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Rough Neutrosophic TOPSIS for Multi-Attribute Group Decision Making

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Abstract: This paper is devoted to present Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method for multi-attribute group decision making under rough neutrosophic environment. The concept of rough neutrosophic set is a powerful mathematical tool to deal with uncertainty, indeterminacy and inconsistency. In this paper, a new approach for multi-attribute group decision making problems is proposed by extending the TOPSIS method under rough neutrosophic environment. Rough neutrosophic set is characterized by the upper and lower approximation operators and the pair of neutrosophic sets that are characterized by truth-membership degree, indeterminacy membership degree, and falsity membership degree. In the decision situation, ratings of alternatives with respect to each attribute are characterized by rough neutrosophic sets that reflect the decision makers' opinion. Rough neutrosophic weighted averaging operator has been used to aggregate the individual decision maker's opinion into group opinion for rating the importance of attributes and alternatives. Finally, a numerical example has been provided to demonstrate the applicability and effectiveness of the proposed approach.

Keywords: Multi-attribute group decision making; Neutrosophic set; Rough set; Rough neutrosophic set; TOPSIS

1 Introduction

Hwang and Yoon [1] put forward the concept of Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in 1981 to help select the best alternative with a finite number of criteria. Among numerous multi criteria decision making (MCDM) methods developed to solve real-world decision problems, (TOPSIS) continues to work satisfactorily in diverse application areas such as supply chain management and logistics [2, 3, 4, 5], design, engineering and manufacturing systems [6, 7], business and marketing management [8, 9], health, safety and environment management[10, 11], human resources management [12, 13, 14], energy management [15], chemical engineering [16], water resources management [17, 18], bi-level programming problem [19, 20], multi-level programmi ng problem [21], medical diagnosis [22], military [23], education [24], others topics [25, 26, 27, 28, 29, 30], etc. Behzadian et al. [31] provided a state-of the-art literature survey on TOPSIS applications and methodologies. According to C. T. Chen [32], crisp data are inadequate to model real-life situations because human judgments including preferences are often vague. Preference information of alternatives provided by the decision makers may be poorly defined, partially known and incomplete. The concept of fuzzy set theory grounded by L. A. Zadeh [33] is capable of dealing with impreciseness in a mathematical form. Interval valued fuzzy set [34, 35, 361, 37] was proposed by several authors independently in 1975 as a generalization of fuzzy set. In 1986, K. T. Atanassov [38] introduced the concept of intuitionistic fuzzy set (IFS) by incorporating non-membership degree as independent entity to deal non-statistical impreciseness. In 2003, mathematical equivalence of intuitionistic fuzzy set (IFS) with interval-valued fuzzy sets was proved by Deschrijver and Kerre [39]. C. T. Chen [32] extended the TOPSIS method in fuzzy environment for solving multi-attribute decision making problems. Boran et al. [12] extended the TOPSIS method for multi-criteria intuitionistic decision making problem. However, fuzzy sets and interval fuzzy sets are not capable of all types of uncertainties in different real physical problems involving indeterminate information.

In order to deal with indeterminate and inconsistent information, the concept of neutrosophic set [40, 41, 42, 43] proposed by F. Smarandache is useful. In neutrosophic set each element of the universe is characterized by the truth degree, indeterminacy degree and falsity degree lying in the non-standard unit interval $]^{-}0, 1^{+}[$. However, it is difficult to apply directly the neutrosophic set in real engineering and scientific applications. Smarandache [43] then Wang et al. [44] introduced single-valued neutrosophic set (SVNS) to face real scientific and engineering fields involving imprecise, incomplete, and inconsistent information. SVNS is a subclass of NS can also represent each element of universe with the truth values, indeterminacy values and falsity values lying in the real unit interval [0, 1]. SVNS has caught attention to the researchers on various topics such as, medical diagnosis [45], similarity measure [46, 47, 48, 49, 50], decision making [51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69], educational problem [70, 71], conflict resolution [72], social problem [73, 74], optimization [75, 76, 77, 78, 79,. 80], etc.

Pawlak [81] proposed the notion of rough set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful mathematical tool for dealing with uncertainty or incomplete information. Broumi et al. [82, 83] proposed new hybrid intelligent structure called rough neutrosophic set by combining the concepts of single valued neutrosophic set and rough set. The theory of rough neutrosophic set [82, 83] is also a powerful mathematical tool to deal with incompleteness. Rough neutrosophic set can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. In rough neutrosophic environment, Mondal and Pramanik [84] proposed rough neutrosophic multi-attribute decision-making based on grey relational analysis. Mondal and Pramanik [85] also proposed rough neutrosophic multi-attribute decision-making based on rough accuracy score function. Pramanik and Mondal [86] proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Pramanik and Mondal [87] also proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. The same authors [88] proposed some similarity measures namely Dice and Jaccard similarity measures in rough neutrosophic environment and applied them for multi attribute decision making problem. Pramanik and Mondal [89] studied decision making in rough interval neutrosophic environment in 2015. Mondal and Pramanik [90] studied cosine, Dice and Jaccard similarity measures for interval rough neutrosophic sets and they also presented their applications in decision making problem. So decision making in rough neutrosophic environment appears to be a developing area of study.

The objective of the study is to extend the concept of TOPSIS method for multi-attribute group decision making (MAGDM) problems under single valued neutrosophic rough neutrosophic environment. All information provided by different domain experts in MAGDM problems about alternative and attribute values take the form of rough neutrosophic set. In a group decision making process, rough neutrosophic weighted averaging operator needs to be used to aggregate all the decision makers' opinions into a single opinion to select best alternative.

The remaining part of the paper is organized as follows: section 2 presents some preliminaries relating to neutrosophic set, section 3 presents the concept of rough neutrosophic set. In section 4, basics of TOPSIS method are discussed. Section 5 is devoted to present TOPSIS method for MAGDM under rough neutrosophic environment. In section 6, a numerical example is provided to show the effectiveness of the proposed approach. Finally, section 7 presents the concluding remarks and scope of future research.

2 Neutrosophic sets and single valued neutrosophic set [43, 44]

2.1 Definition of Neutrosophic sets [40, 41, 42, 43]

Definition 2.1.1. [43]:

Assume that V be a space of points and v be a generic element in V. Then a neutrosophic set G in V is characterized by a truth membership function T_G, an indeterminacy membership function I_G and a falsity membership function F_G. The functions T_G, I_G and F_G are real standard or non-standard subsets of $]^-0, 1^+[$ i.e. $T_G: V \to]^-0, 1^+[$, $I_G: V \to]^-0, 1^+[$, $F_G: V \to]^-0, 1^+[$, and $^-0 \le T_G(v) + I_G(v) + F_G(v) \le 3^+$.

2.1.2.[43]:

The complement of a neutrosophic set G is denoted by G^c and is defined by

$T_{G^c}(v) = \{1^+\} - T_G(v)$; $I_{G^c}(v) = \{1^+\} - I_G(v)$; $F_{G^c}(v) = \{1^+\} - I_G(v)$

Definition 2.1.3. [43]:

A neutrosophic set G is contained in another neutrosophic set H, $G \subseteq H$ iff the following conditions holds.

$\inf T_G(v) \le \inf T_H(v)$ $\sup T_G(v) \le \sup T_H(v)$

$\inf I_G(v) \ge \inf I_H(v)$, $\sup I_G(v) \ge \sup I_H(v)$

$\inf F_G(v) \ge \inf F_H(v)$, $\sup F_G(v) \ge \sup F_H(v)$

for all v in V.

