CAN QUANTUM MECHANICAL SYSTEMS INFLUENCE THE GEOMETRY OF THE FIBER BUNDLE/SPACE-TIME?

VICTOR ATANASOV & HRISTO DIMOV

Abstract. We suggest that gravitation is an emergent phenomenon which origin is the information signal associated with quantum fields acting like test particles. We have shown how the metric (Lamé) coefficients emerge as position & time operator mean value densities. The scalar curvature of the space-time in the case of a Bose-Einstein condensate or super- fluid/conductor is calculated and an experimentally verifiable prediction of the theory is made.

For the better part of his later life Albert Einstein struggled to unite gravity with electromagnetism within the empirical four-dimensional space-time and explain all of physics in terms of one unified field (the geometric field). Theodor Kaluza (1919) and Oskar Klein (1926) showed a way which involved the expansion of the number of dimensions the embedding curved space-time should have in order for the gravitational field to mask electromagnetism[1]. Later, Kaluza-Klein theory was resurrected in spirit in String theory[2]. Yet, the extra number of dimensions which these theories need in order to circumvent the mathematical difficulties of unification of the geometric field with the quantum/electromagnetic ones, consistently fail to show up in empirical data (amongst other problems) thus rendering these ideas speculative at best.

Quantum Field Theory (QFT) is the framework for understanding the birth of quantum particles (quasiparticles in condensed matter physics as well), that is field quanta, from an underlying physical field. However, the nature and meaning of this underlying physical field is by far the hardest concept in physics. In the present paper we are going to extend the difficulty of the concept of the quantum field by suggesting that it is the quantum field that determines the geometry of the underlying base space/space-time of the fiber bundle its section it represents (from an abstract mathematical point of view in analogy with classical vector fields which are sections of the tangent bundle). Such a proposition is certainly not a novelty since General Theory of Relativity (GTR) is founded on exactly the same premise: the local energy-momentum tensor (depends on fields) sets the geometry of space-time. Indeed, on classical level the geometry of space-time can be traced by a test particle. In GTR particles follow geodesics and if one takes two adjacent geodesics their deviation is governed by the curvature tensor directly. Therefore, the all powerful notion of a test particle is applicable in the case of general relativity as well.

However, the notion of a test particle in the case of quantum fields is missing. What is the particle that probes for the Higgs field, for example? Or other quantum bosonic or fermionic field for that matter? In QFT the focus is on the excitations of the fields, that is the particles themselves. The quantum fields are not defined as force or energetic

Date: 18 April 2017.

aspects of some interaction, par excellence. Therefore, we pose the important question: If QFT is all about particles, then to what fields these particles are test particles (besides the trivial answer: electromagnetic)? Here, we would argue that it is the geometric field that the quantum particles are test particles to. In addition, we would argue that quantum fields can set the geometry of the base space/space-time at infinitesimal scale (and vice-versa). Similar ideas are discussed in[3].

Quantum fields have the meaning of probability distributions. Probability is not associated with any material source. It is a pure information field regarding some stochastic process. Therefore, the quantum field acting like a test particle is nothing but an information field. It gives an information measure on the existence of a set of geometric points. The quantum field acts like an information signal that reveals the geometric field. Since the quantum field manifests as a test particle it can also interact with the geometric field.

An all powerful axiom of quantum mechanics states that observables are represented by hermitian operators. However, there is an observable that is not represented by a hermitian operator, and that is the evolutionary parameter called time. Time is not the eigenvalue of an operator which is in stark contrast to the particle's position x, which is the eigenvalue of the position operator $\hat{x}$. This is the main reason why it is so difficult to incorporate relativity into quantum mechanics. There are two ways to put time and space on equal footing: i.) promote time to an operator; or ii.) lower position from its stature as an operator and reduce it as a label, like time. The latter approach took over QFT as it is the easier way over the problem with time. However, doing so the entire aspect of geometry-quantum system interaction akin to GTR is obscured. This is the line of thought we want to explore in the present paper.

Upon the reduction of position to a label, something peculiar happens, namely a new purpose (instead of the geometric one) needs to be assigned and this is the purpose of being a label to something. And that something is operators and quantum fields. Consider assigning a field to each point x in space, call these functions $\psi(x)$ and their set over the coordinate base space forms a section of a fiber bundle. This section is called a quantum field. In the Heisenberg picture these operators are also time dependent and this is accounted for with the help of the total energy operator $\hat{H} : \psi(x,t) = e^{i\hat{H}t/\hbar}\psi(x,0)e^{-i\hat{H}t/\hbar}$. In this way, position and time are now on an equal footing as labels on operators; neither is itself the eigenvalue of an operator. These two different approaches to relativistic quantum theory, might yield different results but this is not the case. It is a choice motivated by convenience and calculation efficiency that rendered position and time as labels on operators.

