

# Regular and Totally Regular Interval Valued Neutrosophic Hypergraphs

Ali Hassan<sup>1</sup>, Muhammad Aslam Malik<sup>2</sup>, Florentin Smarandache<sup>3</sup>

<sup>1</sup> University of Punjab, Lahore, Pakistan  
alihassan.iiui.math@gmail.com

<sup>2</sup> University of Punjab, Lahore, Pakistan  
aslam@math.pu.edu.pk

<sup>3</sup> University of New Mexico, Gallup, NM, USA  
smarand@unm.edu

## Abstract

In this paper, we define the regular and the totally regular interval valued neutrosophic hypergraphs, and discuss the order and size along with properties of the regular and the totally regular single valued neutrosophic hypergraphs. We extend work to completeness of interval valued neutrosophic hypergraphs.

## Keywords

interval valued neutrosophic hypergraphs, regular interval valued neutrosophic hypergraphs, totally regular interval valued neutrosophic hypergraphs.

## 1 Introduction

Smarandache [8] introduced the notion of neutrosophic sets (NSs) as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic sets are characterized by a truth-membership function ( $t$ ), an indeterminacy-membership function ( $i$ ) and a falsity membership function ( $f$ ) independently, which are within the real standard or non-standard unit interval  $]0, 1+[$ .

In order to conveniently use NS in real life applications, Smarandache [8] and Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets.

The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set.

The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the

unit interval  $[0, 1]$ . More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on <http://fs.gallup.unm.edu/NSS/>.

Hypergraph is a graph in which an edge can connect more than two vertices, and can be applied to analyse architecture structures and to represent system partitions. J. Mordesen and P. S. Nasir gave the definitions for fuzzy hypergraphs. R. Parvathy and M. G. Karunambigai's paper introduced the concept of intuitionistic fuzzy hypergraphs and analysed its components. The regular intuitionistic fuzzy hypergraphs and the totally regular intuitionistic fuzzy hypergraphs were introduced by I. Pradeepa and S. Vimala [38].

In this paper, we extend the regularity and the totally regularity on interval valued neutrosophic hypergraphs.

## 2 Preliminaries

Definition 2.1.

Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ . A single valued neutrosophic set  $A$  (SVNS  $A$ ) is characterized by truth membership function  $T_A(x)$ , indeterminacy membership function  $I_A(x)$  and a falsity membership function  $F_A(x)$ . For each point  $x \in X$ ;  $T_A(x), I_A(x), F_A(x) \in [0, 1]$ .

Definition 2.2.

Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ . An interval valued neutrosophic set  $A$  (IVNS  $A$ ) is characterized by truth membership function  $T_A(x)$ , indeterminacy membership function  $I_A(x)$  and a falsity membership function  $F_A(x)$ . For each point  $x \in X$ ;  $T_A(x) = [TL_A(x), TU_A(x)]$ ,  $I_A(x) = [IL_A(x), IU_A(x)]$  and  $F_A(x) = [FL_A(x), FU_A(x)]$  are contained in  $[0, 1]$ .

Definition 2.3.

A hypergraph is an ordered pair  $H = (X, E)$ , where:

- (1)  $X = \{x_1, x_2, \dots, x_n\}$  a finite set of vertices.
- (2)  $E = \{E_1, E_2, \dots, E_m\}$  a family of subsets of  $X$ .
- (3)  $E_j$  for  $j = 1, 2, 3, \dots, m$  and  $\cup_j(E_j) = X$ .

The set  $X$  is called set of vertices and  $E$  is the set of edges (or hyperedges).

Definition 2.4.

An interval valued neutrosophic hypergraph is an ordered pair  $H = (X, E)$ , where:

- (1)  $X = \{x_1, x_2, \dots, x_n\}$  a finite set of vertices.

(2)  $E = \{ E_1, E_2, \dots, E_m \}$  a family of IVNSs of  $X$ .

(3)  $E_j \neq O = ([0,0], [0,0], [0,0])$  for  $j= 1,2,3,\dots,m$  and  $\cup_j \text{Supp}(E_j)= X$ .

The set  $X$  is called set of vertices and  $E$  is the set of IVN-edges (or IVN-hyperedges).

Example 2.5.

