Preliminary Evidence That a Neoclassical Model of Physics (L3) Might Be Correct
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Abstract

Today’s standard model of physics treats the physical masses of elementary particles as given, and assumes that they have a bare radius of zero, as in the older classical physics of Lorentz. Many physicists have studied the properties of the Yang-Mills-Higgs model of continuous fields in hopes that it might help to explain where elementary particles (and their masses) come from in the first place. This paper reviews some of the important prior work on Yang-Mills-Higgs and solitons in general, but it also shows that stable particles in that model cannot have intrinsic angular momentum (spin). It specifies four extensions of Yang-Mills Higgs, the Lagrangians L1 through L4, which are closer to the standard model of physics, and shows that one of the four (L3) does predict/explain spin from a purely neoclassical theory. The paper begins by summarizing the larger framework which has inspired this work, and ends by discussing two possibilities for further refinement.

I. Goal of this paper

I.1. The General Goal And Framework

This paper offers a possible solution to the age-old quest to try to learn what the ultimate laws of physics really are.

The search for the “law of everything,” the underlying laws of physics from which everything else in the universe follows as a kind of emergent behavior, has been a core goal of science for centuries. In the modern world of quantum mechanics, this quest has actually split up into two different quests connected to each other: (1) the quest for the proper formulation of quantum mechanics itself; and (2) the quest for the specific Lagrangian function which, when inserted into the proper formulation, gives the concrete and specific form of the law of everything.

This paper is not intended as a discussion or debate about the proper formulation of quantum mechanics, or of how to model to cosmos in general. It is a report on the quest for the correct Lagrangian function, to be used within one particular formulation, which I would call the “neoclassical” formulation of the laws of physics. In this formulation, I make a sharp distinction between the physics which governs everyday life (not counting gravity or nuclear explosions) and the deeper physics which the everyday physics emerges from.

This paper will focus on the deeper level, and propose a formulation so simple that many modern physicists would find the idea shocking. For the sake of the modern physicist, I will briefly review the larger framework and what motivates it in section 1.2. The nonphysicist should be reassured that many of the technical details cited in 1.2 are not really needed in the rest of this paper. Section 1.3 will state the more specific goal of this paper in mathematical terms.
## 1.2. MQED versus the proposed deeper level

Everyday life, our world seen at a resolution no finer than 3 femtometers, is governed in my view by Markovian Quantum Electrodynamics (MQED) [1,2,3,4], a new variation of QED which is still essentially within the mainstream of quantum field theory. In MQED it is assumed that the dynamics of the cosmos are still described correctly and exactly by the usual Maxwell-Dirac “Schrodinger equation” over the usual Fock-Hilbert space. As in the original canonical form of QED, from the 1950’s [5], everything is governed by the normal form (based on normal product) of the Maxwell-Dirac Hamiltonian, which does not contain the massive zero-point energy terms assumed in many other mainstream forms of QED. MQED can correctly predict phenomena like those of cavity QED, which usually assumes zero-point terms, by imposing strict new requirements on the models used to describe macroscopic objects and measurement, described in detail with examples and proposed decisive experiments in [1,2,3,4]. I developed MQED in great part as a response to the diversity of models, devices, questions and experiments which I oversaw when directing a research area called “QMHP” at the US National Science Foundation [34].

The database of experiments which underlies QED in general is far richer and far more complex than all the rest of the database of physics put together; for example, it includes all the advanced work done by the electronics and photonics industries and high-precision engineering for those areas. Logically, I would plead for the reader to evaluate, study and use MQED on its own merits, and to avoid entangling the clear issues of MQED with the much greater heresy and controversy about what lies below the level of MQED.

Yet in the end, to understand how the cosmos works, we do need to address the question of how to integrate our understanding of electrodynamics with our understanding of gravity and of the nuclear force(s). We do need to examine a variety of ways to perform that integration, before we can be ready to propose decisive experiments (as I have already done to try to tighten up our understanding of QED) to be sure about which way is correct.

Mainstream physics mostly tries to perform this integration on two tracks: (1) there is work to further develop the Standard Model of Physics, which is defined as the combination of Electroweak Theory (EWT), an extension of QED designed to predict weak nuclear decay and neutrinos as well as QED proper [7,8], and Quantum Chromodynamics (QCD), the current form of the quark theory intended to describe strong nuclear forces and most of the menagerie of new particles discovered since the middle of the twentieth century [8,9]; (2) a wide variety of theories of quantum gravity attempt to unify the standard model with gravity.