Definition 2.1.4. [44]:

Assume that V be a universal space of points, and v be a generic element of V. A single-valued neutrosophic set P is characterized by a true membership function $T_P(v)$, a falsity membership function $I_P(v)$, and an indeterminacy function $F_P(v)$. Here, $T_P(v)$, $I_P(v)$, $F_P(v) \in [0, 1]$.

$P = \sum \langle T_P(v), F_P(v), I_P(v) \rangle / v, \ \forall v \in V$

It is obvious that for a SVNS P,

$0 \le \sup T_P(v) + \sup F_P(v) + \sup I_P(v) \le 3, \ \forall v \in V$

Definition 2.1.5. [44]:

The complement of a SVNS set P is denoted by P^C and is defined as follows:

$T_P^{C}(v) = F_P(v)$; $I_P^{C}(v) = 1 - I_P(v)$; $F_P^{C}(v) = T_P(v)$

Definition 2.1.6. [44]:
A SVNS P_G is contained in another SVNS P_H, denoted as $P_G \subseteq P_H$ if the following conditions hold.

$T_{P_G}(v) \le T_{P_H}(v)$; $I_{P_G}(v) \ge I_{P_H}(v)$; $F_{P_G}(v) \ge F_{P_H}(v)$, $\forall\ v \in V$.

Definition 2.1.7. [44]:
Two SVNSs P_G and P_H are equal, i.e., $P_G = P_H$, iff $P_G \subseteq P_H$ and $P_G \supseteq P_H$

Definition 2.1.8. [44]:
The union of two SVNSs P_G and P_H is a SVNS P_Q, written as $P_Q = P_G \cup P_H$.

Its truth, indeterminacy and falsity membership functions are as follows:

$T_{P_Q}(v) = \max(T_{P_G}(v), T_{P_H}(v))$;

$I_{P_Q}(v) = \min(I_{P_G}(v), I_{P_H}(v))$;

$F_{P_G}(v) = \min(F_{P_G}(v), F_{P_H}(v))$, $\forall\ v \in V$.

Definition 2.1.9. [44]:
The intersection of two SVNSs P_G and P_H is a SVNS P_C written as $P_C = P_G \cap P_H$. Its truth, indeterminacy and falsity membership functions are as follows:

$T_{P_C}(v) = \min(T_{P_G}(v), T_{P_H}(v))$;

$P_C(v) = \max(I_{P_G}(v), I_{P_H}(v))$;

$F_{P_C}(v) = \max(F_{P_G}(v), F_{P_H}(v))$, $\forall\ v \in V$.

Definition 2.1.10. [44]:
Wang et al. [44] defined the following operation for two SVNS P_G and P_H as follows:

$$P_G \otimes P_H = \left\langle \begin{array}{l} T_{P_G}(v).T_{P_H}(v),\ I_{P_G}(v) + I_{P_H}(v) - I_{P_G}(v).I_{P_H}(v), \\ F_{P_G}(v) + F_{P_H}(v) - F_{P_G}(v).F_{P_H}(v) \end{array} \right\rangle,$$

$\forall\ v \in V$.

Definition 2.1.11. [91]
Assume that

$$P_G = \left\{ \begin{array}{l} \left(v_1 / \left(T_{P_G}(v_1), I_{P_G}(v_1), F_{P_G}(v_1)\right)\right), \cdots, \\ \left(v_n / \left(T_{P_G}(v_n), I_{P_G}(v_n), F_{P_G}(v_n)\right)\right) \end{array} \right\}$$

$$P_H = \left\{ \begin{array}{l} \left(v_1 / \left(T_{P_H}(v_1), I_{P_H}(v_1), F_{P_H}(v_1)\right)\right), \cdots, \\ \left(v_n / \left(T_{P_H}(v_n), I_{P_H}(v_n), F_{P_H}(v_n)\right)\right) \end{array} \right\}$$

be two SVNSs in v = $\{v_1, v_2, v_3, \ldots, v_n\}$

Then the Hamming distance [91] between two SVNSs P_G and P_H is defined as follows:

$$d_P(P_G, P_H) = \sum_{i=1}^{n} \left\langle \begin{array}{l} \left|T_{P_G}(v) - T_{P_H}(v)\right| + \left|I_{P_G}(v) - I_{P_H}(v)\right| \\ + \left|F_{P_G}(v) - F_{P_H}(v)\right| \end{array} \right\rangle \quad (1)$$

and normalized Hamming distance [91] between two SVNSs P_G and P_H is defined as follow

$$^{N}d_P(P_G, P_H) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{array}{l} \left|T_{P_G}(v) - T_{P_H}(v)\right| + \left|I_{P_G}(v) - I_{P_H}(v)\right| \\ + \left|F_{P_G}(v) - F_{P_H}(v)\right| \end{array} \right\rangle \quad (2)$$

with the following two properties

i. $0 \le d_P(P_G, P_H) \le 3$

ii. $0 \le {}^{N}d_P(P_G, P_H) \le 1$

Distance between two SVNSs:
Majumder and Samanta [91] studied similarity and entropy measure by incorporating Euclidean distances of SVNSs.

Definition 2.1.12. [91]: (Euclidean distance)

Let $P_G = \left\{ \begin{array}{l} \left(v_1 | \left\langle T_{P_G}(v_1),\ I_{P_G}(v_1),\ F_{P_G}(v_1) \right\rangle\right), \cdots, \\ \left(v_m | \left\langle T_{P_G}(v_m),\ I_{P_G}(v_m),\ F_{P_G}(v_m) \right\rangle\right) \end{array} \right\}$ and

$P_H = \left\{ \begin{array}{l} \left(v_1 | \left\langle T_{P_H}(v_1),\ I_{P_H}(v_1),\ F_{P_H}(v_1) \right\rangle\right), \cdots, \\ \left(v_m | \left\langle T_{P_H}(v_m),\ I_{P_H}(v_m),\ F_{P_H}(v_m) \right\rangle\right) \end{array} \right\}$ be two SVNSs

for $v_i \in V$, where i = 1, 2, . . . , n. Then the Euclidean distance between two SVNSs P_G and P_H can be defined as follows:

$$D_{euclid}(P_G, P_H) = \left\langle \sum_{i=1}^{n} \left(\begin{array}{l} \left(T_{P_G}(v_i) - T_{P_H}(v_i)\right)^2 + \left(I_{P_G}(v_i) - I_{P_H}(v_i)\right)^2 \\ + \left(F_{P_G}(v_i) - F_{P_H}(v_i)\right)^2 \end{array} \right) \right\rangle^{0.5} \quad (3)$$

and the normalized Euclidean distance [39] between two SVNSs P_G and P_H can be defined as follows:

$$D^{N}_{euclid}(P_G, P_H) = \frac{1}{3n} \left\langle \sum_{i=1}^{n} \left(\begin{array}{l} \left(T_{P_G}(v_i) - T_{P_H}(v_i)\right)^2 + \left(I_{P_G}(v_i) - I_{P_H}(v_i)\right)^2 \\ + \left(F_{P_G}(v_i) - F_{P_H}(v_i)\right)^2 \end{array} \right) \right\rangle^{0.5} \quad (4)$$

Definition 2.1.13. (Deneutrosophication of SVNS) [69]:
Deneutrosophication of SVNS P_G can be defined as a process of mapping P_G into a single crisp output $\theta^* \in V$ i.e. $f: P_G \to \theta^*$ for $v \in V$. If P_G is discrete set then the vector $P_G = \left\{ v \,|\, \left\langle T_{P_G}(v), I_{P_G}(v), F_{P_G}(v) \right\rangle \,|\, v \in V \right\}$ is reduced to a single scalar quantity $\theta^* \in V$ by deneutrosophication. The obtained scalar quantity $\theta^* \in V$ best represents the aggregate distribution of three membership degrees of neutrosophic element $\left\langle T_{P_G}(v), I_{P_G}(v), F_{P_G}(v) \right\rangle$

3 Definitions on rough neutrosophic set [82, 83]

Rough set theory has been developed based on two basic components. The components are crisp set and equivalence relation. The rough set logic is based on the approximation of sets by a couple of sets. These two are known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Rough neutrosophic sets [82, 83] are the generalization of rough fuzzy sets [92, 923, 94] and rough intuitionistic fuzzy sets [95].

