$

for all $\{\, x_1, x_2, \dots, x_r\}$ subsets of *X.*

Since f is bijective, then we have $f^{-1}(y) = x$ for all $y \ \in Y$.

Thus, we get:

$$\min[T_{E_j}\left(f^{-1}(y)\right)] \ = \min[T_{F_j}(y)], \tag{58}$$

$$\max[I_{E_j}\left(f^{-1}(y)\right)] \ = \max[I_{F_j}(y)], \tag{59}$$

$$\max[F_{E_j}\left(f^{-1}(y)\right)] \ = \max[F_{F_j}(y)], \tag{60}$$

for all $x \in X$, and:

$$R_T\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) \ = S_T(y_1\,, y_2, \dots, y_r), \tag{61}$$

$$R_I\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) \ = S_I(y_1\,, y_2, \dots, y_r), \tag{62}$$

$$R_F\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) \ = S_F(y_1\,, y_2, \dots, y_r), \tag{63}$$

for all $\{\, y_1, y_2, \dots, y_r\}$ subsets of *Y.*

Hence, we have a bijective map $f^{-1} : Y \ \to X$, which is an isomorphism from K to H.

- Transitive.

Let $f\colon X \to Y$ and $g\colon Y \to Z$ be two isomorphism of SVNHGs of H onto K and K onto *M,* respectively. Then gof is a bijective mapping from X to *Z*, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \ \in X$.

Since f is an isomorphism, then, by definition, $f(x) = y$ for all $x \ \in X$, which satisfies:

$$\min[T_{E_j}(x)] \ = \min[T_{F_j}\big(f(x)\big)]\,, \tag{64}$$

$$\max[I_{E_j}(x)] \ = \max[I_{F_j}\big(f(x)\big)], \tag{65}$$

$$\max[F_{E_j}(x)] \ = \max[F_{F_j}\big(f(x)\big)], \tag{66}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (67)$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (68)$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (69)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Since $g : Y \to Z$is an isomorphism, then, by definition, $g(y) = z$ for all $y \in Y$, satisfying the conditions:

$$\min[T_{F_j}(y)] = \min\left[T_{G_j}(g(y))\right], \quad (70)$$

$$\max[I_{F_j}(y)] = \max[I_{G_j}(g(y))], \quad (71)$$

$$\max[F_{F_j}(y)] = \max[F_G(g(y))], \quad (72)$$

for all $x \in X$, and:

$$S_T(y_1, y_2, \dots, y_r) = W_T(g(y_1), g(y_2), \dots, g(y_r)), \quad (73)$$

$$S_I(y_1, y_2, \dots, y_r) = W_I(g(y_1), g(y_2), \dots, g(y_r)), \quad (74)$$

$$S_F(y_1, y_2, \dots, y_r) = W_F(g(y_1), g(y_2), \dots, g(y_r)), \quad (75)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of *Y.*

Thus, from above equations, we conclude that:

$$\min[T_{E_j}(x)] = \min[T_{G_j}(g(f(x)))], \quad (76)$$

$$\max[I_{E_j}(x)] = \max[I_{G_j}(g(f(x)))], \quad (77)$$

$$\max[F_{E_j}(x)] = \max[F_{G_j}(g(f(x)))], \quad (78)$$

for all $x \in X$, and:

$$R_T(x_1, \dots, x_r) = W_T(g(f(x_1)), \dots, g(f(x_r))), \quad (79)$$

$$R_I(x_1, \dots, x_r) = W_I(g(f(x_1)), \dots, g(f(x_r))), \quad (80)$$

$$R_F(x_1, \dots, x_r) = W_F(g(f(x_1)), \dots, g(f(x_r))), \quad (81)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, gof is an isomorphism between H and M. Hence, the isomorphism between SVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between SVNHGs satisfies the partial order relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be SVNHGs with underlying sets X, Y and *Z,* respectively.

- Reflexive.

Consider the map (identity map) $f: X \rightarrow X$, defined as follows $f(x)=x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{E_j}(f(x))], \quad (82)$$

$$\max[I_{E_j}(x)] = \max[I_{E_j}(f(x))], \quad (83)$$

$$\max[F_{E_j}(x)] = \max[F_{E_j}(f(x))], \quad (84)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq R_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (85)$$

$$R_I(x_1, x_2, \dots, x_r) \geq R_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (86)$$

$$R_F(x_1, x_2, \dots, x_r) \geq R_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (87)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.
Hence f is a weak isomorphism of SVNHG H to itself.