Long ago, Einstein proposed a much simpler approach to unification. In effect, he proposed that the entire cosmos could be modeled as a set of functions (“fields”) defined over ordinary Minkowski space, and he proposed that the great complexities of quantum mechanics, Fock-Hilbert space and QED could all be derived as the emergent statistical outcome of something which is much simpler at its core.

This approach was largely discredited for many years, because it was proven both in mathematics and in experiment that the forms of QED then taken for granted could not possibly be derived from something as simple as what Einstein had in mind. The situation was complicated by the fact that Einstein and the Copenhagen school of physics both relied on common sense assumptions about measurement and time which did not follow from
either type of mathematical model of the dynamics of the universe. However, it turns out that Einstein's general approach still makes sense, and can be reconciled in principle with what we really know in QED, if one throws out those extraneous assumptions about measurement [1,2].

Does it really make sense to explore (or even to tolerate) what we thought was an old discredited heresy, simply because new information suggests there might possibly be a simpler and better way forward after all? I would ask the reader to consider the analogy to the field of neural networks and deep learning, which were also viewed as a discredited heresy for many years, and which experienced two massive waves of intolerance, until finally [10-12] it became more generally known that they really can work, if one puts in the right kind of effort.

This paper reports on where we stand so far in putting in the right kind of effort to revive the core program of Albert Einstein, which I will call the “neoclassical” approach. The neoclassical approach relies strictly on “classical field theory,” and is different only because of the different larger framework, which will not be discussed further in this paper.

1.3. Definition and Proposal for a “Quasibosonic” theory of everything

1.3.1 General Mathematical Form

The proposal is that the state $S^*$ of the entire cosmos, across all space-time, can be specified by specifying the values of a set of functions over Minkowski space, and by specifying the usual metric tensor and such of general relativity [13]. The value of these functions at any point $x_\mu$ in space-time can be written as $\phi(x_\mu)$, where $\phi$ is a “vector” in the mathematical sense. The mathematical vector $\phi$ is actually a concatenation of objects which are allowed under special relativity, such as covariant vectors, scalar, spinors and tensors [13,14].

More specifically, we assume that the dynamics and the conservation laws of the cosmos can all be derived from a classical Lagrangian function, by applying the Lagrange-Euler equations, Hamilton relations and Noether theorem reviewed briefly in sections 2.2 and 2.3 of Mandl and Shaw [1], and in [6]. We define a quasibosonic theory as one which assumes that the true Lagrangian of the cosmos is:

$$\mathcal{L}_c = \sqrt{-g}(g_{\mu\nu}R^{\mu\nu} - 2\kappa\mathcal{L})d^4x_\mu$$  \hspace{1cm} (1)

where the core Lagrangian (addressing what the standard model of physics addresses) has the form:

$$\mathcal{L} = \sum_{j,k} C_{jk} \dot{\phi}_j \dot{\phi}_k + \sum_j K_j f_j (\phi, \nabla \phi) + g(\phi, \nabla \phi, \nabla^2 \phi)$$  \hspace{1cm} (2)

where $C$ is a nonnegative symmetric matrix and $g$ is a nonpositive function of the fields $\phi$ and of their first and second derivatives with respect to space, and where $\mathcal{L}$ is also required to be invariant under Lorentz transforms (i.e. to obey special relativity).

Equation 1, taken from Carmeli [16], is the modern, straightforward extension of the Lagrangian developed by John Wheeler of Princeton, for his “already unified field theory,”
which won him his Nobel Prize [13]. In essence, equation 1 assumes that general relativity is the correct theory of gravity.

It is a great thing that gravity researchers have been very creative in developing alternative theories of gravity, probing all aspects of general relativity, but at the present time it is more reasonable than ever to include general relativity in a new proposed model of the cosmos. When the standard model issues are addressed by equation 2, equation 1 is the straightforward way to unify those issues with gravity; there is certainly no need (or basis as yet) to assume additional dimensions of space-time or the like, in order to unify gravity and the standard model, when we use equation 2 directly to address the standard model issues. Because general relativity is now so well-established, this paper focuses entirely on the search for $\mathcal{L}$ and for ways to evaluate and use candidates for what $\mathcal{L}$ might be in more specific terms.

Equation 2 is a very general form, allowing for many alternative possibilities. For example, the Lagrangians assumed in EWT and in QCD can be written in this form; if we use those Lagrangians as classical field theories, those field theories are quasibosonic. The well-studied Yang-Mills-Higgs system [17,18] and the special case which gives rise to the famous BPS monopole [19,20] are also within this larger family. In the past, I have discussed two other candidate Lagrangians in this family [21] which I now call “L1” and “L2.”

