On synchronization and the relativity principle

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Abstract

Lorentz transformation allows two ways to compare time measures from two moving clocks. We show that the more realistic way leads to discover that absolute rest plays a hidden role and prescribes a restriction on the relativity principle.

Let $K_1$ and $K_2$ two cartesian frames moving along the $x$ axis with the speeds $v_1$ and $v_2$ in a third frame $K_0$ at rest.

![Figure 1](image)

If $(x_0, y_0, z_0, t_0)$ are the space-time coordinates of an event in the frame $K_0$, its coordinates $(x_1, y_1, z_1, t_1)$ and $(x_2, y_2, z_2, t_2)$ in the frames $K_1$ et $K_2$ are deducted via Lorentz transformations:

\[
\begin{align*}
    x_1 &= \frac{x_0 - v_1 t_0}{\sqrt{1 - (v_1/c)^2}}, \\
    t_1 &= \frac{t_0 - (v_1 x_0/c^2)}{\sqrt{1 - (v_1/c)^2}}, \\
    x_2 &= \frac{x_0 - v_2 t_0}{\sqrt{1 - (v_2/c)^2}}, \\
    t_2 &= \frac{t_0 - (v_2 x_0/c^2)}{\sqrt{1 - (v_2/c)^2}},
\end{align*}
\]

(1)

with $y_0 = y_1 = y_2$ and $z_0 = z_1 = z_2$.

The inverse transformation of the first one in (1) is:

\[
\begin{align*}
    x_0 &= \frac{x_1 + v_1 t_1}{\sqrt{1 - (v_1/c)^2}}, \\
    t_0 &= \frac{t_1 + (v_1 x_1/c^2)}{\sqrt{1 - (v_1/c)^2}}.
\end{align*}
\]

(2)

The replacement of $x_0$ and $t_0$ from (2) in the second transformation of (1) gives:

\[
\begin{align*}
    x_2 &= \frac{x_1 - qt_1}{\sqrt{1 - (q/c)^2}}, \\
    t_2 &= \frac{t_1 - (q x_1/c^2)}{\sqrt{1 - (q/c)^2}},
\end{align*}
\]

(3)

where $q = \frac{v_2 - v_1}{1 - (v_2 v_1/c^2)}$.

Since the clock linked to the origin of $K_2$ is characterized by $x_2 = 0$, which means that $x_1 = q t_1$, by replacing in the expression of $t_2$ in (3) one finds:

\[
\frac{t_2}{t_1} = \frac{1}{\sqrt{1 - \frac{q^2}{c^2}}}
\]

(5)

Let suppose that $v_1 = -v_2$. Consequently, relations (4) and (5) lead to:

\[
\frac{t_2}{t_1} = \frac{1}{\sqrt{1 - \left(\frac{2 v_2 c}{c^2 + v_2^2}\right)^2}}.
\]

(6)
From Eq. (6), the synchronization is conserved \((t_2 = t_1)\) only in the case \(v_2 = v_1 = 0\).

But according to Special Relativity results (chap. 1.4 in [1]), if the clock linked to the origin of \(K_0\) measure a duration \(t_0\), the moving clocks linked to \(K_1\) and \(K_2\) origins must measure:

\[
t_1 = t_0 \sqrt{1 - \frac{v_1^2}{c^2}}, \quad t_2 = t_0 \sqrt{1 - \frac{v_2^2}{c^2}}.
\]

From the law of speed addition, the speed of the \(K_2\)-clock in relation to the \(K_1\)-clock is:

\[
u = \frac{v_2 - v_1}{1 - (v_1 v_2/c^2)} = q.
\]

Eqs. (7) give:

\[
\frac{t_2}{t_1} = \sqrt{\frac{1 - (v_2/c)^2}{1 - (v_1/c)^2}}.
\]

The synchronization is conserved \((t_2 = t_1)\) if \(v_2 = \pm v_1\) with the possibility of \(v_1\) and \(v_2\) \(\neq 0\) : When the two clocks are moving with the same non null speed in relation to another frame but in two opposite directions. Clearly, this logical issue is very realistic. One can imagine two synchronous clocks starting motion in opposite directions and returning after the same journey length at the same speed. Obviously, their relative speed is not necessary null:

\[
u = \frac{-2v_1}{1 + (v_1/c)^2} = \frac{2v_2}{1 + (v_2/c)^2}.
\]

It is very natural to expect that both the two clocks will measure the same duration. This situation is in disagreement with Einstein’s deduction: All synchronous clocks do not remain so after the accomplishment of galilean relative motions (chap. 1.4 in [1]).

Moreover, together Eqs. (8) and (9) lead to:

\[
\frac{t_2}{t_1} = \frac{\sqrt{1 - (u/c)^2}}{1 + (uv_1/c^2)} = \frac{1 - (uv_2/c^2)}{\sqrt{1 - (u/c)^2}}
\]

Clearly, relation (5) is recovered from Eqs. (10) only in the case \(v_1 = 0\) with \(t_1 > t_2\) and the case \(v_2 = 0\) corresponds to \(t_1 < t_2\).

Eqs. (10) are equivalent to:

\[
v_1 = \frac{c^2}{u} \left(1 - \frac{t_1}{t_2} \sqrt{1 - (u/c)^2}\right), \quad v_2 = \frac{c^2}{u} \left(1 - \frac{t_2}{t_1} \sqrt{1 - (u/c)^2}\right).
\]

From Eq. (8) a fixed value of \(u\) is possible with an infinity choices of \((v_1,v_2)\). But according to relations (11), for any fixed value of \(t_2/t_1\) and \(u\), the values of \(v_1\) and \(v_2\) are unique. Thus the comparison of the measures \(t_1\) and \(t_2\) does not depend only on the relative speed \(u\) (equal to \(q\)) as from Eq. (5), but also on the speed \(v_1\) (or \(v_2\)) whose uniqueness significate that they are in relation to the absolute rest frame. Interestingly, the absolute rest reappears with his fundamental importance, so hidden in Einstein’s theory.

As a conclusion, the “totally motionless” frame is always needed to compare time measures and, thus, is not actually equivalent to “moving” galilean frames. This fact prescribes a neat restriction on Einstein’s equivalence postulate (relativity principle).

Reference