# Multi Criteria Decision Making Based on Projection and Bidirectional Projection Measures of Rough Neutrosophic Sets

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## ABSTRACT

*In this paper, we define projection and bidirectional projection measures between rough neutrosophic sets. Then two new multi criteria decision making methods are proposed based on neutrosophic projection and bidirectional projection measures respectively. Then the proposed methods are applied for solving multiple criteria group decision making problems. Finally, two numerical examples are provided to demonstrate the applicability and effectiveness of the proposed methods.*

KEYWORDS: Rough neutrosophic set; projection measure; bidirectional projection measure.

## Section 1. INTRODUCTION

The concept of fuzzy set theory made its first appearance in the literature in two nearly simultaneous publications by Zadeh (1965) and Klaua (1965). Zadeh's work caught much more attention of the researchers than Klau's pure mathematical treatment. Zadeh (1965) defined fuzzy set by introducing membership function to deal non-statistical uncertainty. Atanassov (1983, 1986) defined intuitionistic fuzzy sets by introducing non-membership function as independent component. Smarandache (1998, 1999, 2002, 2005, 2010) introduced indeterminacy membership function as independent component and defined neutrosophic set. Smarandache (1998) paved the way to define single valued neutrosophic set (SVNS) (Wang et al., 2010) to deal realistic problems. SVNSs (Wang et al.,2010) have been widely studied and applied in different fields such as medical diagnosis (Ye, 2015b), multi criteria/multi attribute decision making (Sodenkamp, 2013; Ye, 2013a, 2013b, 2014a, 2014b, 2015a,; Biswas et al. 2014a, 2014b, 2015a, 2015b, 2016a, 2016b, 2017a, 2017b; Kharal, 2014; Liu et al., 2014; Liu & Li, 2017; Liu & Wang, 2014; Sahin & Liu, 2015; Peng et al., 2016; Pramanik et al., 2015, 2016; Broumi & Smarandache, 2013; Mondal & Pramanik, 2015d, 2015e), educational problem (Mondal & Pramanik, 2014b, 2015a), conflict resolution (Pramanik & Roy 2014), social problem (Pramanik & Chakrabarti, 2013, Mondal & Pramanik, 2014a), optimization (Das & Roy, 2015; Hezam et al, 2015; Abdel-Baset et al., 2016; Pramanik, 2016a, 2016b; Sarkar et al., 2016), clustering analysis (Ye, 2014a, 2014b), image processing (Cheng & Guo, 2008; Guo & Cheng, 2009; Guo et al., 2014), etc.

Pawlak (1982) proposed the concept of rough set. Rough set is an extension of the classical set theory (Cantor, 1874). It is very useful in dealing with incompleteness.

Broumi et al. (2014a, 2014b) proposed the concept of rough neutrosophic set (RNS) by combining the concept of rough set (Pawlak, 1982) and neutrosophic set (Smarandache, 1998). Rough neutrosophic set is very useful to deal with uncertain, inconsistent and incomplete information. Yang et al. (2016) introduced single valued neutrosophic rough sets on two-universes and presented an algorithm for multi criteria decision making (MCDM). Mondal and Pramanik (2015b) presented rough multi-attribute decision making based on grey relational analysis. Pramanik and Mondal (2015a) defined cosine similarity measure of rough neutrosophic sets and presented a MCDM approach in medical diagnosis. Mondal and Pramanik (2015c) presented MADM method using rough accuracy score function. Pramanik and Mondal (2015c) proposed cotangent similarity measure under rough neutrosophic environment. Pramanik and Mondal (2015b) further proposed some similarity measures namely, Dice similarity measure and Jaccard similarity measure in rough neutrosophic environment and their applications in MADM problems. Mondal et al. (2016a) defined several trigonometric Hamming similarity measures such as cosine, sine, cotangent similarity measures and proved some of their properties. In the same study (Mondal et al., 2016a) also presented MADM models based on Hamming similarity measures. Mondal et al. (2016b) proposed rough neutrosophic variational coefficient similarity measure and presented its application in multi attribute decision making. Mondal et al. (2016c) presented rough neutrosophic TOPSIS for multi-attribute group decision making problems.