### 1.3.2. Treatment and Derivation of Elementary Particles

Most traditional physics, whether quantum or Lorentzian classical physics, simply takes the existence of elementary particles for granted, and assumes that the menu of masses, lifetimes, and other properties is a fact of nature which cannot be explained. Thus, for example, all forms of QED generally assume that the electron is a point particle and a point source of electric charge, such that the mass-energy of the electron would be infinite, except for a negative infinite mass-energy ($\delta M$) implicitly assumed to exist at the core of the electron, an assumption invoked at the time of renormalization[5,15,21] necessary to make actual predictions. Many important observed properties of elementary particles [22,23] are simply not explained at all. Our goal here is to explain and predict the menu of particles by modeling them as stable “vortices” or “solitons” of a continuous field theory, in which mass-energy is never negative at any point in space-time.

More precisely, our long-term goal here is to find a Lagrangian $\mathcal{L}$ which possesses stable and metastable “soliton” or “vortex” states, such that the elementary particles of physics can be identified with these “solitons” or bound states of them, or, in a few cases (mainly light) with flows of energy between solitons which are quantized only because of the boundary conditions under which they are created and absorbed.

Even more precisely, our long-term goal is to model the most elementary particles, like the electron or an upgraded model of the quark, as mathematical objects which fulfill our definition of a “chaoiton” [24]. In mathematics, the word “soliton” has been defined precisely[25] as something which is too restrictive to fit the electron, but the phrase “solitary wave” [26] does not require the high level of stability which is required when we try to model the electron. A mathematical “soliton” would not change its direction of movement even after a collision, but a physical electron usually does. On the other hand, a physical electron cannot just leak out energy in the way that a solitary wave can. The chaoiton has gross properties which fit those of actual electrons. It also has the possibility of...
being a static stable state, or an oscillatory state, or even a chaotic state, so long as it possesses the level of stability required in our definition [24].

In physics, the word “soliton” has been used more loosely [27], and applied to different kinds of objects, some of which would meet the stability requirements for a chaitoiton, and some of which would be unacceptable because of how they would leak out energy.

In order to fulfill our goal, we have considered three possible types of chaitoiton to use as a model of the electron and of its closest cousins (like an upgraded model of the quark):

(1) The ordinary nontopological soliton, as might exist when all the field components $\phi_j$ are allowed to vary between $-\infty$ and $+\infty$, but are required to go to zero at distances ever further from the core location of the chaitoiton [28];

(2) The skyrmion family of solitons [17,27], in which some field components are defined to be objects which are “topologically nontrivial,” such as a set of unitary matrices which have the topology of a sphere;

(3) The Higgs family of topological solitons [17,18,19,20], in which the boundary conditions are relaxed so that only the energy density is required to go to zero at infinity, and in which the function “$g$” of equation 2 contains terms which give energy greater than 0 whenever certain field components deviate from a set of preferred values (like a vector in $\mathbb{R}^3$, for which the preferred values are the unit sphere as in [18,19]).

Presumably Einstein would have been quite happy with any of these three possibilities, if they resulted in predictions which fit nature, but at the present time class (3) seems far more promising. MRS even suggested that chaitoitons of the first class are mathematically impossible, at least for quasibosonic field theories [27]; our work suggests they might be wrong on that point [28], but it is a difficult unresolved area, and the work on topological solitons also provides a nice clean pathway to explaining why electric charge is so universally quantized.

The Higgs family is far more tractable than the Skyrme family, and is much closer to the standard model of physics today. Instead of imposing “nontrivial topology” apriori on certain field components, at all points in space, it merely takes advantage of energy terms to impose topology at the far horizon of the chaitoiton. Those terms are called “Higgs terms.” They look similar in a way to the Higgs terms which appear in EWT, but they are used here in a very different way.

In summary, our operational goal here is much narrower than the larger goal I have stated so far. As a practical approach to achieving the larger goal, I have been searching for candidate Lagrangians $\mathcal{L}$ which contain Higgs terms in the function “$g$” of equation 2, such that the resulting set of chaitoitons matches the properties required to describe the electron and its cousins.

More precisely, I have been searching for Lagrangians $\mathcal{L}$ which predict chaitoitons which possess at least two topological charges, one which fits electric charge and one which I will call “nuclear charge,” and which possesses intrinsic angular momentum, to explain “spin.” Matching today’s QCD exactly would require a model with more charges, but it is hoped that two will suffice (when other new properties of the system are accounted for), and in any
case the study of models with two charges provides a pathway to guide us to models with more charges if those turn out to be necessary. This paper provides an update on my search for such Lagrangians, presenting new analysis which narrows the search.