Let us now discuss a completely different approach, namely let us assume that it is the operators (plus their algebra) and the quantum fields that are given as a universal quantum mechanical (mathematical) structure of the Universe prior to the geometric field. In this case, we only need the following things: operators and abstract "ket" and the conjugate "bra" vectors belonging to some separable (in order to have causality) Hilbert space. In fact, the "ket"-quantum field is a section of a fiber bundle which fibers are the separable Hilbert spaces. These ingredients generate quantum numbers, which can be infinitely many or even continuous.

$$\hat{\mathcal{O}} \; |ket\rangle = \text{quantum numbers } \; |ket\rangle \tag{1}$$

For example one such quantum number is the position x of the quantum test particle. The continuous set of all quantum numbers gives rise to an integration domain $\mathcal{D}$ and integration measure which we usually use to calculate the norm. In addition, this framework can set the geometry of the integration domain which is the base space and as a result gravitation becomes an emergent phenomenon. One can think of this as gravitation being an information signal produced by all the quantum fields in the Universe. Such a statement can be supported by the lack of a true energy-momentum tensor for the gravitational field, therefore one can question the material nature of the gravitational field altogether. Indeed, we are left with assuming it is an information signal.

Now let us focus on the quantum mechanical position operator. What is its meaning in the case of a quantum condensate? If it were a quantum particle, the ready answer is the average expectation value for the position where we would find the particle if we make an experiment. Let us take a free quantum particle moving in one direction with a particular momentum p_x then the uncertainty in its momentum would be vanishing and according to Heisenberg's uncertainty relation $\Delta p_x \Delta x \geq \hbar/2$ the uncertainty in localising it would be infinite $\Delta x \to \infty$. The wave-function that describes this state of the quantum particle is the plane wave solution

$$\psi(x,t) = e^{i\frac{p_x x}{\hbar} - i\omega t}, \qquad 1 = \int dx\, \psi^* \psi$$

where the amplitude is determined by the normalisation condition and ω is the circular frequency. In this state the location of the particle is given by

$$l_x = \int dx'\, \psi^* \hat{x'} \psi, \tag{2}$$

that is the size of the integration domain. Note, the uncertainty in localising the particle is not infinite, but rather as large as the size of the integration domain. Next, suppose the size of the integration domain $\mathcal{D}$ is infinitely small, then according to the Mean value theorem

$$dl_x = \int_{\mathcal{D}} dx'\, \psi^* \hat{x'} \psi = \psi^* \hat{x} \psi\, dx = h_x dx. \tag{3}$$

Here $h_x = \psi^* \hat{x} \psi$ coincides with the dimensionless Lamé metric coefficient. One might think this coefficient is simply a re-parametrisation of the $x-$coordinate. However, a re-parametrisation of the base space would not change its scalar curvature, because it is an invariant. Therefore, if we obtain a non-vanishing scalar curvature from these Lamé metric coefficients, we would know with certainty that it is the interaction with the quantum field that produces the curved base space.

In this way we have seen how the quantum mechanical position operator gives rise to the metric coefficients at infinitesimal scale:

$$ds^2 = -c^2 h_t^2 dt^2 + h_x^2 dx^2 + h_y^2 dy^2 + h_z^2 dz^2, \tag{4}$$

where

$$h_t = \psi^* \hat{t} \psi, \quad h_x = \psi^* \hat{x} \psi, \quad h_y = \psi^* \hat{y} \psi, \quad h_z = \psi^* \hat{z} \psi \tag{5}$$

The metric is always a bilinear symmetric form acting locally on the tangent space. In the tetrad formalism this form can be diagonalised and as a result the Lamé coefficients are the tetrad fields and the metric has the form stated above. However, the Lorentz causality encoded in the metric is not derivable within this formalism. It is chosen according to empirical data. Note, the introduction of a time operator is necessary to complete the four dimensional space-time metric. In addition, it also helps define the geometry of the space-time.

For example, suppose there is only one non-constant Lamé metric coefficient, that is the x coordinate one, which depends on the evolutionary t coordinate: $h_x = h_x(t)$. The Ricci scalar curvature is given by

$$R = \frac{1}{2}\frac{1}{c^2 h_t^2 h_x^2}\frac{\partial^2 h_x^2}{\partial t^2}. \tag{6}$$

Alternatively, we may assume that the y part depends only on the x coordinate: $h_y = h_y(x)$, while the rest of the metric coefficients are constant, then the Ricci scalar curvature is given by $R = -1/(2h_y^2 h_x^2)\, \partial^2 h_y^2/\partial x^2$.