Pramanik and Mondal (2015d) studied interval neutrosophic multi-attribute decision-making method based on GRA. Mondal and Pramanik (2015f) developed MADM methods based on cosine similarity measure, Dice similarity measure and Jaccard similarity measures under interval rough neutrosophic environment.

Mondal and Pramanik (2015g) proposed tri-complex rough neutrosophic similarity measure and presented its application in multi-attribute decision making problems. Mondal et al. (2016d) defined rough neutrosophic hyper-complex set and presented its application to multi-attribute decision making problem.

Yue & Jia (2015) proposed a method for multi attribute group decision making (MAGDM) problems based on normalized projection measure, in which the attribute values are offered by decision makers in hybrid form with crisp values and interval data. Yue (2012a) studied a new method for MAGDM based on determining the weights of decision makers using an extended projection method with interval data. Yue (2012a) Xu and Da (2004) and Xu (2005) studied projection method for decision making in uncertain environment with preference information. Yue (2012b) described a model to obtain the weights of DMs with crisp values using a projection method. Yue (2017) defined new projection measures in real number and interval settings and proposed group decision-making with hybrid decision information, including real numbers and interval data. Zheng et al. (2010) proposed an improved grey relational projection method by combining grey relational analysis (GRA) and technique for order of preference by similarity to ideal solution (TOPSIS) to select the optimum building envelope.

Yang et al. (2014) develop projection method for material selection problem in fuzzy environment. Xu and Hu (2010) developed two projection based models for MADM in intuitionistic fuzzy and interval valued intuitionistic fuzzy environment. Zeng et al. (2013) provided weighted projection algorithm for intuitionistic fuzzy MADM problems and interval-valued intuitionistic fuzzy MADM problems.

Chen and Ye (2016) developed the projection based model for solving neutrosophic MADM problem and applied it to select clay-bricks in construction field.

Dey et al. (2016b) defined weighted projection measure with interval neutrosophic environment and applied it to solve MADM problems with interval valued neutrosophic information. Ye (2015c) developed a projection measure-based multiple attribute decision making method with interval neutrosophic information and credibility information.

To overcome the shortcomings of the general projection measure, Ye (2016) introduced a bidirectional projection measure between single valued neutrosophic numbers and developed MADM method for selecting problems of mechanical design schemes under a single valued neutrosophic environment. Ye (2015d) also presented the bidirectional projection method for multiple attribute group decision making with neutrosophic numbers.

Dey et al. (2016a) proposed a new approach to neutrosophic soft MADM using grey relational projection method. Yue (2012b) presented a projection method to obtain weights of the experts in a group decision making problem. Yue (2013) proposed a projection based approach for partner selection in a group decision making problem with linguistic value and intuitionistic fuzzy information.

Dey et al. (2017) defined projection, bidirectional projection and hybrid projection measures between bipolar neutrosophic sets and presented bipolar neutrosophic projection based models for multi-attribute decision making problems.

Literature review reflects that no studies have been made on multi-attribute decision making using projection and bidirectional projection measures under rough neutrosophic environment. In this paper, we propose projection and bidirectional projection measures under rough neutrosophic environment. We also present two numerical examples to show the effectiveness and applicability of the proposed measures.

Rest of the paper is organized as follows: Section 2 describes preliminaries of neutrosophic number, SVNS and rough neutrosophic set (RNS). Section 3 describes the projection measure and bidirectional projection measure between neutrosophic numbers. Section 4 presents definition and properties of proposed projection measure and bidirectional projection measure between RNSs. In section 5 we describe a numerical example. Finally, section 6 presents the conclusion.

## Section 2. PRELIMINARIES

In this Section, we provide some basic definitions regarding SVNSs, RNSs which are useful for developing the paper.

### 2.1 Neutrosophic set

In 1998, Smarandache offered the following definition of neutrosophic set [1].

