On the Planck Fine-structure Constant

Rodolfo A. Frino
Electronics Engineer
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In this paper I introduce a new Planck constant – the Planck fine-structure constant –. Then, from the relativistic model of the hydrogen atom I prove that this new constant is consistent with the existence of hydrogen, and hence, consistent with the appearance of life in the universe.

Keywords: *Planck's constant, Planck unit, fine-structure constant, electromagnetic coupling constant, atomic structure constant, Planck charge, Planck fine-structure constant, Planck electromagnetic coupling constant, Planck atomic structure constant.*

Contents

1. Introduction

The fine-structure constant, commonly denoted α , was introduced by the German physicist Arnold Sommerfeld in the year 1916. This constant is one of the most enigmatic constants of nature. The American physicist Richard Feynman referred to the mysterious aspect of this constant with these words:

"It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!" [1]

The Austrian-born physicist Wolfgang Pauli referred to this constant as an unsolved mystery:

"The theoretical determination of the fine structure constant is certainly the most important of the unsolved problems of modern physics." [2]

The American physicist John Wheeler referred to the fundamental aspect of the fine-structure constant:

"But some numbers, called dimensionless numbers, have the same numerical value no matter what units of measurement are chosen. Probably the most famous of these is the "fine-structure constant," .... Physicists love this number not just because it is dimensionless, but also because it is a combination of three fundamental constants of nature." [3]

The British mathematician and physicist Roger Penrose referred to the relationship this constant has with the strength of the electromagnetic interactions:

"There are considerable mysteries surrounding the strange values that Nature's actual particles have for their mass and charge. For example, there is the unexplained 'fine structure constant' ... governing the strength of electromagnetic interactions, ..." [4]

The American physicist Frank Wilczek referred to the limits of the predictability power of quantum electrodynamics (QED):

"QED reduces ... "all of chemistry and most of physics," to one basic interaction, the fundamental coupling of a photon to electric charge. The strength of this coupling remains, however, as a pure number, the so-called fine-structure constant, which is a parameter of QED that QED itself is powerless to predict." [5]

Despite the fact that, so far, no theory has been successful enough to predict the value of the fine-structure constant, QED has achieved a great success in explaining electromagnetic interactions between electrons, positrons and photons. In the next section I shall define a new Planck constant (*) with a remarkable property that will give us a better understanding of the universe.

(*) Because most Planck units (if not all) have precise physical meanings, I prefer to call them constants rather than units. However we have to keep in mind that these constants are made of other constants.

2. The Planck Fine-structure Constant

The fine-structure constant is defined by the following formula

$$\alpha = \frac{e^2}{2\epsilon_0 h c} \tag{2.1}$$

In Appendix 2, I have derived the value of the Planck charge independently from the definition of the fine-structure constant. Thus, according to Appendix 2 the Planck charge is given by

$$Q_P = \sqrt{2\epsilon_0 hc} \tag{2.2}$$

Now, let me define a new constant (or unit if you like): the Planck fine-structure constant, that I shall denote α_P , as

$$\alpha_P = \frac{Q_P^2}{2\epsilon_0 h c} \tag{2.3}$$

Where I have replaced the elementary change, *e*, with the Planck charge, Q_P . From equations (2.2) and (2.3) we get

$$\alpha_P = \frac{\left(\sqrt{2\epsilon_0 hc}\right)^2}{2\epsilon_0 h c} \tag{2.4}$$

Which gives

$$\alpha_P = \frac{2\epsilon_0 hc}{2\epsilon_0 h c} \tag{2.5}$$

Finally

Planck fine-structure constant

$$\alpha_P = 1 \tag{2.6}$$

Thus, we find that the value of the Planck fine-structure constant is 1. But what does this mean? In other words, what is the physical meaning of this constant? The knowledge that allow us to interpret and understand the Planck fine-structure constant comes from the relativistic model of the hydrogen atom. This is done in the next section.

3. The Relativistic Atomic Model

The relativistic model of the hydrogen atom that I formulated in 2014 [6], predicted that the radius of the hydrogen atom is a function of the quantum number *n* and the fine-structure constant (let me put it this way although the fine-structure constant is not a variable). According to this formulation the atomic radius turned out to be

$$r_n = \left(n\sqrt{\frac{n^2}{\alpha^2} - 1}\right)\frac{h}{2\pi m_0 c} \tag{3.1}$$

$$n = 1, 2, 3, 4$$

Where *n* is the principal quantum number that takes integer values. From this formula we deduce that the hydrogen atom can exist, if and only if, the square root yields a positive number. Then, the condition for the existence of hydrogen is

$$\frac{n^2}{\alpha^2} - 1 > 0 \tag{3-2}$$

which means that

$$\alpha < n \tag{3-3}$$

The lowest value of the second side of inequation (3-3) occurs when $n = 1$ (the lowest quantum number). Thus, we can rewrite the condition for the existence of hydrogen as