2. Some General Properties of Nonbosonic Lagrangians

The standard tools for analyzing Lagrangian field theories include the use of “conjugate fields.” For any specific field or field component $\phi_j$, the conjugate field is defined as:

$$\pi_j(x_\mu) \equiv \frac{\delta \mathcal{L}(x_\mu)}{\delta (\phi_j(x_\mu))}$$  \hspace{1cm} (3)

Here, I follow Mandl [5] in using “$\delta$” rather than “$\partial$” to indicate a simple algebraic partial derivative. Like Mandl, I preserve to reserve “$\partial$” for partial derivatives with respect to space or coordinate systems, because of how vitally important it is to distinguish between different types of partial derivative [29].

2.1 Mass-energy and Lack of Oscillation

Applying equation 3 to equation 2, we easily derive:

$$\pi_j = \sum_k 2C_{jk} \dot{\phi}_k + K_jf_j(\phi, \nabla \phi)$$  \hspace{1cm} (4)

The standard tools also include the well-known Hamiltonian energy density defined by:

$$\mathcal{H} \equiv \sum_j \pi_j \dot{\phi}_j - \mathcal{L}$$  \hspace{1cm} (5)

where the usual measure $d^3x$ is assumed implicitly as in almost all of physics. By substituting equations (4) and (2) into (5), we derive:

$$\mathcal{H} = \sum_{j,k} C_{jk} \dot{\phi}_j \dot{\phi}_k - g(\phi, \nabla \phi, \nabla^2 \phi)$$  \hspace{1cm} (6)

In [24], I proved that any choaiton must be a state $S$ (or set of states) of minimum energy in any system like this, where the energy density is nonnegative definite. This is essentially the same concept of (strong) stability for a soliton state $S$ used in physics [28]. Note that a state $S$ is defined here is defined as a set of values for $\phi(x)$ for all points $x$ in $\mathbb{R}^3$ and of values for $\dot{\phi}_j$ for all field components $j$ for which $\pi_j$ is not identically zero.

Even when the boundary conditions do not require $\phi \to 0$ as $|x| \to \infty$, equation 6 always makes it possible to find another state $S$ in the neighborhood of a proposed choaiton but with lower energy, whenever $\dot{\phi}_j$, has a nonzero value (and is one of the state variables). Thus we can immediately deduce that any choaiton, even a topological choaiton, in a quasibosonic system must possess zero $\dot{\phi}_j$ for all such $j$. For all practical purposes, this tells
us that we should search for static chaoitons, not oscillatory ones, to have any hope of finding an acceptable model of the electron within the class of quasibosonic systems, which are still the most promising place to look by far. This indication will become even stronger as we proceed.

Those familiar with the algebra of Dirac fields may look for an additional term in equation 6, linear rather than only quadratic in the time derivatives of a spinor. Those terms do not apply here, since even the spinors discussed in this paper are the spinors of classical mathematics [14], built up from complex numbers which are commutative under multiplication.

2.2. Angular Momentum

Another important standard tool is the definition of the momentum density \( p(x) \) of a Lagrangian system, extracted from equation (2.51b) of [15]:

\[
p_j(x) = T_{0j}(x) = \sum_k \pi_k(x) \frac{\partial \varphi_k(x)}{\partial x_j} \quad (j=1,2,3) \tag{7}
\]

and the angular momentum density vector \( \mathbf{M}(x) \) which may be written simply as the vector product:

\[
\mathbf{M}(x) = \mathbf{x} \times p(x) = (x_2 p_3 - x_3 p_2, x_3 p_1 - x_1 p_3, x_1 p_2 - x_2 p_1) \tag{8}
\]

or equivalently as a matrix \( M_{0k} = x_k T_{0k} - x_k T_{0j} \) as in equation 2.54 of [15], in the case (as here) where we do not assume exogenous angular momentum from sources other than the field theory itself.

For the case of chaoitons in a quasibosonic theory, where \( \dot{\varphi}_j \) must be zero for all \( j \) in which it is a state variable, we may substitute from equation 4 into equation 7 to derive:

\[
p_j(x) = \sum_k K_{k,j} f_k(\varphi, \nabla \varphi) \frac{\partial \varphi_k(x)}{\partial x_j} \tag{9}
\]

Clearly, if the functions \( f_k \) equal zero at all points in space for any chaoiton state, then momentum \( p \) and angular momentum \( M \) will also equal zero at all points in space. Total angular momentum is just the integral of \( \mathbf{M}(x) \) over all space, and it too would equal zero in that case. Such chaoitons have zero intrinsic angular momentum, and are thus unacceptable as models of the electron in Lagrangian field theory.