- Anti-symmetric.

Let f be a weak isomorphism between H onto K, and g be a weak isomorphic between K and H, that is $f: X \rightarrow Y$ is a bijective map defined by $f(x) = y$ for all $x \in X$, satisfying the conditions:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \quad (88)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \quad (89)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \quad (90)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (91)$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (92)$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (93)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.
Since g is also a bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[T_{F_j}(y)] = \min[T_{E_j}(g(y))], \quad (95)$$

$$\max[I_{F_j}(y)] = \max[I_{E_j}(g(y))], \quad (96)$$

$$\max[F_{F_j}(y)] = \max[F_{E_j}(g(y))], \quad (97)$$

for all $y \in Y$, and:

$$R_T(y, y_2, \dots, y_r) \leq S_T(g(y_1), g(y_2), \dots, g(y_r)), \quad (98)$$

$$R_I(y_1, y_2, \dots, y_r) \geq S_I(f(y_1), f(y_2), \dots, f(y_r)), \quad (99)$$

$$R_F(y_1, y_2, \dots, y_r) \geq S_F(f(y_1), f(y_2), \dots, f(y_r)), \quad (100)$$

for all $\{ y_1, y_2, \dots, y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only when H and K SVNHGs have same number of edges and the corresponding edge have same weight, hence H is identical to K.

- Transitive.

Let $f: X \to Y$ and $g: Y \to Z$ be two weak isomorphism of SVNHGs of H onto K and K onto M, respectively. Then gof is a bijective mapping from X to Z, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is a weak isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{F_j}\big(f(x)\big)], \quad (101)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}\big(f(x)\big)], \quad (102)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}\big(f(x)\big)], \quad (103)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (104)$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (105)$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (106)$$

for all $\{ x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g: Y \to Z$ is a weak isomorphism, then, by definition, $g(y) = z$ for all $y \in Y$ satisfying the conditions:

$$\min[T_{F_j}(y)] = \min[T_{G_j}\big(g(y)\big)], \quad (107)$$

$$\max[I_{F_j}(y)] = \max[I_{G_j}\big(g(y)\big)], \quad (108)$$

$$\max[F_{F_j}(y)] = \max[F_G\big(g(y)\big)], \quad (109)$$

for all $x \in X$, and:

$$S_T(y_1, y_2, \dots, y_r) \leq W_T(g(y_1), g(y_2), \dots, g(y_r)), \quad (110)$$

$$S_I(y_1, y_2, \dots, y_r) \geq W_I(g(y_1), g(y_2), \dots, g(y_r)), \quad (111)$$

$$S_F(y_1, y_2, \dots, y_r) \geq W_F(g(y_1), g(y_2), \dots, g(y_r)), \quad (112)$$

for all $\{ y_1, y_2, \dots, y_r\}$ subsets of Y.

Thus, from above equations, we conclude that:

$$\min[T_{E_j}(x)] = \min[T_{G_j}(g(f(x)))], \quad (113)$$

$$\max[I_{E_j}(x)] = \max[I_{G_j}(g(f(x)))], \quad (114)$$

$$\max[F_{E_j}(x)] = \max[F_{G_j}(g(f(x)))], \quad (115)$$

for all $x \in X$, *and:*

$$R_T(x_1, \dots, x_r) \leq W_T(g(f(x_2)), \dots, g(f(x_r))), \quad (116)$$

$$R_I(x_1, \dots, x_r) \geq W_I(g(f(x_2)), \dots, g(f(x_r))), \quad (117)$$

$$R_F(x_1, \dots, x_r) \geq W_F(g(f(x_2)), \dots, g(f(x_r))) \quad (118)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Therefore gof is a weak isomorphism between H and M.

Hence, a weak isomorphism between SVNHGs is a partial order relation.

4 Conclusion

Theoretical concepts of graphs and hypergraphs are highly used by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science.

In this paper, the isomorphism between SVNHGs is proved to be an equivalence relation and the weak isomorphism to be a partial order relation. Similarly, it can be proved that a co-weak isomorphism in SVNHGs is a partial order relation.

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