Condition for the existence of hydrogen

$$\alpha < 1 \tag{3-4}$$

But because the fine-structure constant is defined as

$$\alpha = \frac{e^2}{2\epsilon_0 h c} \tag{3-5}$$

Then we can write

$$\frac{e^2}{2\epsilon_0 h c} < 1 \tag{3-6}$$

Solving this inequation for the speed of light, we get

$$c > \frac{e^2}{2\epsilon_0 h} \tag{3-7}$$

Now we define the constant c_H as

$$c_H \equiv \frac{e^2}{2\epsilon_0 h} \tag{3-8}$$

$$c_H = 2\,187691.266 \frac{m}{S}$$

c_H is the speed of the electron in its fundamental or lowest energy level. This velocity is also known as v_1. However, since this constant is a threshold for the speed of light (in vacuum) to ensure the existence of the hydrogen atom, we shall use a new nomenclature, c_H, to indicate that this constant is a new constant of nature.
Now inequation (3-7) can be rewritten as follows

$$c > c_H \tag{3-9}$$

This inequation reveals one of the most important conditions for the existence of hydrogen in the universe

A necessary condition for the existence of hydrogen atoms is that the fine-structure constant to be less than 1:

Condition for the existence of hydrogen $$\alpha < 1 \qquad (3\text{-}10 = 3.4)$$

Since α *= 0.007 297 352 57, this condition is satisfied.*

Comparing eq. (2.6) with the last inequality, ineq. (3.4), we find out that the Planck fine-structure constant represents the limit for the existence of the hydrogen atom. In other words, if the fine-structure constant were greater or equal than the Planck fine-structure constant, then hydrogen wouldn't have formed. In turn, this means that water wouldn't have formed either. And without water life wouldn't have arisen, at least, as we know it.

4. Conclusions

The findings presented in this paper are summarized in the following table

CONDITION	Would hydrogen have formed?	Would life have arisen? (as we know it)
$\alpha < \alpha_P$ (4.1) or, equivalently $\alpha < 1$	Yes	Yes
$\alpha >= \alpha_P$ (4.2) or, equivalently $\alpha >= 1$	No	No

Table 1: *The necessary condition for the formation of hydrogen atoms and for life to arise.*

Thus, condition (4.1) is a necessary condition for the formation of hydrogen. In other words, this inequation is telling us that the universe "knows" what the necessary condition for the existence of life is. This condition is written in the value of the Planck fine-structure constant:

Planck fine-structure constant $$\alpha_P = 1 \qquad (4.3)$$

This is a remarkable result that I did not see when I wrote the relativistic model of the hydrogen atom.

In summary, the formation of hydrogen atoms requires that the fine-structure constant to be less than 1. Since the value of this constant is *0.007 297 352 57,* approximately, nature satisfies this condition beautifully. However, and more importantly,

nature shows us, in a very simple mathematical form, that the value of the Planck fine-structure constant, is exactly the boundary that separates universes with life forms from those without them.

Appendix 1
Nomenclature

I shall use the following nomenclature for the constants and variables used in this paper

α = fine-structure constant (electromagnetic coupling constant, atomic structure constant)
c = speed of light in vacuum
h = Planck's constant
e = elementary charge
G = Newton's gravitational constant
ϵ_0 = permittivity of vacuum
c_H = speed of the electron in its fundamental or lowest energy level
n = principal quantum number (or simply quantum number)
Q_P = Planck electric charge
α_P = Planck fine-structure constant (Planck atomic structure constant)
QED = quantum electro-dymnamics
F = electrostatic force
F_P = Planck force
k = Coulomb's constant
q_1 = electric charge 1
q_2 = electric charge 2
q = electric charge
r = Section 3: radius of the hydrogen atom. Appendix 2: distance between the electric charges
r_n = radius of the hydrogen atom
m_0 = electron rest mass
v_1 = orbital velocity of the electron for the ground level (n =1)
L_P = Planck length

For the rest of the symbol used in this paper please refer to reference [6]

Appendix 2
Derivation of the Planck Charge from the Coulomb's Law

The idea is to use the Coulomb's law which is given by

$$F = k\frac{q_1 q_2}{r^2} \tag{A2.1}$$

Where the constant *k* is the Coulomb's constant

$$k = \frac{1}{4\pi\epsilon_0} \tag{A2.2}$$

We shall assume that we have two identical charges, *q*, separated by a distance equal to the planck length, L_P . We also assume that the force between them is equal to the Planck force, F_P . Using these values we write the above law as follows

$$F_P = k\frac{q^2}{L_P^2} \tag{A2.3}$$

Where the Planck length is given by

$$L_P = \sqrt{\frac{hG}{2\pi c^3}} \tag{A2.4}$$

and the Planck force is given by

$$F_P = \frac{c^4}{G} \tag{A2.5}$$

The problem is to find the expression of the charge, *q*, that satisfies equation (A2.3). Thus, we solve eq. (A2.3) for *q*. This yields

$$q = L_P\sqrt{\frac{F_P}{k}} \tag{A2.6}$$

Now we replace the values of L_P , F_P and *k* by equations (A2.4), (A2.5) and (A2.2), respectively. This produces

$$q = \sqrt{\frac{4\pi\epsilon_0 hG}{2\pi c^3}\frac{c^4}{G}} \tag{A2.7}$$

Which, after simplification, yields

$$q = \sqrt{2\epsilon_0\, h\, G} \tag{A2.8}$$

Now we use this value of charge as the definition of the Planck charge

Planck charge
$$Q_P \equiv \pm\sqrt{2\epsilon_0\, h\, G} \tag{A2.9}$$

This way we have found the expression for the Planck charge independently from the formula for the fine-structure constant.

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