Consider an interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{ a, b, c, d \}$  and  $E = \{ P, Q, R \}$ , defined by:

$$P = \{(a, [0.8, 0.9], [0.4, 0.7], [0.2, 0.7]), (b, [0.7, 0.9], [0.5, 0.8], [0.3, 0.9])\},$$

$$Q = \{(b, [0.9, 1.0], [0.4, 0.5], [0.8, 1.0]), (c, [0.8, 0.9], [0.4, 0.5], [0.2, 0.7])\},$$

$$R = \{(c, [0.1, 0.9], [0.5, 0.7], [0.4, 1.0]), (d, [0.1, 1.0], [0.9, 1.0], [0.5, 0.9])\}.$$

Proposition 2.6.

The Interval Valued Neutrosophic Hypergraph (IVNHG) is the generalization of fuzzy hypergraph, intuitionistic fuzzy hypergraphs, interval valued fuzzy hypergraph, interval valued intuitionistic fuzzy hypergraph and single valued neutrosophic hypergraph.

### 3 Regular and Totally Regular IVNHGs

Definition 3.1.

The open neighbourhood of a vertex  $x$  in the interval valued neutrosophic hypergraphs (IVNHGs) is the set of adjacent vertices of  $x$ , excluding that vertex, and it is denoted by  $N(x)$ .

Definition 3.2.

The closed neighbourhood of a vertex  $x$  in the interval valued neutrosophic hypergraphs (IVNHGs) is the set of adjacent vertices of  $x$ , including that vertex, and it is denoted by  $N[x]$ .

Example 3.3.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{ a, b, c, d, e \}$  and  $E = \{ P, Q, R, S \}$ , defined by:

$$P = \{(a, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.8])\},$$

$$Q = \{(c, [0.1, 0.7], [0.2, 0.8], [0.3, 0.9]), (d, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7])\},$$

$$(e, [0.7, 0.9], [0.8, 0.9], [0.9, 1.0])\},$$

$$R = \{(b, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9]), (c, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7])\},$$

$$S = \{(a, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7]), (d, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9])\}.$$

Then, the open neighbourhood of a vertex  $a$  is  $b$  and  $d$ .

The closed neighbourhood of a vertex  $b$  is  $b, a$  and  $c$ .

Definition 3.4.

Let  $H = (X, E)$  be an IVNHG; the open neighbourhood degree of a vertex  $x$  is denoted and defined by:

$$deg(x) = ([deg_{TL}(x), deg_{TU}(x)], [deg_{IL}(x), deg_{IU}(x)], [deg_{FL}(x), deg_{FU}(x)]), \quad (1)$$

where:

$$deg_{TL}(x) = \sum_{x \in N(x)} TL_E(x), \quad (2)$$

$$deg_{IL}(x) = \sum_{x \in N(x)} IL_E(x), \quad (3)$$

$$deg_{FL}(x) = \sum_{x \in N(x)} FL_E(x), \quad (4)$$

$$deg_{TU}(x) = \sum_{x \in N(x)} TU_E(x), \quad (5)$$

$$deg_{IU}(x) = \sum_{x \in N(x)} IU_E(x), \quad (6)$$

$$deg_{FU}(x) = \sum_{x \in N(x)} FU_E(x). \quad (7)$$

Example 3.5.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{a, b, c, d, e\}$  and  $E = \{P, Q, R, S\}$ , defined by:

$$P = \{(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\},$$

$$Q = \{(c, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]), (e, [0.7, 0.8], [0.8, 0.9], [0.9, 1.0])\},$$

$$R = \{(b, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (c, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\},$$

$$S = \{(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\}.$$

Then, the open neighbourhood of a vertex  $a$  is  $b$  and  $d$ .

Therefore, the open neighbourhood degree of a vertex  $a$  is  $([0.8, 1.0], [1.0, 1.2], [1.2, 1.4])$ .

Definition 3.6.