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A$, an indeterminacy membership function $I_A$ and a falsity membership function $F_A$. The functions $T_A$ , $I_A$ and $F_A$ are real standard or non-standard subsets of $]^-0, 1^+[$ that is $T_A$:X$\rightarrow$ $]^-0, 1^+[$, $I_A$:X$\rightarrow$ $]^-0, 1^+[$ and $F_A$:X$\rightarrow$ $]^-0, 1^+[$. It should be noted that there is no restriction on the sum of $T_A(x)$ , $I_A(x)$ and $F_A(x)$ i.e. $0^- \le T_A(X) + I_A(X) + F_A(X) \le 3^+$.

Definition 2.1.2: (complement) The complement of a neutrosophic set A is denoted by c(A) and is defined by $T_{c(A)}(x) = \{1^+\}-T_A(x)$, $I_{c(A)}(x) =\{1^+\}-I_A(x)$, $F_{c(A)}(x) =\{1^+\}-F_A(x)$.
Definition 2.1.3: (Containment) A neutrosophic set A is contained in the other neutrosophic set B, denoted by $A \subseteq B$ iff

$$\inf T_A(x) \le \inf T_B(x), \sup T_A(x) \le \sup T_B(x), \inf I_A(x) \ge \inf I_B(x), \sup I_A(x) \ge \sup I_B(x)$$
$$\text{and} \quad \inf F_A(x) \ge \inf F_B(x), \sup F_A(x) \ge \sup F_B(x) \; \forall x \in X$$

Definition 2.1.4: (Single-valued neutrosophic set). Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function $T_A(x)$, a falsity membership function $F_A(x)$ and indeterminacy function $I_A(x)$ with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0,1] \;\; \forall \text{ x in X}.$$

When X is continuous, a SNVS S can be written as follows:

$$A = \int_x < T_A(x), F_A(x), I_A(x) > / x \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows:

$$A = \sum < T_A(x), F_A(x), I_A(x) > / x \forall x \in X$$

For a SVNS S, $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

Definition2.1.5: The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1-I_A(x)$, $F_{c(A)}(x) = T_A(x)$.
Definition 2.1.6: A SVNS A is contained in the other SVNS B, denoted as $A \subseteq B$ iff,

$$T_A(x) \le T_B(x), I_A(x) \ge I_B(x) \text{ and } F_A(x) \ge F_B(x), \forall x \in X.$$

**2.2 Rough neutrosophic set** (Broumi et al., 2014a, 2014b)

Broumi et al., (2014a, 2014b) defined hybrid intelligent structure called Rough neutrosophic set.

Definition 2.2.1: Let Y be a non-null set and R be an equivalence relation on Y. Let P be a neutrosophic set in Y with the membership function $T_P$, indeterminacy membership function $I_P$ and falsity membership function $F_P$. The lower and the upper approximations of P in the approximation space (Y, R) denoted by $\underline{N(P)}$ and $\overline{N(P)}$ are respectively defined as:

$$\underline{N(P)} = \left\langle < x, T_{\underline{N(P)}}(x), I_{\underline{N(P)}}(x), F_{\underline{N(P)}}(x) > / y \in [x]_R, x \in Y \right\rangle$$

and

$$\overline{N(P)} = \left\langle < x, T_{\overline{N(P)}}(x), I_{\overline{N(P)}}(x), F_{\overline{N(P)}}(x) > / y \in [x]_R, x \in Y \right\rangle$$

where,

$$T_{\underline{N(P)}}(x) = \wedge z \in [x]_R T_P(Y), I_{\underline{N(P)}}(x) = \wedge z \in [x]_R I_P(Y), F_{\underline{N(P)}}(x) = \wedge z \in [x]_R F_P(Y) \text{ and}$$

$$T_{\overline{N(P)}}(x) = \vee z \in [x]_R T_P(Y), I_{\overline{N(P)}}(x) = \vee z \in [x]_R I_P(Y), F_{\overline{N(P)}}(x) = \vee z \in [x]_R F_P(Y).$$

So,

$$0 \le T_{\underline{N(P)}}(x) + I_{\underline{N(P)}}(x) + F_{\underline{N(P)}}(x) \le 3$$

and

$0 \le T_{\overline{N(P)}}(x) + I_{\overline{N(P)}}(x) + F_{\overline{N(P)}}(x) \le 3.$

Here $\vee$ and $\wedge$ denote "max" and "min" operators respectively. $T_P(y)$, $I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of Y with respect to P.