Definition 3.1.
Assume that S be a non-null set and ρ be an equivalence relation on S. Assume that E be neutrosophic set in S with the membership function T_E, indeterminacy function I_E and non-membership function F_E. The lower and the upper

approximations of E in the approximation (S, ρ) denoted by $\underline{L}(E)$ and $\overline{U}(E)$ are respectively defined as follows:

$$\underline{L}(E) = \left\langle < v, T_{\underline{L}(E)}(v), I_{\underline{L}(E)}(v), F_{\underline{L}(E)}(v) > / s \in [v]_\rho, v \in S \right\rangle \quad (5)$$

$$\overline{U}(E) = \left\langle < v, T_{\overline{U}(E)}(v), I_{\overline{U}(E)}(v), F_{\overline{U}(E)}(v) > / s \in [v]_\rho, v \in S \right\rangle \quad (6)$$

Here, $T_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho\, T_E(s)$, $I_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho\, I_E(s)$, $F_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho\, F_E(s)$, $T_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho\, T_E(s)$, $I_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho\, T_E(s)$, $F_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho\, I_E(s)$.

So, $0 \le T_{\underline{L}(E)}(v) + I_{\underline{L}(E)}(v) + F_{\underline{L}(E)}(v) \le 3$

$0 \le T_{\overline{U}(E)}(v) + I_{\overline{U}(E)}(v) + F_{\overline{U}(E)}(v) \le 3$

The symbols $\vee$ and $\wedge$ indicate “max” and “min’’ operators respectively. $T_E(s)$, $I_E(s)$ and $F_E(s)$ represent the membership , indeterminacy and non-membership of S with respect to E. $\underline{L}(E)$ and $\overline{U}(E)$ are two neutrosophic sets in S.

Thus the mapping $\underline{L}$, $\overline{U}$: N(S) $\rightarrow$ N(S) are, respectively, referred to as the lower and upper rough neutrosophic approximation operators, and the pair $(\underline{L}(E), \overline{U}(E))$ is called the rough neutrosophic set in (S, ρ).

$\underline{L}(E)$ and $\overline{U}(E)$ have constant membership on the equivalence classes of ρ if $\underline{L}(E) = \overline{U}(E)$; i.e. $T_{\underline{L}(E)}(v) = T_{\overline{U}(E)}(v)$, $I_{\underline{L}(E)}(v) = I_{\overline{U}(E)}(v)$, $F_{\underline{L}(E)}(v) = F_{\overline{U}(E)}(v)$ for any v belongs to S.

E is said to be definable neutrosophic set in the approximation (S, ρ). It is obvious that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 3.2.

If N(E) = ($\underline{L}(E), \overline{U}(E)$) be a rough neutrosophic set in (S, ρ), the complement of N(E) is the rough neutrosophic set and is denoted as $\sim N(E) = (\underline{L}(E)^C, \overline{U}(E)^C)$,where $\underline{L}(E)^C, \overline{U}(E)^C$ are the complements of neutrosophic sets of $\underline{L}(E), \overline{U}(E)$ respectively.

$\underline{L}(E)^c = \left\langle < v, T_{\underline{L}(E)}(v), 1 - I_{\underline{L}(E)}(v), F_{\underline{L}(E)}(v) > / \ v \in S \right\rangle$ and

$\overline{U}(E)^c = \left\langle < v, T_{\overline{U}(E)}(v), 1 - I_{\overline{U}(E)}(v), F_{\overline{U}(E)}(v) > / \ v \in S \right\rangle$

Definition 3. 3.

If $N(E_1)$ and $N(E_2)$ be two rough neutrosophic sets in S, then the following definitions hold:

$N(E_1) = N(E_2) \Leftrightarrow \underline{L}(E_1) = \underline{L}(E_2) \wedge \overline{U}(E_1) = \overline{U}(E_2)$

$N(E_1) \subseteq N(E_2) \Leftrightarrow \underline{L}(E_1) \subseteq \underline{L}(E_2) \wedge \overline{U}(E_1) \subseteq \overline{U}(E_2)$

$N(E_1) \bigcup N(E_2) = < \underline{L}(E_1) \bigcup \underline{L}(E_2), \overline{U}(E_1) \bigcup \overline{U}(E_2) >$

$N(E_1) \bigcap N(E_2) = < \underline{L}(E_1) \bigcap \underline{L}(E_2), \overline{U}(E_1) \bigcap \overline{U}(E_2) >$

$N(E_1) + N(E_2) = < \underline{L}(E_1) + \underline{L}(E_2), \overline{U}(E_1) + \overline{U}(E_2) >$

$N(E_1) \,.\, N(E_2) = < \underline{L}(E_1) \,.\, \underline{L}(E_2), \overline{U}(E_1) \,.\overline{U}(E_2) >$

If α, β, γ be rough neutrosophic sets in (S, ρ), then the following properties are satisfied.

Properties I:

1. $\sim \alpha(\sim \alpha) = \alpha$
2. $\alpha \cup \beta = \beta \cup \alpha$, $\beta \cup \alpha = \alpha \cup \beta$
3. $(\gamma \cup \beta) \cup \alpha = \gamma \cup (\beta \cup \alpha)$, $(\gamma \cap \beta) \cap \alpha = \gamma \cap (\beta \cap \alpha)$
4. $(\gamma \cup \beta) \cap \alpha = (\gamma \cup \beta) \cap (\gamma \cup \alpha)$, $(\gamma \cap \beta) \cup \alpha = (\gamma \cap \beta) \cup (\gamma \cap \alpha)$

Properties II:

De Morgan’s Laws are satisfied for rough neutrosophic sets

1. $\sim (N(E_1) \cup N(E_2)) = (\sim N(E_1)) \cap (\sim N(E_2))$
2. $\sim (N(E_1) \cap N(E_2)) = (\sim N(E_1)) \cup (\sim N(E_2))$

Properties III:

If E_1 and E_2 are two neutrosophic sets of universal collection (U) such that $E_1 \subseteq E_2$, *then* 1. $N(E_1) \subseteq N(E_2)$

2. $N(E_1 \bigcap E_2) \subseteq N(E_2) \bigcap N(E_2)$
3. $N(E_1 \bigcup E_2) \supseteq N(E_2) \bigcup N(E_2)$

Properties IV:

1. $\underline{L}(E) = \sim \overline{U}(\sim E)$
2. $\overline{U}(E) = \sim \underline{L}(\sim E)$
3. $\underline{L}(E) \subseteq \overline{U}(E)$

4 TOPSIS

TOPSIS is used to determine the best alternative from the compromise solutions. The best compromise solution should have the shortest Euclidean distance from the positive ideal solution (PIS) and the farthest Euclidean distance from the negative ideal solution (NIS). The TOPSIS method can be described as follows. Let K = {K_1, K_2, …,K_m} be the set of alternatives, L = {L_1, L_2, …, L_n} be the set of criteria and P = $\{p_{ij} : i = 1, 2, \ldots, m ; j = 1, 2, \ldots, n\}$ be the performance ratings with the criteria weight vector W = { w_1, w_2, …, w_n }.