3. Angular Momentum in the Yang-Mills-Higgs System

3.1. Key Calculations From Previous Work

Eardley and Moncrief [18] provide wonderfully clear and rigorous results on the general Yang-Mills-Higgs system, for which they write the Lagrangian in Einstein notation as their equation 2.9:

\[
\mathcal{L} = Tr[-\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} (D_\mu \phi) \cdot (D^\mu \phi) - P(\phi) \tag{10}
\]
where $\phi$ is an “isovector,” a set of three real numbers $\phi^a$ which do not change under Lorentz rotations, and where the force tensor $F$ is a function of the underlying vector field $A_{\mu}^a$.

Here the mathematical vector $\phi$ has 15 components, the three components of the Higgs field $\phi$ and the twelve components of the gauge field $A_{\mu}^a$ as $\mu=0,1,2,3$ and $a=1,2,3$. Following equation 2.12 of [18], the Hamiltonian density here is

$$\mathcal{H} = \frac{1}{2} \sum_j TrE_j E_j + \frac{1}{4} \sum_{jk} Tr F_{jk} F_{jk} + \frac{i}{2} \pi^\phi \cdot \pi^\phi + \frac{1}{2} \sum_j (D_j \phi) \cdot (D_j (\phi)) + P(\phi) + Tr(A_{0} \zeta)$$

(11)

where $\pi^\phi$ is just the set of three numbers $\pi_a$ defined by equation 4 for the three field variables $\phi_1, \phi_2$ and $\phi_3$, where $\zeta$ turns out to be irrelevant for our purposes here, where $E_j$ is defined for $j=1, 2$ or 3 as:

$$E_j = F_{0j} = \frac{\partial A_j}{\partial t} - \frac{\partial A_0}{\partial x_j} + A_0 \times A_j$$

(12)

Note that $E_j$ is actually an isovector, a set of three real numbers $E_j^1, E_j^2$ and $E_j^3$. Likewise $A_{\mu}$ for any of the four allowed values of $\mu$, either 0 (representing time) or 1, 2 or 3 (a direction in space) is an isovector made up of three real numbers. Equation 2.13 of [18] includes:

$$\pi^\phi = \frac{\partial \phi}{\partial t} + A_0 \phi$$

(13)

Applying equation 4 to this system (as Eardley and Moncrief did in deriving their equation 2.12), we can easily see that $\pi_\mu^a$ for the A field variables for $\mu=1,2,3$ is just $E_\mu$.

### 3.2 Additional Comments and Explanation of Yang-Mills-Higgs

The work of Eardley and Moncrief, as important as it is, is just one paper in a huge literature on the Yang-Mills-Higgs system, and on the special limiting case known as the BPS monopole. [19]. The topological charge is generally interpreted as a magnetic charge, following an ansatz proposed by Hasenfratz and ‘tHooft [30]; however, the system itself is abstract, and electromagnetism is not an explicit part of the system proper. The field $A_{\mu}^a$ is an example of a type of field studied very widely in physics, called an SU(2) gauge field.

Intuitively, the topology created by the Higgs term (the last term in Eq. 10) makes it possible to “tie a knot in space, which cannot possibly be untied, because of the topology.” More precisely, the Higgs term creates topology such that there exist field states with “topological charge” different from zero, such that there exists no pathway of small perturbations of those particular field states of finite energy connecting those field states to the vacuum; in other words, there is no way that those field states could dissipate away to a vacuum state, through any sequences of small perturbations.

How does the Higgs term do that? Think of the proposed variational soliton as an equilibrium state of the fields $Q$ and $A$, centered on the origin, where $x=0$, or, in polar coordinates, $r=0$. As $r$ goes to infinity, $|Q|$ must go to $F$, or else the proposed state would have infinite energy. We are only interested in perturbations which lower our energy; thus we only need to consider perturbations which maintain this condition, that $|Q|$ goes to $F$, such that $Q$ goes to a vector $Q/|Q|$ on the sphere with radius 1. Crudely, for large enough 1, the function $Q/|Q|$ takes on values on the sphere of radius 1. The points of distance $r$ from the origin also form a sphere. Thus, as $r$ goes to
infinity, the function $Q$ is a mapping from the sphere $(S^2)$ to the sphere $S^2$. The Wikipedia article “Homotopy groups of sphere” has an excellent clear and concise overview of what this implies. In brief, $\pi_2(S^2) = \mathbb{Z}$. This means that the mappings from the sphere to the sphere are a disconnected set, indexed by the integers ($\mathbb{Z}$). The vacuum states are values of the function $Q$ (mappings) with one $Z$ index. The BPS monopole has values of $Q$ with a different $Z$ index. There is no way to get from the BPS monopole to the vacuum by small perturbations on a path where the states have finite energy. The “topological charge” of any state in the BPS system is simply the $Z$ index of that state.

