Selectivity in sets and the duality P vs NP

Haroun Boutamani

Abstract

This paper addresses the concept of selectivity using a main set and an auxiliary set, the auxiliary serves the purpose of deciding which elements do or do not fit together within the output sets chosen based on the elements of the main set.

This paper also presents an approach of comparing sets based on a defined relation, the sets' properties, and the elementary properties being total compatibility and total incompatibility.

The problem classes P and NP are two sets considered under the criteria of distinguishability, then the possibility of comparison of the two sets is discussed. The proposed comparison presents a distinction between P and NP.

Text

Let A be a set of n elements;

And let B be a set of v elements; B is a set of pairs out of A.

$B \subseteq AxA$

Definition 1.

Incompatibility is a relation R-incompatibility(x,y) such as $\{x, y\} \in AxA$, $\{x, y\} \in B$ And $\{x,y\} \notin f_B(A)$, x | y .

Definition 2.

Compatibility is a relation R-compatibility(x, y) such as $\{x, y\} \in AxA$, $\{x, y\} \in B$ And $\{x,y\} \in f_B(A)$, x | y .

B is a set of selectivity of A.

If $B \neq \Phi$ then A_B is a selective set.

A_Φ is the non selective set.

- Every element of the selective set A has a number of elementary compatibilities and a number of elementary incompatibilities.

For example an element a of the selective set A may be written the following way:

a(number of elementary compatibilities, number of elementary incompatibilities)

And we can read : a has x elementary compatibilities it is written

$$compa(a) = x$$

We can also read : a has y elementary incompatibilities

$$incompa(a) = y$$

For any given element a of the selective set A of n elements:

$$compa(a) + incompa(a) = n - 1$$

- Any element that has n-1 elementary compatibilities is considered totally compatible in A.
- Any element that has n-1 elementary incompatibilities is considered totally incompatible in A.

Theorem 1.

Let A be a selective set of n elements, and B a set of selectivity of A.

The set A does not contain a totally compatible element and a totally incompatible element.

Proof.

If the selective set A contains at least one totally compatible element, then every other element of A has at least one elementary compatibility, thus no other element of A can be totally incompatible.

And if the selective set A contains at least one totally incompatible element, then every other element has less than n-1 elementary compatibilities, thus no other element of A can be totally compatible.

Theorem 2.

Let A_B be a selective set of n elements.

If A_B contains at least one totally incompatible element or contains at least one totally compatible element then A_B is a distinguishable set.

Proof.

$\exists$ a $\in$ A, such as

((compa(a) = n-1 and incompa(a) = 0)

Or

(compa(a) = 0 and incompa(a)=n-1))

Then:

((order(A) – compa(a) – 1 = 0)

Or

(order(A) – incompa(a) - 1 = 0)) (1)

We have zero equations containing exclusively and exhaustively the variables order(A) which is a set property and the variables compa(a) and incompa(a) which are elementary properties. The said equations lead to 0 or 1 thus the set A_B is distinguishable.

Theorem 3.

Let A_B be a selective set.

If A_B contains no totally incompatible elements and no totally compatible elements then it is indistinguishable.

Proof.
$\forall a \in A$, such as

(compa(a) $\neq$ n-1 and incompa(a) $\neq$ 0 and compa(a) $\neq$ 0 and incompa(a) $\neq$ n-1)

Then:

((order(A) – compa(a) – 1 $\neq$ 0)

And

(order(A) – incompa(a) - 1 $\neq$ 0)) (2)

We have a zero equation including exclusively and exhaustively order(A) which is a set property and compa(a) and incompa(a) which are elementary properties, said equations lead to neither 0 or 1 as a result, then A_B is indistinguishable.

Theorem 4.

Let A_B, C_D be selective sets of n, m elements, respectively.

If A_B, C_D are distinguishable then they are comparable.

Proof.

A_B, C_D are distinguishable.