The procedure of TOPSIS method is presented with following steps:

Step 1. Normalization the decision matrix

Calculation of the normalized value $[\vartheta]_{ij}^N$ is as follows:

For benefit criteria, $\vartheta_{ij} = (\vartheta_{ij} - \vartheta_j^-)/(\vartheta_j^+ - \vartheta_j^-)$,

where $\vartheta_j^+ = \max_i (v_{ij})$ and $\vartheta_j^- = \min_i (v_{ij})$

or setting ϑ_j^+ is the desired level and ϑ_j^- is the worst level.

For cost criteria, $\vartheta_{ij} = (\vartheta_j^- - \vartheta_{ij})/(\vartheta_j^- - \vartheta_j^+)$

Step 2. Weighted normalized decision matrix

In the weighted normalized decision matrix, the upgraded ratings are calculated as follows:

$\eta_{ij} = w_j \times \eta_{ij}$ for i = 1, 2, . . . , m and j = 1, 2, . . . , n. Here w_j is the weight of the j-th criteria such that $w_j \geq 0$ for j = 1, 2, . . . , n and $\sum_{j=1}^{n} w_j = 1$

Step 3. The positive and the negative ideal solutions

The positive ideal solution (PIS) and the negative ideal solution (NIS) are calculated as follows:

$$PIS = M^{+} = \left\langle \eta_1^{+}, \eta_2^{+}, \cdots \eta_n^{+} \right\rangle =$$

$$\left\langle \left(\max_j \eta_{ij} / j \in C_1 \right), \left(\min_j \eta_{ij} / j \in C_2 \right) : j = 1, 2, \cdots, n \right\rangle \text{ and}$$

$$NIS = M^{-} = \left\langle \eta_1^{-}, \eta_2^{-}, \cdots \eta_n^{-} \right\rangle =$$

$$\left\langle \left(\min_j \eta_{ij} / j \in C_1 \right), \left(\max_j \eta_{ij} / j \in C_2 \right) : j = 1, 2, \cdots, n \right\rangle$$

where $C_1$and C_2 are the benefit and cost type criteria respectively.

Step 4. Calculation of the separation measures for each alternative from the PIS and the NIS

The separation values for the PIS and the separation values for the NIS can be determined by using the n-dimensional Euclidean distance as follows:

$$\delta_i^{+} = \left\langle \sum_{j=1}^{n} \left(\eta_{ij} - \eta_j^{+} \right)^2 \right\rangle^{0.5} \text{ for i = 1, 2, . . . , m.}$$

$$\delta_i^{-} = \left\langle \sum_{j=1}^{n} \left(\eta_{ij} - \eta_j^{-} \right)^2 \right\rangle^{0.5} \text{ for i = 1, 2, . . . , m.}$$

Step 5. Calculation of the relative closeness coefficient to the PIS

The relative closeness coefficient for the alternative K_i with respect to M^+ is

$$\chi_i = \frac{\delta_i^{-}}{(\delta_i^{+} + \delta_i^{-})} \text{ for i = 1, 2, . . . m.}$$

Step 6. Ranking the alternatives

According to relative closeness coefficient to the ideal alternative, larger value of χ_i reflects the better alternative K_i.

5 Topsis method for multi-attribute decision making under rough neutrosophic environment

Assume that a multi-attribute decision-making problem with m alternatives and n attributes. Let K = (K_1, K_2,..., K_m) be a set of alternatives, and L = (L_1, L_2, ..., L_n) be the set of attributes. The rating measured by the decision maker describes the performance of alternative K_i against attribute L_j. Let W = {w_1, w_2 . . . , w_n} be the weight vector assigned for the attributes L_1, L_2, ..., L_n by the decision makers. The values associated with the alternatives for multi-attribute decision-making problem (MADM) with respect to the attributes can be presented as follows.

Table1: Rough neutrosophic decision matrix

$$D = \left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle_{m \times n} =$$

$$\begin{array}{c|cccc} & L_1 & L_2 & \cdots & L_n \\ \hline K_1 & \langle \underline{d}_{11}, \overline{d}_{11} \rangle & \langle \underline{d}_{12}, \overline{d}_{12} \rangle & \ldots & \langle \underline{d}_{1n}, \overline{d}_{1n} \rangle \\ K_2 & \langle \underline{d}_{21}, \overline{d}_{21} \rangle & \langle \underline{d}_{22}, \overline{d}_{22} \rangle & \ldots & \langle \underline{d}_{2n}, \overline{d}_{2n} \rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ K_m & \langle \underline{d}_{m1}, \overline{d}_{m1} \rangle & \langle \underline{d}_{m2}, \overline{d}_{m2} \rangle & \ldots & \langle \underline{d}_{mn}, \overline{d}_{mn} \rangle \end{array} \quad (7)$$

Here $\left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle$ is the rough neutrosophic number according to the i-th alternative and the j-th attribute.

In a decision-making situation, there exist many attributes of alternatives. Some of them are important and others may be less important. So it is necessary to select the proper attributes for decision-making problems. The most important attributes may be selected with expert opinions.

Definition 5.1. Accumulated geometric operator (AGO) [84]

Assume a rough neutrosophic number in the form: $\left\langle \underline{L}_{ij}(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), \overline{U}_{ij}(\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}) \right\rangle$. We transfer the rough neutrosophic number to SVNSs using the accumulated geometric operator (AGO). The operator is expressed as follows.

$$N_{ij} \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle = \left\langle \underline{L}_{ij} . \overline{U}_{ij} \right\rangle^{0.5} =$$

$$N_{ij} \left\langle (\underline{T}_{ij} \overline{T}_{ij})^{0.5}, (\underline{I}_{ij} \overline{I}_{ij})^{0.5}, (\underline{F}_{ij} \overline{F}_{ij})^{0.5} \right\rangle \quad (8)$$

After using AGO operator [84], the rating of each alternative with respect to each attribute is transferred as SVNS for MADM problem. The rough neutrosophic values (transferred as SVNS) associated with the alternatives for MADM problems can be represented in decision matrix as follows.

$$D = \langle d \rangle_{m \times n} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n} =$$

$$\begin{array}{c|cccc} & L_1 & L_2 & \ldots & L_n \\ \hline K_1 & \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \ldots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ K_2 & \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \ldots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ K_m & \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \ldots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{array} \quad (9)$$

In the matrix $\langle d \rangle_{m \times n} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n}$, T_{ij}, I_{ij} and F_{ij} (i = 1, 2,..., n and j = 1, 2,..., m) denote the degree of truth membership value, indeterminacy membership value and falsity membership value of alternative K_i with respect to attribute L_j.