To illustrate our point we take a model equation for the single-particle wave-function in a Bose-Einstein condensate (gas of bosons that share the same wave-function), which is similar to the Ginzburg-Landau equation, that is the time-independent Gross-Pitaevskii equation which describes the ground state of a bosonic quantum system using the mean-field approximation and an interaction model. The non-linearity in the equation stems from the interaction between the bosons:

$$\mu\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + eE\hat{x}\psi + \lambda\left|\psi\right|^2\psi, \tag{7}$$

where μ is the chemical potential, E is the electric field along the direction x and λ is a coupling constant. We can then express the Lamé metric coefficient h_x as

$$h_x = \psi^*\hat{x}\psi = \psi^*\left(\frac{\mu}{eE} + \frac{1}{eE}\frac{\hbar^2}{2m}\nabla^2 - \frac{\lambda}{eE}\left|\psi\right|^2\right)\psi, \tag{8}$$

where the number of particles is related to the wave-function/order parameter quantum field by $n = \int|\psi|^2 d^3r$, which for sufficiently homogenius field is $n = |\psi|^2/V$. Here V is the volume occupied by the condensate. Now, suppose that the quantum field is giving the base space its geometry, that is the geometry at the particular quantum state, then

$$\begin{aligned} R \propto \quad & \frac{\partial^2}{\partial t^2}\left\{\psi^*\left[\frac{\mu}{eE} + \frac{1}{eE}\frac{\hbar^2}{2m}\nabla^2 - \frac{\lambda'}{eE}n\right]\psi\right\}^2 \\ & = \frac{\partial^2}{\partial t^2}\left\{\left[\frac{\mu}{eEV}n + \frac{1}{eE}\, f\left(\mathrm{div}\vec{J}, J^2, \frac{1}{|\psi|}, \Delta|\psi|\right) - \frac{\lambda'}{eEV}n^2\right]\right\}^2, \end{aligned} \tag{9}$$

where λ' is the coupling constant per unit volume. Here $f\left(\mathrm{div}\vec{J}, J^2, \frac{1}{|\psi|}, \Delta|\psi|\right)$ is a function containing the current $\vec{J}$ and its derivatives as well as the amplitude of the order parameter and its derivatives. Note, at a stationary state, the geometry of the base space is strictly flat $R = 0$. Curvature can be induced only if there is dynamics in the quantum field, number of particles or electric field/current. *This represents an experimentally verifiable prediction of the present framework.*

An alternative view towards the geometric field as being an information field, comes from the Heisenberg uncertainty relations, encoded in the operators' algebra. For example, the interaction of the quantum field with the geometric field can be inferred from the uncertainty relation between the two non-commuting variables of a quantum condensate: the number n_s of Cooper pairs/bosons and the phase θ of the superconducting wave-function or order parameter[4]: $\delta n_s \delta\theta \geq 1/2$. Suppressing the variation in the number of particles, that is either the entire condensed matter system has made the super- fluid/conducting transition $n_s = \max \Rightarrow \delta n_s = 0$, or the condensate is being destroyed $n_s = 0 \Rightarrow \delta n_s = 0$, the phase variable should experience amplified fluctuations. Provided a relation between the phase of the quantum condensate and the curvature of the base space/space-time exists[5], we can safely assume that in a state in which the phase cannot be specified, the geometry of space-time cannot be specified to being exactly flat. Therefore, energy can be channeled into the base space geometry and create curved space-time configuration.

In conclusion, we would like to point out an alternative view towards time: in newtonian mechanics, time introduces a complete causal order of events, therefore it is tempting to suggest that if a "causal order" operator exists in quantum mechanics it would actually be the time operator. And time would be his eigenvalue, that is quantum number. In this paper we have suggested that the geometric (gravitational) field is an emergent phenomenon associated with the information content (quantum fields as probability distributions) in the Universe. In effect, quantum fields are information signals that reveal the geometric field. We have shown how the metric (Lamé) coefficients emerge as position & time operator mean value densities. The scalar curvature of the base space/space-time in the case of a Bose-Einstein condensate is calculated and an experimentally verifiable prediction of the theory is made. The prediction involves the creation of curvature of space-time via manipulation of the dynamics of the quantum field, number of particles or electric field/current.

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Faculty of Physics, Sofia University, 5 boul. J. Bourchier, 1164 Sofia, Bulgaria

E-mail address: `vatanaso@phys.uni-sofia.bg`