Let  $H = (X, E)$  be an IVNHG; the closed neighbourhood degree of a vertex  $x$  is denoted and defined by:

$$deg[x] = ([deg_{TL}[x], deg_{TU}[x]], [deg_{IL}[x], deg_{IU}[x]], [deg_{FL}[x], deg_{FU}[x]]], \tag{8}$$

where:

$$deg_{TL}[x] = deg_{TL}(x) + TL_E(x), \tag{9}$$

$$deg_{IL}[x] = deg_{IL}(x) + IL_E(x), \tag{10}$$

$$deg_{FL}[x] = deg_{FL}(x) + FL_E(x), \tag{11}$$

$$deg_{TU}[x] = deg_{TU}(x) + TU_E(x), \tag{12}$$

$$deg_{IU}[x] = deg_{IU}(x) + IU_E(x), \tag{13}$$

$$deg_{FU}[x] = deg_{FU}(x) + FU_E(x). \tag{14}$$

Example 3.7.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{a, b, c, d, e\}$  and  $E = \{P, Q, R, S\}$ , defined by:

$$P = \{(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\},$$

$$Q = \{(c, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]), (e, [0.7, 0.8], [0.8, 0.9], [0.9, 1.0])\},$$

$$R = \{(b, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (c, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\},$$

$$S = \{(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\}.$$

The closed neighbourhood of a vertex  $a$  is  $a, b$  and  $d$ .

Hence the closed neighbourhood degree of a vertex  $a$  is  $([0.9, 1.2], [1.2, 1.5], [1.5, 1.8])$ .

Definition 3.8.

Let  $H = (X, E)$  be an IVNHG; then  $H$  is said to be a  $n$ -regular IVNHG if all the vertices have the same open neighbourhood degree,

$$n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]). \tag{15}$$

Definition 3.9.

Let  $H = (X, E)$  be an IVNHG; then  $H$  is said to be a  $m$ -totally regular IVNHG if all the vertices have the same closed neighbourhood degree,

$$m = ([m_1, m_2], [m_3, m_4], [m_5, m_6]). \tag{16}$$

Proposition 3.10.

A regular IVNHG is the generalization of regular fuzzy hypergraphs, regular intuitionistic fuzzy hypergraphs, regular interval valued fuzzy hypergraphs and regular interval valued intuitionistic fuzzy hypergraphs.

Proposition 3.11.

A totally regular IVNHG is the generalization of the totally regular fuzzy hypergraphs, totally regular intuitionistic fuzzy hypergraphs, totally regular interval valued fuzzy hypergraphs and totally regular interval valued intuitionistic fuzzy hypergraphs.

Example 3.12.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{a, b, c, d\}$  and  $E = \{P, Q, R, S\}$ , defined by:

$$P = \{(a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\},$$

$$Q = \{(b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\},$$

$$R = \{(c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\},$$

$$S = \{(d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}.$$

Here, the open neighbourhood degree of every vertex is  $([1.6, 1.8], [0.4, 0.6], [0.6, 0.8])$ , hence  $H$  is regular IVNHG and the closed neighbourhood degree of every vertex is  $([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$ . Hence  $H$  is both a regular and a totally regular IVNHG.

Theorem 3.13.

Let  $H = (X, E)$  be an IVNHG which is both a regular and a totally regular IVNHG; then  $E$  is constant.

Proof.

Suppose  $H$  is a  $n$ -regular and a  $m$ -totally regular IVNHG. Then,

$$\text{deg}(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \quad (17)$$

$$\text{deg}[x] = m = ([m_1, m_2], [m_3, m_4], [m_5, m_6]), \quad (18)$$

for all  $x \in E_i$ .

Consider

$$\text{deg}[x] = m, \quad (19)$$

hence, by definition,

$$\text{deg}(x) + E_i(x) = m; \tag{20}$$

this implies that

$$E_i(x) = m - n, \tag{21}$$

for all  $x \in E_i$ .

Hence  $E$  is constant.

Remark 3.14.

The converse of above theorem need not to be true in general.

Example 3.15.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{a, b, c, d\}$  and  $E = \{P, Q, R, S\}$ , defined by:

$$P = \{ (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \},$$

$$Q = \{ (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \},$$

$$R = \{ (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \},$$

$$S = \{ (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}.$$

Here  $E$  is constant, but  $\text{deg}(a) = ([1.6, 1.8], [0.4, 0.6], [0.6, 0.8])$  and  $\text{deg}(d) = ([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$ , i.e  $\text{deg}(a)$  and  $\text{deg}(d)$  are not equals, hence  $H$  is a not regular IVNHG. Next,  $\text{deg}[a] = ([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$  and  $\text{deg}[d] = ([3.2, 3.6], [0.8, 1.2], [1.2, 1.6])$ , hence  $\text{deg}[a]$  and  $\text{deg}[d]$  are not equals, hence  $H$  is not a totally regular IVNHG.