The ratings of each alternative with respect to the attributes are explained by the neutrosophic cube [97] proposed by Dezert. The vertices of neutrosophic cube are (0, 0, 0), (1, 0, 0), (1, 0, 1), (0, 0, 1), (0, 1, 0), (1, 1, 0), (1, 1, 1) and (0, 1, 1). The acceptance ratings [61] in neutrosophic cube are classified in three types namely,

I. Highly acceptable neutrosophic ratings,

II. Manageable neutrosophic rating

III. Unacceptable neutrosophic ratings.

Definition 5.2. (Highly acceptable neutrosophic ratings) [61]

In decision making process, the sub cube (Θ) of a neutrosophic cube (Ω) (i.e. $\Theta \subset \Omega$) reflects the field of highly acceptable neutrosophic ratings (Ψ). Vertices of Λ are defined with the eight points (0.5, 0, 0),(1, 0, 0),(1, 0, 0.5), (0.5, 0, 0.5), (0.5, 0, 0.5), (1, 0, 0.5), (1, 0.5, 0.5) and (0.5, 0.5, 0.5). U includes all the ratings of alternative considered with the above average truth membership degree, below average indeterminacy degree and below average falsity membership degree for multi-attribute decision making. So, Ψ has a great role in decision making process and can be defined as follows:

$\Psi = \left\langle (\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5} \right\rangle$ where $0.5 < (\underline{T}_{ij}\overline{T}_{ij})^{0.5} < 1$, $0 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} < 0.5$ and $0 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} < 0.5$, for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Definition 5.3. (Unacceptable neutrosophic ratings) [61]

The field Σ of unacceptable neutrosophic ratings Λ is defined by the ratings which are characterized by 0% membership degree, 100% indeterminacy degree and 100% falsity membership degree. Hence, the set of unacceptable ratings Λ can be considered as the set of all ratings whose truth membership value is zero.

$\Lambda = \left\langle (\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5} \right\rangle$ where $(\underline{T}_{ij}\overline{T}_{ij})^{0.5} = 0$, $0 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} \le 1$ and $0 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} \le 1$, for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

In decision making situation, consideration of Λ should be avoided.

Definition 5.4. (Manageable neutrosophic ratings) [61]

Excluding the field of high acceptable ratings and unacceptable ratings from a neutrosophic cube, tolerable neutrosophic rating field Φ $(= \Omega \cap \neg\Theta \cap \neg\Sigma)$ is determined. The tolerable neutrosophic rating (Δ) considered membership degree is taken in decision making process. Δ can be defined by the expression as follows:

$\Delta = \left\langle (\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5} \right\rangle$ where $0 < (\underline{T}_{ij}\overline{T}_{ij})^{0.5} < 0.5$, $0.5 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} < 1$ and $0.5 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} < 1$.

for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Definition 5.5.

Fuzzification of transferred rough neutrosophic set (SVNS) $N = \langle T_N(v), I_N(v), F_N(v) \rangle$ for any v ∈ V can be defined as a process of mapping N into fuzzy set F = $\{v / \mu_F(v) / v \in V\}$ i.e. f: N $\rightarrow$ F for v ∈ V. The representative fuzzy membership degree $\mu_F(v) \in [0,1]$ of the vector $\{v / \langle T_N(v), I_N(v), F_N(v) \rangle, v \in V\}$ is defined from the concept of neutrosophic cube. It can be obtained by determining the root mean square of 1-T_N(v), I_N(v), and F_N(v) for all v ∈ V. Therefore the equivalent fuzzy membership degree is defined as follows:

$$\mu_F(v) = \begin{cases} 1 - \left\langle \left\{ (1 - T_N(v))^2 + (I_N(v))^2 + (F_N(v))^2 \right\} / 3 \right\rangle^{0.5} & \forall v \in \psi \cup \Delta \\ 0 & \forall v \in \Lambda \end{cases} \quad (10)$$

Now the steps of decision making using TOPSIS method under rough neutrosophic environment are stated as follows.

Step 1. Determination of the weights of decision makers

Assume that a group of k decision makers having their own decision weights involved in the decision making. The importance of the decision makers in a group may not be equal. Assume that the importance of each decision maker is considered with linguistic variables and expressed it by rough neutrosophic numbers.

Assume that $\left\langle \underline{N}_k(\underline{T}_k, \underline{I}_k, \underline{F}_k), \overline{N}_k(\overline{T}_k, \overline{I}_k, \overline{F}_k) \right\rangle$ be a rough neutrosophic number for the rating of k−th decision maker. After using AGO operator, we obtain $E_k = \langle T_k, I_k, F_k \rangle$ as a single valued neutrosophic number for the rating of k−th decision maker. Then, according to equation (10) the weight of the k−th decision maker can be written as:

$$\xi_k = \frac{1 - \left\langle \left\{ (1 - T_k(v))^2 + (I_k(v))^2 + (F_k(v))^2 \right\} / 3 \right\rangle^{0.5}}{\sum_{k=1}^{r} \left\langle 1 - \left\{ (1 - T_k(v))^2 + (I_k(v))^2 + (F_k(v))^2 \right\} / 3 \right\rangle^{0.5}} \quad (11)$$

and $\sum_{k=1}^{r} \xi_k = 1$

Step 2. Construction of the aggregated rough neutrosophic decision matrix based on the assessments of decision makers

Assume that $D^k = \left\langle \underline{d}_{ij}^{(k)}, \overline{d}_{ij}^{(k)} \right\rangle_{m \times n}$ be the rough neutrosophic decision matrix of the k−th decision maker. According to equation (11), $D^k = \left(d_{ij}^{(k)} \right)_{m \times n}$ be the single-valued neutrosophic decision matrix corresponding to the rough neutrosophic decision matrix and $\xi = (\xi_1, \xi_2, ..., \xi_k)^T$ be the weight vector of decision maker such that each $\xi_k \in [0, 1]$. In the group decision making process, all the individual assessments need to be accumulated into a group opinion to make an aggregated single valued neutrosophic decision matrix. This aggregated matrix can be obtained by using rough neutrosophic aggregation operator as follows:

$D = (d_{ij})_{m \times n}$ where,

$$(d_{ij})_{m \times n} = RNWA_{\xi}\left(d_{ij}^1, d_{ij}^2, \cdots, d_{ij}^r \right) = \xi_1 d_{ij}^1 \oplus \xi_2 d_{ij}^2 \oplus \cdots \oplus \xi_r d_{ij}^r$$

$$= \left\langle 1 - \prod_{k=1}^{r} (1 - T_{ij}^{(r)})^{\xi_k}, \prod_{k=1}^{r} (I_{ij}^{(r)})^{\xi_k}, \prod_{k=1}^{r} (F_{ij}^{(r)})^{\xi_k}, \right\rangle \quad (12)$$

Here, $d_{ij}^r = \left\langle \underline{d}_{ij}^r . \overline{d}_{ij}^r \right\rangle^{0.5}$

Now the aggregated rough neutrosophic decision matrix has been defined as follows:

$$(d_{ij})_{m\times n} = \left\langle (\underline{T}_{ij}.\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}.\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}.\overline{F}_{ij})^{0.5} \right\rangle_{m\times n}$$

$$\begin{array}{c|cccc} & L_1 & L_2 & \dots & L_n \\ \hline K_1 & \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \dots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ K_2 & \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \dots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ . & \dots & \dots & \dots & \dots \\ . & \dots & \dots & \dots & \dots \\ K_m & \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \dots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{array} \quad (13)$$

Here, $d_{ij}=\langle T_{ij}, I_{ij}, F_{ij}\rangle = \left\langle (\underline{T}_{ij}.\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}.\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}.\overline{F}_{ij})^{0.5} \right\rangle$ is the aggregated element of rough neutrosophic decision matrix D for i = 1, 2, . . . m and j = 1, 2, . .. n.