The BPS monopole has another remarkable property important to our goals here. It “looks like” an exact point source of magnetism [19], even though its energy density is positive at all points and its total energy is finite (unlike the picture of the electron in today’s standard model of physics without renormalization). Thus if we model the electron by this general sort of model, we can expect to explain how the electron appears to be a perfect point source of a Coulomb field, without violating the goals of positive energy density and finite mass [21].

### 3.3. Angular Momentum of Possible Chaoitons

From section 2, we know that all possible chaoitons in this system (an example of a quasibosonic system) must be static in all the underlying field variables, the components of $A$ and $\phi$, because they all have nonzero conjugate variables in principle. But Eardley and Moncrief [18] also point out that we can represent any such chaoiton equivalently, without loss of generality, in the Weyl gauge, sometimes called the temporal gauge, in which $A_0 = 0$.

When we insert these conditions into equation 13, we immediately see that $\pi^\phi$ must be identically zero in such states. Likewise, when we insert them into equation 12, we see that the conjugate field values for all the nonzero field components must also be zero. From equation 7, this tells us that $p(x) = 0$ at all points $x$ in any such state, and thus we find that all chaoitons in the Yang-Mills-Higgs system must have zero intrinsic angular momentum.

### 4. Angular Momentum In Previously Proposed Lagrangians

In previous work [21], we asked whether we could devise a quasibosonic Lagrangian which would yield topological solitons with two conserved and quantized topological charges, capable of modeling a set of elementary particles which may possess both electric charge and another kind of charge, and help us begin to explain the menu of known basic particles [22,23]. We asked how to build such a theory as close as possible to the existing Standard Model of Physics, so as to maximize the probability it might actually be true.

This led to two alternative possible systems, which I would now call L1 and L2.

#### 4.1. Specification of L1, previously called “The Conservative Alternative”

The Lagrangian $L_1$ was defined by:

$$L_E = L_W + L_B + L_l + L_r - V$$

where
\[ \mathcal{L}_W = \frac{1}{4} F_{\mu \nu} \cdot F^{\mu \nu} (\text{i.e. } \frac{1}{4} F^{a}_{\mu \nu} F^{a \mu \nu}) \]  
(15)

\[ F_{\mu \nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g W_\mu \times W_\nu \]  
(16)

\[ \mathcal{L}_B = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu), \]  
(17)

\[ \mathcal{L}_i = c_i \left( \partial_\mu - \frac{i}{2} g \tau \cdot W_\mu - \frac{i}{2} g' B_\mu \right) \rho^2 \]  
(18)

\[ \mathcal{L}_s = c_s \left( \partial_\mu - \frac{i}{2} g' B_\mu \right) Q^2 \]  
(19)

\[ V = f_1 (Q^2 - F_1^2) + f_2 (\phi^2 - F_2^2) \]  
(20)

where the underlying fields are the two gauge fields \( W_\mu^a \) and \( B_\mu \), and the two Higgs fields \( Q^a \) and \( \phi^a \), where \( g, g', f_1 \) and \( f_2 \) are parameters, where \( \tau \) is the usual set of 4 Pauli matrices in EWT, and where \( f_1 \) and \( f_2 \) are smooth monotonic functions with the property that \( f(0)=0 \).

Equations 14 through 17 are exactly the same equations used in EWT. (For example, see section 8.4 of Taylor [7].) Equations 18 and 19 are the same coupling terms used in EWT, except that instead of coupling to fermions like electrons they couple here to the Higgs field which generate the solitons; in other words, if we model the electron and its cousins as solitons, we naturally adapt the coupling to couple to the solitons. Equation 20 is the biggest change; while EWT has a Higgs term, and recent experiments do affirm the existence of some kind of Higgs field, a relatively small change in the form of that term can explain how it is that we see stable particles. Though it is too early to be sure, the empirical evidence so far is consistent with the theory that here indeed two Higgs fields or the equivalent [31].

### 4.2. Key calculations for the L1 system

Note that \( W_\mu^a \) is an SU(2) gauge field, just like \( A_\mu^a \) in section 3. Its algebra is essentially the same. The algebra of \( B_\mu \) is essentially the same as that of the electromagnetic field, which is a well-known U(1) gauge theory. Each of the two isovectors, \( Q \) and \( \phi \), are used in Higgs terms in equation 20; they generate topological charges by exactly the same mechanism as the one which generates a topological charge in the Yang-Mills-Higgs system.