We use (1) . $\exists\, a \in A$, $\exists\, c \in C$ such as:

(((order(A) – compa(a) – 1 = 0) Or (order(A) – incompa(a) - 1 = 0))

And

((order(C) – compa(c) – 1 = 0) Or (order(C) – incompa(c) - 1 = 0)))

Then:

((order(A) – compa(a) – 1 = 0)

Or

(order(A) – incompa(a) - 1 = 0)

Or

(order(C) – compa(c) – 1 = 0)

Or

(order(C) – incompa(c) - 1 = 0))

We have equations including exclusively and exhaustively the variables order(A), order(C) which are sets' properties and the variables compa(a), incompa(a), compa(c), incompa(c) which are elementary properties.

The outcome is 0 or 1 thus the selective sets A_B and C_D are comparable.

Theorem 5.

Let A_B, C_D be selective sets.

If either of A_B, C_D is indistinguishable then A_B and C_D are not comparable.

Proof.

Either of A_B, C_D is indistinguishable.

We use (2).
$\forall a \in A, \forall c \in C$.

(((order(A) – compa(a) – 1 ≠ 0) and (order(A) – incompa(a) - 1 ≠ 0))

Or

((order(C) – compa(C) – 1 ≠ 0) and (order(C) – incompa(C) - 1 ≠ 0)))

Then:

((order(A) – compa(a) – 1 ≠ 0)

Or

(order(A) – incompa(a) - 1 ≠ 0)

Or

(order(C) – compa(C) – 1 ≠ 0)

Or

(order(C) – incompa(C) - 1 ≠ 0))

We have zero equations including exclusively and exhaustively the variables order(A), order(C) which are sets' properties and the variables compa(a), incompa(a), compa(c), incompa(c) which are elementary properties.

None of the mentioned equations either leads to result 0 or 1, thus the sets A_B and C_D are not comparable.

Theorem 6.

Let A, C be sets, such as $A \subset C$.

If A is selective then C is selective.

Proof.

If A selective then it has a set B of selectivity.

$$B \subseteq AxA$$

We have $A \subset C$ then

$$B \subseteq CxC$$

The set B is a set of selectivity of C.

Theorem 7.

Let A be a selective set of n elements, and B a set of selectivity of A.

And let C be a selective set of m elements, and D a set of selectivity of C.

We define the relation R-incompatibility for both sets A and C.

If the set A contains at least one element that is totally compatible and C contains at least one element that is totally incompatible, that would mean that A is distinct from C.

A distinct from C $\Leftrightarrow$ $((\exists\ x \in A, x \notin C) \text{ or } (\exists\ y \in C, y \notin A))$

A distinct from C $\Leftrightarrow$

$$A \neq C$$

$$A \not\subset C$$

$$C \not\subset A$$

Proof.

According to Theorem1. any given set does not contain one totally compatible element and one totally incompatible element, this is sufficient to prove not equality.

We also have :

$$\exists\ x \in A, x \notin C \Leftrightarrow A \not\subset C$$

And we have :

$$\exists\ y \in C, y \notin A \Leftrightarrow C \not\subset A$$

<u>The P vs NP problem :</u>

The problem has been formulated independently by Stephen Cook and Leonid Levin.

Problem statement:

"The P versus NP problem is to determine whether every language accepted by some nondeterministic algorithm in polynomial time is also accepted by some (deterministic) algorithm in polynomial time". [1]

<u>P vs NP :</u>

We define the compatibility relation R-compatibility for the two problem classes P and NP, as follows: the two problems L and K are compatible means there is polynomial relation between their expressed complexities.

This particular compatibility relation basically refers to class P problems.

According to the defined compatibility relation we may notice that:

All class P problems are totally compatible which satisfies the criteria for P to be distinguishable.

At least some class NP problems are totally incompatible according to the defined relation, which satisfies the criteria for NP to be distinguishable.

P is distinguishable and NP is distinguishable $\Leftrightarrow$ P and NP are comparable.

<u>Comparison between P and NP</u>

We may notice that at least one class NP problem is totally incompatible.

We may also notice that all class P problems are totally compatible.

This gives us:

P is distinct from NP

$$P \neq NP$$

$$P \not\subset NP$$

$$NP \not\subset P$$

<u>References:</u>

[1] The P versus NP problem, Stephen Cook.