Step 3. Determination of the attribute weights

In the decision-making process, all attributes may not have equal importance. So, every decision maker may have their own opinion regarding attribute weights. To obtain the group opinion of the chosen attributes, all the decision makers' opinions need to be aggregated. Assume that $\left\langle \underline{w}^{j}_{(k)}, \overline{w}^{j}_{(k)} \right\rangle$ be rough neutrosophic number (RNN) assigned to the attribute L_j by the k–th decision maker. According to equation (8) w^j_k be the neutrosophic number assigned to the attribute L_j by the k–th decision maker. Then the combined weight W = (w_1, w_2 . . . , w_n) of the attribute can be determined by using rough neutrosophic weighted aggregation (RNWA) operator

$$w_j = RNWA_{\xi}(w^{(1)}_j, w^{(2)}_j, \cdots, w^{(r)}_j) = \xi_1 w^{(1)}_j \oplus \xi_2 w^{(2)}_j \oplus \cdots \oplus \xi_r w^{(r)}_j$$

$$= \left\langle 1-\prod_{k=1}^{r}(1-T^{(r)}_j)^{\xi_k}, \prod_{k=1}^{r}(I^{(r)}_j)^{\xi_k}, \prod_{k=1}^{r}(F^{(r)}_j)^{\xi_k} \right\rangle \quad (14)$$

$$Here,\ \xi^r_{ij} = \left\langle \underline{d}_{ij}.\overline{d}_{ij} \right\rangle;\ \ w_j = \left\langle T^{(r)}_j, I^{(r)}_j, F^{(r)}_j \right\rangle =$$

$\left\langle (\underline{T}^{(r)}_j.\overline{T}^{(r)}_j)^{0.5}, (\underline{I}^{(r)}_j.\overline{I}^{(r)}_j)^{0.5}, (\underline{F}^{(r)}_j.\overline{F}^{(r)}_j)^{0.5} \right\rangle$ for j = 1, 2, . . . n.

W = (w_1, w_2 . . . , w_n) (15)

Step 4. Aggregation of the weighted rough neutrosophic decision matrix

In this section, the obtained weights of attribute and aggregated rough neutrosophic decision matrix need to be further fused to make the aggregated weighted rough neutrosophic decision matrix. Then, the aggregated weighted rough neutrosophic decision matrix can be defined by using the multiplication properties between two neutrosophic sets as follows:

$$D\otimes W = D^W = \left\langle d^{w_j}_{ij} \right\rangle_{m\times n} = \left\langle T^{w_j}_{ij}, I^{w_j}_{ij}, F^{w_j}_{ij} \right\rangle_{m\times n} =$$

$$\begin{array}{c|cccc} & L_1 & L_2 & \dots & L_n \\ \hline K_1 & \langle T^{w_1}_{11}, I^{w_1}_{11}, F^{w_1}_{11}\rangle & \langle T^{w_2}_{12}, I^{w_2}_{12}, F^{w_2}_{12}\rangle & \dots & \langle T^{w_n}_{1n}, I^{w_n}_{1n}, F^{w_n}_{1n}\rangle \\ K_2 & \langle T^{w_1}_{21}, I^{w_1}_{21}, F^{w_1}_{21}\rangle & \langle T^{w_2}_{22}, I^{w_2}_{22}, F^{w_2}_{22}\rangle & \dots & \langle T^{w_n}_{2n}, I^{w_n}_{2n}, F^{w_n}_{2n}\rangle \\ . & \dots & \dots & \dots & \dots \\ . & \dots & \dots & \dots & \dots \\ K_m & \langle T^{w_1}_{m1}, I^{w_1}_{m1}, F^{w_1}_{m1}\rangle & \langle T^{w_2}_{m2}, I^{w_2}_{m2}, F^{w_2}_{m2}\rangle & \dots & \langle T^{w_n}_{mn}, I^{w_n}_{mn}, F^{w_n}_{mn}\rangle \end{array} \quad (16)$$

Here, $d^{w_j}_{ij} = \left\langle T^{w_j}_{ij}, I^{w_j}_{ij}, F^{w_j}_{ij} \right\rangle$ is an element of the aggregated weighted rough neutrosophic decision matrix D^W for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Step 5. Determination of the rough relative positive ideal solution (RRPIS) and the rough relative negative ideal solution (RRNIS) for rough neutrosophic sets

After transferring RNS decision matrix, assume $D_N = \left\langle d^W_{ij} \right\rangle_{m\times n} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m\times n}$ be a SVNS based decision matrix, where, T_{ij}, I_{ij} and F_{ij} are the membership degree, indeterminacy degree and non-membership degree of evaluation for the attribute L_j with respect to the alternative K_i. In practical, two types of attributes namely, benefit type attribute and cost type attribute exist in multi-attribute decision making problems.

Definition 5.6.

Let $C_1$and C_2 be the benefit type attribute and cost type attribute respectively. G^+_N is the relative rough neutrosophic positive ideal solution (RRNPIS) and G^-_N is the relative rough neutrosophic negative ideal solution (RRNNIS).

Then G^+_N can be defined as follows:

$$G^+_N = \left\langle d^{w+}_1, d^{w+}_2, \cdots, d^{w+}_n \right\rangle \quad (17)$$

Here $d^{w+}_j = \left\langle T^{w+}_j, I^{w+}_j, F^{w+}_j \right\rangle$ for j = 1, 2, . . . , n.

$$T^{w+}_j = \{(\max_i\{T^{w_j}_{ij}\}/ j\in C_1), (\min_i(T^{w_j}_{ij})/ j\in C_2)\}$$

$$I^{w+}_j = \{(\min_i\{I^{w_j}_{ij}\}/ j\in C_1), (\max_i(I^{w_j}_{ij})/ j\in C_2)\}$$

$$F^{w+}_j = \{(\min_i\{F^{w_j}_{ij}\}/ j\in C_1), (\max_i(F^{w_j}_{ij})/ j\in C_2)\}$$

Then G^-_N can be defined as follows:

$$G^-_N = \left\langle d^{w-}_1, d^{w-}_2, \cdots, d^{w-}_n \right\rangle \quad (18)$$

Here $d^{w-}_j = \left\langle T^{w-}_j, I^{w-}_j, F^{w-}_j \right\rangle$ for j = 1, 2, . . . , n.

$$T^{w-}_j = \{(\min_i\{T^{w_j}_{ij}\}/ j\in C_1), (\max_i(T^{w_j}_{ij})/ j\in C_2)\}$$

$$I^{w-}_j = \{(\max_i\{I^{w_j}_{ij}\}/ j\in C_1), (\min_i(I^{w_j}_{ij})/ j\in C_2)\}$$

$$F^{w-}_j = \{(\max_i\{F^{w_j}_{ij}\}/ j\in C_1), (\min_i(F^{w_j}_{ij})/ j\in C_2)\}$$

Step 6. Determination of the distance measure of each alternative from the RRNPIS and the RRNNIS for rough neutrosophic sets

The normalized Euclidean distance measure of all alternative $\left\langle T^{w_j}_{ij}, I^{w_j}_{ij}, F^{w_j}_{ij} \right\rangle$ from the RRNPIS $\left\langle d^{w+}_1, d^{w+}_2, ..., d^{w+}_n \right\rangle$ for i = 1, 2, …, m and j = 1, 2, …, n can be written as follows:

$$\delta^{i+}_{euclid}(d^{w_j}_{ij}, d^{w+}_{ij}) =$$

$$\left\langle \frac{1}{3n} \sum_{j=1}^{n} \begin{pmatrix} (T^{w_j}_{ij}(v_j) - T^{w+}_j(v_j))^2 + (I^{w_j}_{ij}(v_j) - I^{w+}_j(v_j))^2 \\ + (F^{w_j}_{ij}(v_j) - F^{w+}_j(v_j))^2 \end{pmatrix} \right\rangle^{0.5} \quad (19)$$

