# A Short Note on the de Broglie Wavelengths of Composite Objects


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## ABSTRACT

The de Broglie wavelength of an object is inversely proportional to the object's mass and relative velocity multiplied by Planck's constant. A composite object, such as an atom composed of a proton and electron, possesses a total mass and therefore a composite de Broglie wavelength. Mass is simply additive, although the relation for combining de Broglie wavelengths has never been stated. In this paper, we derive the relation for combining the de Broglie wavelengths of an object's component parts into the composite de Broglie wavelength of the object as a whole.


## Introduction

In 1924, Louis de Broglie, then a French graduate student, proposed in his doctoral dissertation that the wave-particle duality then known to exist for radiation was also a characteristic of matter. At the time, his suggestion was highly speculative, since there was yet no experimental evidence for wave-like behavior of electrons or any other particles. De Broglie described his realization with these words:

> After the end of World War I, I gave a great deal of thought to the theory of quanta and to the wave-particle dualism ... It was then that I had a sudden inspiration. Einstein's wave-particle dualism was an absolutely general phenomenon extending to all physical nature.

Since the universe consists entirely of matter and radiation, de Broglie's hypothesis is a fundamental statement about the grand symmetry of nature.

## The de Broglie Hypothesis

De Broglie stated his proposal with the following simple equations for the frequency and wavelength of electron waves, which are referred to as the *de Broglie relations*:

$$f = \frac{E}{h}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where $E$ is the total energy, $p$ is the momentum, and $\lambda$ is called the *de Broglie wavelength* of the particle. For photons, these same equations result directly from Einstein's quantization of radiation $E = hf$ and the equation $E^2 = (pc)^2 + (mc^2)^2$ for a particle of zero rest energy $E = pc$ as follows:

$$E = pc = hf = \frac{hc}{\lambda}$$

By a more indirect approach using relativistic mechanics, de Broglie was able to demonstrate that the de Broglie relations also apply to particles with mass.

## Particle-Wave Interference

In a brief note in the August 14, 1925 issue of the journal *Naturwissenschaften*, Walter Elsasser, at the time a student of Franck's (of the Frank-Hertz experiemnt), proposed that the wave effects of low-velocity electrons might be detected by scattering them from single crystals. The first such measurements of the wavelengths of electrons were made in 1927 by Davisson and Germer, who were studying electron reflection from a nickel target at Bell Telephone Laboratories.

The wave properties of neutral atoms and molecules were first demonstrated by Stern and Estermann in 1930 with beams of helium atoms and hydrogen molecules diffracted from a lithium fluoride crystal.

The constructive and destructive interference exhibited in particle beams may lead one to believe that de Broglie waves combine in composite objects via some type of Fourier synthesis. But that turns out not to be the case. Rather, a nonlinear relation can be simply derived that relates a composite de Broglie wavelength to its component de Broglie wavelengths.

## The Two-Body Case

In combining the parts of an object into a composite object, the total mass is a simple addition of the component masses:

$$m_c = m_1 + m_2$$

where $m_c$ is the composite or total mass. The de Broglie relations of each part can be given as:

$$m_1 = \frac{h}{\lambda_1 v} \qquad m_2 = \frac{h}{\lambda_2 v}$$

whereas for the composite de Broglie wavelength we have

$$\lambda_c = \frac{h}{(m_1 + m_2)v} = \frac{h}{(\frac{h}{\lambda_1 v} + \frac{h}{\lambda_2 v})v} = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

## The Three-Body Case

In solving for the 3-body case, we assume that 2 parts combine as before, then we combine with a third part, i.e.,

$$\lambda_c = \frac{\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} \cdot \lambda_3}{\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} + \lambda_3} = \frac{\lambda_1 \lambda_2 \lambda_3}{\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3}$$

## The General Case

We now state the general relation for the composition of de Broglie wavelengths for an arbitrary number of component wavelengths:

$$\lambda_c = \frac{\prod_{n=1}^{N} \lambda_n}{\sum_{\substack{j=1 \\ i_1 < i_2 < \cdots < i_{N-1}}}^{\binom{N}{N-1}} \lambda_{i_1} \lambda_{i_2} \cdots \lambda_{i_{N-1}}}$$

This relation has many deep implications for the meaning of mass and a theory of quantum gravity.

## References


1. Paul Tipler and Ralph Llewellyn. *Modern Physics*. W.H. Freeman and Company, New York, 1999.
2. De Broglie, Louis. *Matter and Light: The New Physics*. Dover, New York, 1939.
3. Resnick, R. and D. Halliday. *Basic Concepts in Relativity and Early Quantum Theory*. 2d ed., Wiley, New York, 1992.