Thus NS mappings $\underline{N}$ , $\overline{N}$: N(Y)-> N(Y) are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair ( $\underline{N}(P)$, $\overline{N(P)}$ ) is called the rough neutrosophic set in (Y, R).

Definition 2.2.2 If N(P) = ( $\underline{N(P)}$, $\overline{N(P)}$ ) is a rough neutrosophic set in (Y, R) , the rough complement of N(P) is the rough neutrosophic set denoted by ~(N(P)) and defined as: ~(N(P)) = (( $\underline{N(P)}^c$), ( $\overline{N(P)}$ )$^c$),

where ( $\underline{N(P)}$ )$^c$ and ( $\overline{N(P)^c}$ ) are the complements of neutrosophic sets $\underline{N(P)}$ and $\overline{N(P)}$ respectively.

**Section 3. PROJECTION AND BIDIRECTIONAL PROJECTION MEASURE OF ROUGH NEUTROSOPHIC SETS**

Existing projection and bidirectional projection measure are not capable of dealing with MCDM problems in rough neutrosophic environment. Therefore, new projection and bidirectional projection measures between RNSs are proposed.

Assume that M and N are two RNSs represented by

$M=\{<(\underline{T_M(x_i)}, \underline{I_M(x_i)}, \underline{F_M(x_i)}),(\overline{T_M(x_i)}, \overline{I_M(x_i)}, \overline{F_M(x_i)})>: i = 1, 2, \dots, n\}$

and

$N=\{<(\underline{T_N(x_i)}, \underline{I_N(x_i)}, \underline{F_N(x_i)}),(\overline{T_N(x_i)}, \overline{I_N(x_i)}, \overline{F_N(x_i)})>: i = 1, 2, \dots, n\}.$

Then, the inner product of M and N denoted by M.N can be defined as:

$$M.N = \sum_{i=1}^{n} [\underline{T_1(x_i)}.\underline{T_2(x_i)} + \underline{I_1(x_i)}.\underline{I_2(x_i)} + \underline{F_1(x_i)}.\underline{F_2(x_i)} + \overline{T_1(x_i)}.\overline{T_2(x_i)} + \overline{I_1(x_i)}.\overline{I_2(x_i)} + \overline{F_1(x_i)}.\overline{F_2(x_i)}].$$

The modulus of M can be defined as

$$\|M\| = \sqrt{\sum_{i=1}^{n} [\underline{T_1(x_i)}^2 + \underline{I_1(x_i)}^2 + \underline{F_1(x_i)}^2 + \overline{T_1(x_i)}^2 + \overline{I_1(x_i)}^2 + \overline{F_1(x_i)}^2 ]}$$

and the modulus of N can be defined as

$$\|N\| = \sqrt{\sum_{i=1}^{n} [\underline{T_2(x_i)}^2 + \underline{I_2(x_i)}^2 + \underline{F_2(x_i)}^2 + \overline{T_2(x_i)}^2 + \overline{I_2(x_i)}^2 + \overline{F_2(x_i)}^2 ]}.$$

Definition4.1. The projection of M on N can be defined as:

$$Proj(M)_N = \frac{1}{\|N\|} M.N.$$

Definition4.2.The bidirectional projection measure between the RNSs M and N is defined as:

$$BProj(M, N) = \frac{1}{1 + \left| \|M\| - \|N\| \right| M.N} = \frac{\|M\| \, \|N\|}{\|M\| \, \|N\| + \left| \|M\| - \|N\| \right| M.N}.$$

Here also the bidirectional projection measure satisfies the following properties:

(1) BProj(M, N) = BProj(N, M);

(2) $0 \le BProj(M,N) \le 1$;

(3) BProj(M, N) = 1, iff M = N.