We conclude that  $H$  is neither a regular and nor a totally regular IVNHG.

Theorem 3.16.

Let  $H = (X, E)$  be an IVNHG; then  $E$  is constant on  $X$  if and only if the following are equivalent:

- (1)  $H$  is a regular IVNHG;
- (2)  $H$  is a totally regular IVNHG.

Proof.

Suppose  $H = (X, E)$  is an IVNHG and  $E$  is constant in  $H$ , i.e.:

$$E_i(x) = c = ([c_1, c_2], [c_3, c_4], [c_5, c_6]), \tag{22}$$

for all  $x \in E_i$ .

Suppose  $H$  is a  $n$ -regular IVNHG; then

$$\deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \quad (23)$$

for all  $x \in E_i$ .

Consider

$$\deg[x] = \deg(x) + E_i(x) = n + c, \quad (24)$$

for all  $x \in E_i$ .

Hence,  $H$  is a totally regular IVNHG.

Next, suppose that  $H$  is a  $m$ -totally regular IVNHG; then:

$$\deg[x] = m = ([m_1, m_2], [m_3, m_4], [m_5, m_6]), \quad (25)$$

for all  $x \in E_i$ . i.e.:

$$\deg(x) + E_i(x) = m, \quad (26)$$

for all  $x \in E_i$ .

This implies that

$$\deg(x) = m - c, \quad (27)$$

for all  $x \in E_i$ .

Thus,  $H$  is a regular IVNHG, and consequently (1) and (2) are equivalent.

Conversely.

Assume that (1) and (2) are equivalent, i.e.  $H$  is a regular IVNHG if and only if  $H$  is a totally regular IVNHG.

Suppose by contrary that  $E$  is not constant, that is  $E_i(x)$  and  $E_i(y)$  not equals for some  $x$  and  $y$  in  $X$ . Let  $H = (X, E)$  be a  $n$ -regular IVNHG; then

$$\deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \quad (28)$$

for all  $x \in E_i$ .

Consider:

$$\deg[x] = \deg(x) + E_i(x) = n + E_i(x), \quad (29)$$

$$\deg[y] = \deg(y) + E_i(y) = n + E_i(y), \quad (30)$$

since  $E_i(x)$  and  $E_i(y)$  are not equals for some  $x$  and  $y$  in  $X$ , hence  $\deg[x]$  and  $\deg[y]$  are not equals, thus  $H$  is not a totally regular IVNHG, which is a contradiction to our assumption.

Next, let  $H$  be a totally regular IVNHG, then

$$\deg[x] = \deg[y]. \quad (31)$$

That is

$$\deg(x) + E_i(x) = \deg(y) + E_i(y), \quad (32)$$

$$\deg(x) - \deg(y) = E_i(y) - E_i(x), \quad (33)$$



since RHS of above equation is nonzero, hence LHS of above equation is also nonzero, thus  $deg(x)$  and  $deg(y)$  are not equals, so  $H$  is not a regular IVNHG, which is again a contradiction to our assumption, thus our supposition was wrong, hence  $E$  must be constant, and this completes the proof.

Definition 3.17.

Let  $H = (X, E)$  be a regular IVNHG; then the order of an IVNHG  $H$  is denoted and defined by:

$$O(H) = ([p, q], [r, s], [t, u]), \tag{34}$$

where

$$p = \sum_{x \in X} TL_{E_i}(x), \quad q = \sum_{x \in X} TU_{E_i}(x), \quad r = \sum_{x \in X} IL_{E_i}(x), \tag{35}$$

$$s = \sum_{x \in X} IU_{E_i}(x), \quad t = \sum_{x \in X} FL_{E_i}(x), \quad u = \sum_{x \in X} FU_{E_i}(x), \tag{36}$$

for every  $x \in X$ , and the size of a regular IVNHG is denoted and defined by:

$$S(H) = \sum_{i=1}^n (S_{E_i}), \tag{37}$$

where

$$S(E_i) = ([a, b], [c, d], [e, f]) \tag{38}$$

and

$$a = \sum_{x \in E_i} TL_{E_i}(x), \quad b = \sum_{x \in E_i} TU_{E_i}(x), \quad c = \sum_{x \in E_i} IL_{E_i}(x) \tag{39}$$

$$d = \sum_{x \in E_i} IU_{E_i}(x), \quad e = \sum_{x \in E_i} FL_{E_i}(x), \quad f = \sum_{x \in E_i} FU_{E_i}(x). \tag{40}$$