The normalized Euclidean distance measure of all alternative $\langle T_{ij}^{wj}, I_{ij}^{wj}, F_{ij}^{wj} \rangle$ from the RRNPIS $\langle d_1^{w-}, d_2^{w-}, ..., d_n^{w-} \rangle$ for i = 1, 2, …, m and j = 1, 2, …, n can be written as follows:

$$\delta_{euclid}^{i-}(d_{ij}^{wj}, d_{ij}^{w-}) = \frac{1}{3n}\left\langle \sum_{j=1}^{n} \begin{pmatrix} (T_{ij}^{wj}(v_j) - T_j^{w-}(v_j))^2 + (I_{ij}^{wj}(v_j) - I_j^{w-}(v_j))^2 \\ + (F_{ij}^{wj}(v_j) - F_j^{w-}(v_j))^2 \end{pmatrix} \right\rangle^{0.5} \quad (20)$$

Step 7. Determination of the relative closeness co-efficient to the rough neutrosophic ideal solution for rough neutrosophic sets

The relative closeness coefficient of each alternative K_i with respect to the neutrosophic positive ideal solution G_N^+ is defined as follows:

$$\chi_i^* = \frac{\langle \delta_{euclid}^{i-}(d_{ij}^{wj}, d_{ij}^{w-}) \rangle}{\langle \delta_{euclid}^{i-}(d_{ij}^{wj}, d_{ij}^{w-}) + \delta_{euclid}^{i+}(d_{ij}^{wj}, d_{ij}^{w+}) \rangle} \quad (21)$$

Here $0 \le \chi_i^* \le 1$.

Step 8. Ranking the alternatives

According to the relative closeness coefficient values larger the values of χ_i^* reflects the better alternative K_i for i = 1, 2, …, n.

6 Numerical example

In order to demonstrate the proposed method, logistic center location selection problem has been described here. Suppose that a new modern logistic center is required in a town. There are three locations K_1, K_2, K_3. A committee of three decision makers or experts D_1, D_2, D_3 has been formed to select the most appropriate location on the basis of six parameters obtained from expert opinions, namely, cost (L_1), distance to suppliers (L_2), distance to customers (L_3), conformance to government and law (L_4), quality of service (L_5), and environmental impact (L_6).

Based on the proposed approach the considered problem has been solved using the following steps:

Step 1. Determination of the weights of decision makers

The importance of three decision makers in a selection committee may be different based on their own status. Their decision values are considered as linguistic terms expressed in Table-1. The importance of each decision maker expressed by linguistic term with its corresponding rough neutrosophic values shown in Table-2. The weight of decision maker has been determined with the help of equation (11) as:

$\xi_1 = 0.398$, $\xi_2 = 0.359$, $\xi_3 = 0.243$.

We transfer rough neutrosophic number (RNN) to neutrosophic number (NN) with the help of AGO operator [84] in Table 1, Table 2 and Table 3.

Step 2. Construction of the aggregated rough neutrosophic decision matrix based on the assessments of decision makers

The linguistic term along with RNNs has been defined in Table-3 to rate each alternative with respect to each attribute. The assessment values of each alternative K_i (i = 1, 2, 3) with respect to each attribute L_j (j = 1, 2, 3, 4, 5, 6) provided by three decision makers have been listed in Table-4. Then the aggregated neutrosophic decision matrix can be obtained by fusing all the decision maker opinions with the help of aggregation operator (equation 12) (see Table 5).

Step 3. Determination of the weights of attributes

The linguistic terms shown in Table-1have been used to evaluate each attribute. The importance of each attribute for every decision maker is rated with linguistic terms shown in Table-4. Three decision makers' opinions need to be aggregated to final opinion.

The fused attribute weight vector is determined by using equation (14) as follows:

$$W = \begin{Bmatrix} \langle 0.761, 0.205, 0.195 \rangle, \langle 0.800, 0.181, 0.159 \rangle, \langle 0.737, 0.241, 0.196 \rangle, \\ \langle 0.761, 0.223, 0.169 \rangle, \langle 0.774, 0.203, 0.172 \rangle, \langle 0.804, 0.184, 0.172 \rangle \end{Bmatrix} \quad (23)$$

Step 4. Construction of the aggregated weighted rough neutrosophic decision matrix

After obtaining the combined weights of attribute and the ratings of alternative, the aggregated weighted rough neutrosophic decision matrix can be obtained by using equation (16) as shown in Table-6.

Step 5. Determination of the rough neutrosophic relative positive ideal solution and the rough neutrosophic relative negative ideal solution

The RNRPIS can be calculated from the aggregated weighted decision matrix on the basis of attribute types i.e. benefit type or cost type by using equation (17) as

$$G_N^+ = \begin{bmatrix} \langle 0.670, 0.289, 0.274 \rangle, \langle 0.694, 0.284, 0.252 \rangle, \langle 0.588, 0.388, 0.309 \rangle, \\ \langle 0.607, 0.374, 0.286 \rangle, \langle 0.642, 0.331, 0.303 \rangle, \langle 0.708, 0.270, 0.253 \rangle \end{bmatrix} \quad (25)$$

Here $d_1^{w+} = \langle T_1^{w+}, I_1^{w+}, F_1^{w+} \rangle$ is calculated as:

$T_1^{w+} =$ max [0.670, 0.485, 0.454] = 0.670, $I_1^{w+} =$ min [0.289, 0.449, 0.471] = 0.289,

$F_1^{w+} =$ min [0.274, 0.377, 0.463]= 0.274.

Similarly, other RNRPISs can be calculated.

\The RNRNIS can be calculated from aggregated weighted decision matrix on the basis of attribute types i.e. benefit type or cost type by using equation (18) as follows:

$$G_N^- = \begin{bmatrix} \langle 0.454, 0.471, 0.463 \rangle, \langle 0.588, 0.377, 0.353 \rangle, \langle 0.469, 0.480, 0.309 \rangle, \\ \langle 0.522, 0.441, 0.358 \rangle, \langle 0.524, 0.429, 0.372 \rangle, \langle 0.512, 0.435, 0.414 \rangle \end{bmatrix} \quad (26)$$

Here, $d_1^{w-} = \langle T_1^{w-}, I_1^{w-}, F_1^{w-} \rangle$ is calculated as

T_1^{w} = min [0.670, 0.485, 0.454] = 0.454, I_1^{w-} = max [0.289, 0.449, 0.471] = 0.471,
F_1^{w-} = max [0.274, 0.377, 0.463] = 0.463.

Other RNRNISs can be calculated in similar way.

Step 6. Determination of the distance measure of each alternative from the RRNPIS and the RRNNIS and relative closeness co-efficient

Normalized Euclidean distance measures defined in equation (19) and equation (20) are used to determine the distances of each alternative from the RRNPIS and the RNNIS. With these distances relative closeness co-efficient is calculated by using equation (21). These results have been listed in Table7.

Step 7. Determination of the relative closeness co-efficient to the rough neutrosophic ideal solution for rough neutrosophic sets

The values of relative closeness coefficient of each alternative K_1 , K_2 , K_3 with respect to the rough neutrosophic positive ideal solution G_N^{+} is defined as follows:

Table8. Distance measure and relative closeness co-efficient of each alternative

Alternativ es (K_i)	δ_{euclid}^{i+}	δ_{euclid}^{i-}	χ_i^{*}
K_1	0.0078	0.1248	0.9411
K_2	0.1192	0.0682	0.3639
K_3	0.1025	0.0534	0.3425

(27)

Step 9. Ranking the alternatives

According to the values of relative closeness coefficient of each alternative shown in Table 7, the ranking order of three alternatives have been obtained as

$K_1 \succ K_2 \succ K_3$.

Thus K_1 is the best alternative for logistic centre in town.