Two major questions immediately arise: (1) from equations 17 and 18, these topological charges act as sources for the B and W fields, just as ordinary electric charge acts as a source of electromagnetism, but how do we reconcile the two, with appropriate choices of charge for various elementary particles?; and (2) does this theory, unlike Yang-Mills-Higgs, predict (explain) particles with intrinsic angular momentum (spin)?

Question 1 gets into issues beyond the scope of this paper, but question 2 is already enough to rule out L1 as a law of everything.
Here, as in section 3, the time derivatives must all be zero in a chaoiton state, and we may use the Weyl gauge as before to give us the condition that \( W_0 \) and \( B_0 \) are zero. This immediately tells us that the conjugate fields must also all be zero in a chaoiton state, except possibly for the new terms implied by the new terms, equations 18 and 19, in the Lagrangian:

\[
\pi^\phi \equiv \frac{\delta \mathcal{L}_t}{\delta \dot{\phi}} = 2c_1 (\partial_0 - \frac{i}{2} g \tau \cdot W_0 - \frac{i}{2} g' B_0) \phi
\]

(21)

\[
\pi^Q \equiv \frac{\delta \mathcal{L}_r}{\delta \dot{Q}} = 2c_r (\partial_0 - \frac{i}{2} g' B_0) Q
\]

(22)

But as we inspect these terms, recalling that the time derivatives \( (\partial_0) \) and \( W_0 \) and \( B_0 \) are all zero in such a state, we find that the total conjugate fields are still all zero at all points in space. As in section 3, this tells us that total angular momentum must again be zero.

### 4.3. Specification and Discussion of L2, the Isotwistor Variation

L2 is defined as the Lagrangian specified by equations 14 through 17, but with three new equations to replace 18,19 and 20:

\[
\mathcal{L}_t = c_1 \left| \left( \partial_\mu - \frac{i}{2} g \tau \cdot W_\mu - \frac{i}{2} g' B_\mu \right) \Omega \right|^2
\]

(23)

\[
\mathcal{L}_r = c_r \left| \left( \partial_\mu - \frac{i}{2} g' B_\mu \right) \Omega^* \right|^2
\]

(24)

\[
V = f_1 \left( |\text{det} \Omega|^2 \right) + f_2 \left( (1 - \text{tr}(\Omega \Omega^*))^2 \right)
\]

(25)

Here, there is just one Higgs field, the two-by-two complex matrix \( \Omega \).

The isotwistor variation is more beautiful and geometric looking than L1, but a bit harder to work with. The Higgs term in equation 25 guarantees that as \( |x| \to \infty \), \( \Omega \) will go to matrices of determinant zero and trace one. Because of the zero determinant, \( \Omega \) will go to \( uv^* \) at every point on the sphere of radius \( r \), where \( u \) and \( v \) are vectors in \( \mathbb{C}^2 \), and \( * \) denotes the usual adjoint (transpose conjugate). Because of the unit trace, we may without loss of generality represent \( \Omega \) at each such point as \( uv^* \) for \( u \) and \( v \) both of unit length.

Thus, in the limit as \( r \) goes to infinity, we again have two vector fields, effectively. The topology of these fields is not \( S^2 \). The topology of vectors \( \mathbb{C}^2 \) is just the same as \( \mathbb{R}^4 \); however, with the constraint of unit length, we have the topology of \( S^3 \), the topology of a “hypersphere” in four dimensions. However, \( \pi_3(S^3) = \mathbb{Z} \). (This is called the “Hopf fibration.”) We still get two topological
charges, a u-charge and a v-charge, which can be called a left-hand charge and a right-hand charge, based on how they appear in Eqs. 23 and 24.

Nevertheless, L2 clearly shares the most basic properties of L1. While I have not worked out the algebra in all detail, it seems very unlikely that L1 could meet the requirement of predicting nonzero spin.

5. L3, The Higgs-Spinor Lagrangian Which Passes All Tests So far, and L4

5.1 Specifications of L3 and L4

The L3 Lagrangian would again be based on equations 14 through 17 but with equations 18 through 20 now replaced by:

\[
\mathcal{L}_l = ic_j \mu^\mu \left( \partial_\mu - \frac{i}{2} g \tau \cdot W_\mu - \frac{i}{2} g' B_\mu \right) u
\]

\[
\mathcal{L}_r = ic_r v^\mu \left( \partial_\mu - \frac{i}{2} g' B_\mu \right) v
\]

\[
V = f_1 \left( |u|^2 - 1 \right) + f_2 \left( |v|^2 - 1 \right)
\]

where \(u\) and \(v\) are vectors in \(\mathbb{C}^2\) (spinors), and where \(\tau^\mu\) is the usual four-dimensional version of the Pauli matrices [26]. The logic of sections 3 and 4 still applies; it still shows us that we have two topological charges in this system, a u-charge and a v-charge. This should still allow analytical proof of the existence of variational solitons, even without closed form expressions for them as in limiting cases of the BPS system [19].