Example 3.18.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{a, b, c, d\}$  and  $E = \{P, Q, R, S\}$ , defined by:

$$P = \{ (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \},$$

$$Q = \{ (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \},$$

$$R = \{ (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \},$$

$$S = \{ (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}.$$

Here, the order and the size of  $H$  are given,  $([3.2, 3.6], [.8, 1.2], [1.2, 1.6])$ , and  $([6.4, 7.2], [1.6, 2.4], [2.4, 3.2])$  respectively.

Proposition 3.19.

The size of a  $n$ -regular IVNHG  $H = (H, E)$  is  $\frac{nk}{2}$  where  $|X|=k$ .

Proposition 3.20.

If  $H = (X, E)$  is a  $m$ -totally regular IVNHG, then  $2S(H) + O(H) = mk$ , where  $|X| = k$ .

Corollary 3.21.

Let  $H = (X, E)$  be a  $n$ -regular and a  $m$ -totally regular IVNHG; then  $O(H) = k(m - n)$ , where  $|X|=k$ .

Proposition 3.22.

The dual of a  $n$ -regular and a  $m$ -totally regular IVNHG  $H = (X, E)$  is again a  $n$ -regular and a  $m$ -totally regular IVNHG.

Definition 3.23.

The interval valued neutrosophic hypergraph (IVNHG) is said to be a complete IVNHG if for every  $x$  in  $X$ ,  $N(x) = \{ x : x \text{ in } X - \{x\} \}$ ; that is  $N(x)$  contains all remaining vertices of  $X$  except  $x$ .

Example 3.24.

Consider the interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{ a, b, c, d \}$  and  $E = \{ P, Q, R \}$ , defined by:

$$P = \{(a, [0.4, 0.5], [0.6, 0.7], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$$

$$Q = \{(a, [0.8, 1.0], [0.7, 0.9], [0.3, 0.7]), (b, [0.8, 0.9], [0.2, 0.3], [0.1, 0.9]), (d, [0.8, 0.9], [0.2, 0.5], [0.1, 0.9])\}$$

$$R = \{(c, [0.4, 0.6], [0.9, 1.0], [0.9, 1.0]), (d, [0.7, 0.9], [0.2, 0.7], [0.1, 0.7]), (b, [0.4, 0.6], [0.2, 0.7], [0.1, 0.8])\}$$

Here,  $N(a) = \{b, c, d\}$ ,  $N(b) = \{a, c, d\}$ ,  $N(c) = \{a, b, d\}$ ,  $N(d) = \{a, b, c\}$ . Hence  $H$  is a complete IVNHG.

Remark 3.25.

In a complete IVNHG  $H = (X, E)$ , the cardinality of  $N(x)$  is the same for every vertex.

Theorem 3.26.

Every complete IVNHG  $H = (X, E)$  is both a regular and a totally regular if  $E$  is constant in  $H$ .

Proof.

Let  $H = (X, E)$  be a complete IVNHG; suppose  $E$  is constant in  $H$ .

Consequently:

$$E_i(x) = c = ([c_1, c_2], [c_3, c_4], [c_5, c_6]), \quad (41)$$

for all  $x \in E_i$ ; since IVNHG is complete, then by definition for every vertex  $x$  in  $X$ ,  $N(x) = \{x : x \text{ in } X - \{x\}\}$ , and the open neighbourhood degree of every vertex is same, that is:

$$deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \quad (42)$$

for all  $x \in E_i$ .

Hence, a complete IVNHG is a regular IVNHG.

Also,

$$deg[x] = deg(x) + E_i(x) = n + c \quad (43)$$

for all  $x \in E_i$ .

Hence  $H$  is a totally regular IVNHG.

Remark 3.27.

Every complete IVNHG is totally regular even if  $E$  is not constant.