Conclusion

In general, realistic MAGDM problems adhere to uncertain, imprecise, incomplete, and inconsistent data and rough neutrosophic set theory is adequate to deal with it. In this paper we have proposed rough neutrosophic TOPSIS method for MAGDM. We have also proposed rough neutrosophic aggregate operator and, rough neutrosophic weighted aggregate operator. In the decision making process, the ratings of each alternative with respect to each attribute are presented as linguistic variables characterized by rough neutrosophic numbers. Rough neutrosophic aggregation operator has been used to aggregate all the opinions of decision makers. Rough neutrosophic positive ideal and rough neutrosophic negative ideal solution have been defined to form aggregated weighted decision matrix. Euclidean distance measure has been used to calculate the distances of each alternative from positive as well as negative ideal solutions for relative closeness co-efficient of each alternative. The proposed approach can be applied in pattern recognition, artificial intelligence, medical diagnosis, etc in rough neutrosophic environment.

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Table1. Linguistic Terms (LT) for Rating Attributes

Linguistic Terms	Rough neutrosophic numbers	Neutrosophic numbers
Very good / Very important (VG/VI)	$\langle(0.85, 0.05, 0.05), (0.95, 0.15, 0.15)\rangle$	$\langle 0.899, 0.087, 0.087\rangle$
Good / Important(G /I)	$\langle(0.75, 0.15, 0.10), (0.85, 0.25, 0.20)\rangle$	$\langle 0.798, 0.194, 0.141\rangle$
Fair / Medium(F/M)	$\langle(0.45, 0.35, 0.35), (0.55, 0.45, 0.55)\rangle$	$\langle 0.497, 0.397, 0.439\rangle$
Bad / Unimportant (B / UI)	$\langle(0.25, 0.55, 0.65), (0.45, 0.65, 0.75)\rangle$	$\langle 0.335, 0.598, 0.698\rangle$
Very bad/Very Unimportant (VB/VUI)	$\langle(0.05, 0.75, 0.85), (0.15, 0.85, 0.95)\rangle$	$\langle 0.087, 0.798, 0.899\rangle$

Table 2. Importance of Decision makers expressed with rough neutrosophic numbers (RNN)

DM	D_1	D_2	D_3
LT	VI	I	M
RNN	$\left\langle\begin{matrix}(0.85, 0.05, 0.05),\\(0.95, 0.15, 0.15)\end{matrix}\right\rangle$	$\left\langle\begin{matrix}(0.75, 0.15, 0.10),\\(0.85, 0.25, 0.20)\end{matrix}\right\rangle$	$\left\langle\begin{matrix}(0.45, 0.35, 0.35),\\(0.55, 0.45, 0.55)\end{matrix}\right\rangle$
NN	$\langle 0.899, 0.087, 0.087\rangle$	$\langle 0.798, 0.194, 0.141\rangle$	$\langle 0.497, 0.397, 0.439\rangle$

Table 3. Linguistic terms (LT) for rating the candidates with RNNs

Linguistic terms	RNNs	NNs
Extremely Good/High (EG/EH)	$\langle(1.00, 0.00, 0.00), (1.00, 0.00, 0.00)\rangle$	$\langle 1.000, 0.000, 0.000\rangle$
Very Good/High (VG/VH)	$\langle(0.85, 0.05, 0.05), (0.95, 0.15, 0.15)\rangle$	$\langle 0.899, 0.087, 0.087\rangle$
Good/High (G/H)	$\langle(0.75, 0.15, 0.10), (0.85, 0.25, 0.20)\rangle$	$\langle 0.798, 0.194, 0.141\rangle$
Medium Good/High (MG/MH)	$\langle(0.55, 0.30, 0.25), (0.65, 0.40, 0.35)\rangle$	$\langle 0.598, 0.346, 0.296\rangle$
Medium/Fair (M/F)	$\langle(0.45, 0.45, 0.35), (0.55, 0.55, 0.55)\rangle$	$\langle 0.497, 0.497, 0.439\rangle$
MediumBad/MediumLaw(MB/ML)	$\langle(0.30, 0.60, 0.55), (0.40, 0.70, 0.65)\rangle$	$\langle 0.346, 0.648, 0.598\rangle$
Bad/Law (G/L)	$\langle(0.15, 0.70, 0.75), (0.25, 0.80, 0.85)\rangle$	$\langle 0.194, 0.748, 0.798\rangle$
Very Bad/Low (VB/VL)	$\langle(0.05, 0.80, 0.85), (0.15, 0.90, 0.95)\rangle$	$\langle 0.087, 0.849, 0.899\rangle$
VeryVeryBad/low(VVB/VVL)	$\langle(0.05, 0.95, 0.95), (0.05, 0.85, 0.95)\rangle$	$\langle 0.050, 0.899, 0.950\rangle$

Table4. Assessments of alternatives and attribute weights given by three decision makers

Alternatives (K_i)	Decision Makers	L_1	L_2	L_3	L_4	L_5	L_6
K_1	D_1	VG	G	G	G	G	VG
	D_2	VG	VG	G	G	G	VG
	D_3	G	VG	G	G	VG	G
K_2	D_1	M	G	M	G	G	M
	D_2	G	MG	G	G	MG	G
	D_3	M	G	M	MG	M	M
K_3	D_1	M	VG	G	MG	VG	M
	D_2	M	M	G	G	M	G
	D_3	G	M	M	MG	G	VG

Table 5. Aggregated transferred rough neutrosophic decision matrix

	L_1	L_2	L_3	L_4	L_5	L_6
K_1	$\langle 0.881, 0.106, 0.098\rangle$	$\langle 0.867, 0.126, 0.111\rangle$	$\langle 0.798, 0.194, 0.141\rangle$	$\langle 0.798, 0.194, 0.141\rangle$	$\langle 0.830, 0.160, 0.125\rangle$	$\langle 0.880, 0.106, 0.098\rangle$
K_2	$\langle 0.637, 0.307, 0.292\rangle$	$\langle 0.741, 0.239, 0.184\rangle$	$\langle 0.637, 0.315, 0.292\rangle$	$\langle 0.761, 0.223, 0.169\rangle$	$\langle 0.677, 0.284, 0.242\rangle$	$\langle 0.637, 0.307, 0.292\rangle$
K_3	$\langle 0.597, 0.334, 0.333\rangle$	$\langle 0.735, 0.217, 0.231\rangle$	$\langle 0.748, 0.231, 0.186\rangle$	$\langle 0.686, 0.281, 0.227\rangle$	$\langle 0.787, 0.182, 0.175\rangle$	$\langle 0.755, 0.212, 0.197\rangle$

Table 6. Aggregated weighted rough neutrosophic decision matrix

	L_1	L_2	L_3	L_4	L_5	L_6
K_1	$\langle 0.670, 0.289, 0.274\rangle$	$\langle 0.694, 0.284, 0.252\rangle$	$\langle 0.588, 0.388, 0.309\rangle$	$\langle 0.607, 0.374, 0.286\rangle$	$\langle 0.642, 0.331, 0.303\rangle$	$\langle 0.708, 0.270, 0.253\rangle$
K_2	$\langle 0.485, 0.449, 0.377\rangle$	$\langle 0.593, 0.377, 0.344\rangle$	$\langle 0.469, 0.480, 0.431\rangle$	$\langle 0.579, 0.396, 0.309\rangle$	$\langle 0.524, 0.429, 0.372\rangle$	$\langle 0.512, 0.435, 0.414\rangle$
K_3	$\langle 0.454, 0.471, 0.463\rangle$	$\langle 0.588, 0.359, 0.353\rangle$	$\langle 0.551, 0.416, 0.346\rangle$	$\langle 0.522, 0.441, 0.358\rangle$	$\langle 0.609, 0.348, 0.317\rangle$	$\langle 0.607, 0.357, 0.335\rangle$

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