**Proof:**

**(i)** $BProj(M,N)=\frac{1}{1+\left|\|M\|-\|N\|\right|M.N}=\frac{1}{1+\left|\|N\|-\|M\|\right|N.M}=BProj(N,M)$

**(ii)** As $\frac{1}{1+\left|\|M\|-\|N\|\right|M.N}\geq 0$ and $\frac{1}{1+\left|\|M\|-\|N\|\right|M.N}\leq 1$ so, $0\leq BProj(M,N)\leq 1$

**(iii)** If M = N then $BProj(M,N)=BProj(M,M)=\frac{1}{1+\left|\|M\|-\|M\|\right|M.M}=1$

**Section 4. PROJECTION AND BIDIRECTIONAL PROJECTION BASED DECISION MAKING METHODS FOR MCDM PROBLEMS WITH ROUGH NEUTROSOPHIC INFORMATION**

In this section, we develop projection and bidirectional projection based MCDM models to solve MCDM problems with rough neutrosophic information. Consider $E=\{E_1, \ldots, E_n\}$ be a set of alternatives and $A=\{A_1, \ldots, A_m\}$ be a set of attributes. Now we present two algorithms for MCDM problems involving rough neutrosophic information.

**4.1 PROJECTION BASED DECISION MAKING METHODS FOR MCDM PROBLEMS WITH ROUGH NEUTROSOPHIC INFORMATION**

**Algorithm 1.**

Step 1. The value of alternative $E_i$(i = 1, 2, ….., n) for the attribute $A_j$(j = 1, 2, ……, m) is evaluated by the decision maker in terms of RNSs and the rough neutrosophic decision matrix is constructed as:

$$Z=\langle Z_{ij}\rangle_{n\times m}=\begin{bmatrix} Z_{11} & Z_{12} & \ldots & Z_{1m} \\ Z_{21} & Z_{22} & \ldots & Z_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ Z_{n1} & Z_{m2} & \ldots & Z_{nm} \end{bmatrix} \text{ where } Zij=\langle(\underline{T}_{ij},\underline{I}_{ij},\underline{F}_{ij}),(\overline{T}_{ij},\overline{I}_{ij},\overline{F}_{ij})\rangle \text{ with}$$

$0\leq \underline{T}_{ij}+\underline{I}_{ij}+\underline{F}_{ij}\leq 3$ and $0\leq \overline{T}_{ij},\overline{I}_{ij},\overline{F}_{ij}\leq 3$.

Step 2. Determine the ideal solution $S^*=\{S_1, S_2, \ldots, S_m\}$.

If $A_i$ is benefit type attribute then $S_i=\{(\min_j \underline{T}_{ji},\max_j \underline{I}_{ji},\max_j \underline{F}_{ji}),(\max_j \overline{T}_{ji},\min_j \overline{I}_{ji},\min_j \overline{F}_{ji})\}$.

If $A_i$ is cost type attribute then $S_i=\{(\max_j \underline{T}_{ji},\min_j \underline{I}_{ji},\min_j \underline{F}_{ji}),(\min_j \overline{T}_{ji},\max_j \overline{I}_{ji},\max_j \overline{F}_{ji})\}$.

Step 3. Compute the projection measure between $S^*$ and $Z_i=\langle Z_{ij}\rangle_{nxm}$ for all i = 1, ….., n and j = 1, ….., m. According to the descending order of projection measure $Proj(Z_i)_{S^*}$ for i = 1, …., n alternatives are ranked and highest value of $Proj(Z_i)_{S^*}$ reflects the best option.

## 4.2. BIDIRECTIONAL PROJECTION BASED DECISION MAKING METHODS FOR MCDM PROBLEMS WITH ROUGH NEUTROSOPHIC INFORMATION

**Algorithm 2.**

Step 1. The value of alternative $E_i$(i = 1, 2, ….., n) for the attribute $A_j$(j = 1, 2, ……, m) is evaluated by the decision maker in terms of RNSs and the rough neutrosophic decision matrix is constructed as:

$$Z = <Z_{ij}>_{nxm} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots & Z_{2m} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{m2} & \dots & Z_{nm} \end{bmatrix} \text{ where } Zij = <(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), (\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij})> \text{ with}$$

$0 \le \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \le 3$ and $0 \le \overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij} \le 3$.