Definition 3.28.

An IVNHG is said to be  $k$ -uniform if all the hyper-edges have the same cardinality.

Example 3.29.

Consider an interval valued neutrosophic hypergraphs  $H = (X, E)$ , where  $X = \{a, b, c, d\}$  and  $E = \{P, Q, R\}$ , defined by:

$$P = \{(a, [0.8, 0.9], [0.4, 0.7], [0.2, 0.7]), (b, [0.7, 0.9], [0.5, 0.8], [0.3, 0.9])\},$$

$$Q = \{(b, [0.9, 1.0], [0.4, 0.5], [0.8, 1.0]), (c, [0.8, 0.9], [0.4, 0.5], [0.2, 0.7])\},$$

$$R = \{(c, [0.1, 0.9], [0.5, 0.7], [0.4, 1.0]), (d, [0.1, 1.0], [0.9, 1.0], [0.5, 0.9])\}.$$

## 4 Conclusion

The theoretical concepts of graphs and hypergraphs are highly used in computer science applications. The interval valued neutrosophic hypergraphs are more flexible than the fuzzy hypergraphs and the intuitionistic fuzzy hypergraphs, the interval valued fuzzy hypergraphs and the interval valued intuitionistic fuzzy hypergraphs. The concept of interval valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science.

In this paper, we defined the regular and the totally regular interval valued neutrosophic hypergraphs.

We plan to extend our research work to the irregular interval valued neutrosophic hypergraphs.

## 5 References

- [1] A. V. Devadoss, A. Rajkumar & N. J. P. Praveena. *A Study on Miracles through Holy Bible using Neutrosophic Cognitive Maps (NCMS)*. In: International Journal of Computer Applications, 69(3) (2013).
- [2] A. Nagoor Gani and M. B. Ahamed. *Order and Size in Fuzzy Graphs*. In: Bulletin of Pure and Applied Sciences, Vol 22E (No.1) (2003) 145-148.
- [3] A. N. Gani. A. and S. Shajitha Begum. *Degree, Order and Size in Intuitionistic Fuzzy Graphs*. In: Intl. Journal of Algorithms, Computing and Mathematics, (3)3 (2010).
- [4] A. Nagoor Gani and S.R Latha. *On Irregular Fuzzy Graphs*. In: Applied Mathematical Sciences, Vol. 6, no.11 (2012) 517-523.
- [5] F. Smarandache. *Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies*. In: Neutrosophic Sets and Systems, Vol. 9 (2015) 58- 63.
- [6] F. Smarandache. *Types of Neutrosophic Graphs and Neutrosophic Algebraic Structures together with their Applications in Technology*, Seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania, June, 6th, 2015.
- [7] F. Smarandache. *Symbolic Neutrosophic Theory*. Brussels: Europanova, 2015, 195 p.
- [8] F. Smarandache. *Neutrosophic set - a generalization of the intuitionistic fuzzy set*. In: Granular Computing, 2006 IEEE Intl. Conference, (2006) 38 - 42, DOI: 10.1109/GRC.2006.1635754.
- [9] H. Wang, F. Smarandache, Y. Zhang, R. Sunderraman. *Single Valued Neutrosophic Sets*. In: Multispace and Multistructure, 4 (2010) 410-413.
- [10] H. Wang, F. Smarandache, Zhang, Y.-Q. and R. Sunderraman. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*. Phoenix: Hexis, 2005.
- [11] I. Turksen. *Interval valued fuzzy sets based on normal forms*. In: Fuzzy Sets and Systems, vol. 20(1986) 191-210.
- [12] K. Atanassov. *Intuitionistic fuzzy sets*. In: Fuzzy Sets and Systems. vol. 20 (1986) 87-96.
- [13] K. Atanassov and G. Gargov. *Interval valued intuitionistic fuzzy sets*. In: Fuzzy Sets and Systems, vol. 31 (1989) 343-349.
- [14] L. Zadeh. *Fuzzy sets*. In: Information and Control, 8 (1965) 338-353.
- [15] M. Akram and B. Davvaz. *Strong intuitionistic fuzzy graphs*. In: Filomat, vol. 26, no. 1 (2012) 177-196.
- [16] M. Akram and W. A. Dudek. *Interval-valued fuzzy graphs*. In: Computers & Mathematics with Applications, vol. 61, no. 2 (2011) 289-299.