Step 2. Determine the ideal solution $S^* = \{S_1, S_2, \dots, S_m\}$.

If $A_i$ is benefit type attribute then $S_i = \{(\min_j \underline{T}_{ji}, \max_j \underline{I}_{ji}, \max_j \underline{F}_{ji}), (\max_j \overline{T}_{ji}, \min_j \overline{I}_{ji}, \min_j \overline{F}_{ji})\}$.

If $A_i$ is cost type attribute then $S_i = \{(\max_j \underline{T}_{ji}, \min_j \underline{I}_{ji}, \min_j \underline{F}_{ji}), (\min_j \overline{T}_{ji}, \max_j \overline{I}_{ji}, \max_j \overline{F}_{ji})\}$.

Step 3. Compute the bidirectional projection measure between $S^*$ and $Z_i = <Z_{ij}>_{nxm}$ for all i = 1, ….., n and j = 1, 2, ….., m. According to the descending order of bidirectional projection measure $BProj(Z_i, S^*)$ for i = 1, 2, …., n alternatives are ranked and highest value of $BProj(Z_i, S^*)$ reflects the best option.

## Section 5. NUMERICAL EXAMPLES

**Example 1**: Assume that a decision maker intends to select the most suitable smartphone from the three initially chosen smartphones ($S_1$, $S_2$, $S_3$) by considering four attributes namely: feature $A_1$, price $A_2$, customer care $A_3$, and risk factor $A_4$.

Step1: The decision maker forms the following decision matrix:

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $S_1$ | <(.6,.3,.3), (.8,.1,.1)> | <(.6,.4,.4), (.8,.2,.2)> | <(.6,.4,.4), (.8,.2,.2)> | <(.7,.4,.4), (.9,.2,.2)> |
| $S_2$ | <(.7,.3,.3), (.9,.1,.3)> | <(.6,.3,.3), (.8,.3,.3)> | <(.6,.2,.2), (.8,.4,.2)> | <(.7,.3,.3), (.9,.3,.3)> |
| $S_3$ | <(.6,.2,.2), (.8,.0,.2)> | <(.7,.3,.3), (.9,.1,.1)> | <(.7,.4,.6), (.9,.2,.4)> | <(.6,.3,.2), (.8,.1,.2)> |

Step2: Here $A_2$ and $A_4$ are the cost type attributes.

So, the ideal solution is:
$S^*$= [<(.6,.3,.3), (.9,.0,.1)>, <(.7,.3,.3), (.8,.3,.3)>, <(.6,.4,.6), (.9,.2,.2)>, <(.7,.3,.2), (.8,.3,.3)>].
Step3: Determination of the projection and bidirectional projection measure:

$$\|S^*\| = 6.06, \|S_1\| = 2.387467, \|S_2\| = 2.424871, \|S_3\| = 2.412468$$

$$S_1.S^* = 5.78,\ S_2.S^* = 5.80,\ S_3.S^* = 5.82.$$

$$\mathrm{Proj}(S_1)_{S^*} = 0.953795,\ \mathrm{Proj}(S_2)_{S^*} = 0.957095,\ \mathrm{Proj}(S_3)_{S^*} = 0.960396.$$

$$\Rightarrow \mathrm{Proj}(S_3)_{S^*} > \mathrm{Proj}(S_2)_{S^*} > \mathrm{Proj}(S_1)_{S^*}$$

$$\Rightarrow S_3 > S_2 > S_1.$$

$$\mathrm{BProj}(S_1, S^*) = 0.405320,\ \mathrm{BProj}(S_2, S^*) = 0.410714,\ \mathrm{BProj}(S_3, S^*) = 0.407818.$$

$$\Rightarrow \mathrm{BProj}(S_2, S^*) > \mathrm{BProj}(S_3, S^*) > \mathrm{BProj}(S_1, S^*)$$

$$\Rightarrow S_2 > S_3 > S_1.$$

Here $S_3$ is the best alternative according to projection measure and $S_2$ is the best alternative according to bidirectional projection measure. As bidirectional projection measure is better than projection measure so the decision maker selects the smartphone $S_2$.

**Example 2**: Assume that a decision maker intends to select the most suitable location of modern logistic centre from the three initially chosen locations ($K_1$, $K_2$, $K_3$) by considering six attributes namely: cost $L_1$, distance to suppliers $L_2$, distance to customers $L_3$, conformance to government and law $L_4$, quality of service $L_5$, environmental impact $L_6$.
Step1: The decision maker forms the following decision matrix:

| | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ |
|---|---|---|---|---|---|---|
| $K_1$ | <(.85,.05,.05), (.95,.15,.15)> | <(.75,.15,.10), (.85,.25,.20)> | <(.75,.15,.10), (.85,.25,.20)> | <(.75,.15,.10), (.85,.25,.20)> | <(.75,.15,.10), (.85,.25,.20)> | <(.85,.05,.05),(.95,.15,.15)> |
| $K_2$ | <(.45,.45,.35),(.55,.55,.55)> | <(.75,.15,.10), (.85,.25,.20)> | <(.45,.45,.35), (.55,.55,.55)> | <(.75,.15,.10), (.85,.25,.20)> | <(.75,.15,.10), (.85,.25,.20)> | <(.45,.45,.35), (.55,.55,.55)> |
| $K_3$ | <(.45,.45,.35), (.55,.55,.55)> | <(.85,.05,.05), (.95,.15,.15)> | <(.75,.15,.10), (.85,.25,.20)> | <(.75,.15,.10), (.85,.25,.20)> | <(.85,.05,.05), (.95,.15,.15)> | <(.45,.45,.35), (.55,.55,.55)> |

Step2: Here $L_1$, $L_2$, $L_3$ are cost type attributes
So, the ideal solution is:
$S^*$= [ < (.85,.05,.05), (.55,.55,.55)>, < (.85,.05,.05), (.85,.25,.20)>, <(.75,.15,.10), (.55,.55,.55)>, <(.55,.30,.25), (.85,.25,.20)>, <(.75,.15,.10), (.95,.15,.15)>, <(.45,.45,.35), (.95,.15,.15)>].
Step3: Determination of the projection and bidirectional projection measure:

$\|S^*\| = 2.997916$

$\|K_1\| = 3.004995, \|K_2\| = 2.926602, \|K_3\| = 2.966479$

$K_1.S^* = 8.3475, \ K_2.S^* = 8.1450, \ K_3.S^* = 8.2325.$

$\mathrm{Proj}(K_1)_{S^*} = 2.784434, \ \mathrm{Proj}(K_2)_{S^*} = 2.716897, \ \mathrm{Proj}(K_3)_{S^*} = 2.746074.$

$\Rightarrow \mathrm{Proj}(K_1)_{S^*} > \mathrm{Proj}(K_3)_{S^*} > \mathrm{Proj}(K_2)_{S^*}$

$\Rightarrow K_1 > K_3 > K_2.$

$B\mathrm{Proj}(K_1, S^*) = 0.993481, \ B\mathrm{Proj}(K_2, S^*) = 0.937908, \ B\mathrm{Proj}(K_3, S^*) = 0.971721.$

$\Rightarrow B\mathrm{Proj}(K_1, S^*) > B\mathrm{Proj}(K_3, S^*) > B\mathrm{Proj}(K_2, S^*)$

$\Rightarrow K_1 > K_3 > K_2.$

Hence, $K_1$ is the best alternative.

## Section 6. CONCLUSION

This paper defines projection measure and bidirectional projection measure between rough neutrosophic sets. Two new multi criteria decision making methods have been proposed based on the proposed neutrosophic projection and bidirectional projection measures respectively. Finally, two numerical examples are provided to demonstrate the applicability and effectiveness of the proposed methods. The proposed methods can be extended for solving multi criteria decision making in interval neutrosophic rough environments.

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