- [17] M. Akram. *Interval-valued fuzzy line graphs*. In: Neural Comp. and Applications, vol. 21 (2012) 145-150.
- [18] M. Akram. *Bipolar fuzzy graphs*. In: Information Sciences, vol. 181, no. 24 (2011) 5548-5564.
- [19] M. Akram. *Bipolar fuzzy graphs with applications*. In: Knowledge Based Systems, vol. 39 (2013) 1-8.
- [20] M. Akram and A. Adeel. *m-polar fuzzy graphs and m-polar fuzzy line graphs*. In: Journal of Discrete Mathematical Sciences and Cryptography, 2015.
- [21] M. Akram, W. A. Dudek. *Regular bipolar fuzzy graphs*. In: Neural Computing and Applications, vol. 21, pp. 97-205 (2012).
- [22] M. Akram, W.A. Dudek, S. Sarwar. *Properties of Bipolar Fuzzy Hypergraphs*. In: Italian Journal of Pure and Applied Mathematics, no. 31 (2013), 141-161.
- [23] M. Akram, N. O. Alshehri, and W. A. Dudek. *Certain Types of Interval-Valued Fuzzy Graphs*. In: Journal of Appl. Mathematics, 2013, 11 pages, <http://dx.doi.org/10.1155/2013/857070>.
- [24] M. Akram, M. M. Yousaf, W. A. Dudek. *Self-centered interval-valued fuzzy graphs*. In: Afrika Matematika, vol. 26, Issue 5, pp 887-898, 2015.
- [25] P. Bhattacharya. *Some remarks on fuzzy graphs*. In: Pattern Recognition Letters 6 (1987) 297-302.
- [26] R. Parvathi and M. G. Karunambigai. *Intuitionistic Fuzzy Graphs*. In: Computational Intelligence. In: Theory and applications, International Conference in Germany, Sept 18 -20, 2006.
- [27] R. A. Borzooei, H. Rashmanlou. *More Results On Vague Graphs*, U.P.B. Sci. Bull., Series A, Vol. 78, Issue 1, 2016, 109-122.
- [28] S. Broumi, M. Talea, F. Smarandache, A. Bakali. *Single Valued Neutrosophic Graphs: Degree, Order and Size*, FUZZ IEEE Conference (2016), 8 page.
- [29] S. Broumi, M. Talea, A. Bakali, F. Smarandache. *Single Valued Neutrosophic Graphs*. In: Journal of New Theory, no. 10, 68-101 (2016).
- [30] S. Broumi, M. Talea, A. Bakali, F. Smarandache. *On Bipolar Single Valued Neutrosophic Graphs*. In: Journal of New Theory, no. 11, 84-102 (2016).
- [31] S. Broumi, M. Talea, A. Bakali, F. Smarandache. *Interval Valued Neutrosophic Graphs*. SISOM Conference (2016), in press.
- [32] S. Broumi, F. Smarandache, M. Talea, A. Bakali. *An Introduction to Bipolar Single Valued Neutrosophic Graph Theory*. OPTIROB conference, 2016.
- [33] S. Broumi, M. Talea, A. Bakali, F. Smarandache. *Operations on Interval Valued Neutrosophic Graphs* (2016), submitted.
- [34] S. Broumi, M. Talea, A. Bakali, F. Smarandache. *Strong Interval Valued Neutrosophic Graphs* (2016), submitted.
- [35] S. N. Mishra and A. Pal. *Product of Interval Valued Intuitionistic fuzzy graph*. In: Annals of Pure and Applied Mathematics, Vol. 5, No. 1 (2013) 37-46.
- [36] S. Rahurikar. *On Isolated Fuzzy Graph*. In: Intl. Journal of Research in Engineering Technology and Management, 3 pages.
- [37] W. B. Vasantha Kandasamy, K. Ilanthenral and F. Smarandache. *Neutrosophic Graphs: A New Dimension to Graph Theory*.

- [38] I. Prandeepta and S. Vimala, *Regular and totally regular intuitionistic fuzzy hypergraphs*. In: *International Journal of Mathematics and Applications*, volume 4, issue 1-C (2016), 137-142.