Strictly speaking, there is also a full twistor variation, L4, where \(\Omega\) is a twistor (still in \(\mathbb{C}^2\) by \(\mathbb{C}^2\)), as in L2, but we replace Eqs 23 and 24 with:

\[
\mathcal{L}_l = ic_j Tr(\Omega \tau^\mu (\partial_\mu - \frac{i}{2} g \tau \cdot W_\mu - \frac{i}{2} g' B_\mu )\Omega^*)
\]

\[
\mathcal{L}_r = ic_r Tr(\Omega^* \tau^\mu (\partial_\mu - \frac{i}{2} g' B_\mu )\Omega)
\]

This is the most appealing variation, in some ways, but its mathematical properties are less obvious.

5.2. Key Calculations for L3

As with L1, all of the terms in the conjugate fields which depend on differentiating \(\mathcal{L}_w\) and \(\mathcal{L}_b\) reduce to zero for static states in the Weyl gauge. Thus the only nonzero terms are those which come from differentiating equations 26 and 27. From differentiating equation 26 we get:
\[ \pi^u_j \equiv \frac{\delta A_j}{\delta u_j} = \frac{\delta A_0}{\delta (\partial_0 u_j)} = i(c_i u^* \tau^0)_j \]  

(31)

In other words, we deduce that the vector conjugate to \( u \) is simply:

\[ \pi^u = ic_i u^* \tau^0 \]  

(32)

Likewise from differentiating equation 27 we get:

\[ \pi^v = ic_v v^* \tau^0 \]  

(33)

Very simply, unlike the case with L1 or Yang-Mills-Higgs, we generally expect the conjugate fields for the Higgs fields to be nonzero. Inserting equations 32 and 33 into equation 7, we derive:

\[ p(x) = ic_i u^*(x) \tau^0 \nabla u(x) + ic_v v^*(x) \tau^0 \nabla v(x) \]  

(34)

Inserting equation 34 into equation 8 yields:

\[ M(x) = x \times (ic_i u^*(x) \tau^0 \nabla u(x) + ic_v v^*(x) \tau^0 \nabla v(x)) \]  

(35)

Of course, we would not expect the integral of \( M(x) \) to be nonzero for spherically symmetric states, which do not possess an axis of rotation, but for axisymmetric stable states [32] such as spherical harmonics we would expect to see nonzero angular momentum here. To verify that such states are acceptable as a model of the electron, the next main task is to show that such stable states do exist, for some choice of the topological charges, for the simple Higgs function \( V \) given here or for some more complicated function with the necessary properties (analogous to the general function \( P \) in equation 10).

The Hamiltonian for this system may be derived by inserting \( L_3 \) into equation 6. This yields the sum of two copies of the left hand terms of equation 11 (one for \( W \) and a simplified one for \( B \)), minus the sum of equations 24, 25 and 26 with the time derivatives removed.

It is fascinating to consider the more concrete implications of this model for physics and perhaps even for technology (not unlike the hopes discussed in [17]), but that is for later. It is also intriguing that the statistical description of these spinor fields using distribution functions [6] may well be fermionic, in a straightforward way, in the limit of “large” \( x \) (more than about 3 femtometers!), due to the way the system behaves under Lorentz rotations.

Because the usual fine structure constant of QED, \( \alpha \), is ultimately derived from \( g \) and \( g' \) of EWT, and because the nature of mixing between \( B \) and \( W \) may be different here with a different type of Higgs term, it seems quite plausible that \( L_3 \) may explain the curious regularities observed by MacGregor at Livermore [23] and provide a mathematical grounding for the intuitive explanations suggested by Schilling [33].

It is quite possible that the ratio between angular momentum and mass for the electron in this model, expressed in these units, will not equal one-half. However, that would not be a
problem, if the relation between angular momentum and mass is the same for any emitted photons, resulting in the same value for Planck's constant.

6. Further Possibilities

In general, the scientific method demands that we not be excessively attached to one model, no matter how well it works. Researchers in gravity have given us an excellent example, of the need to try to develop multiple alternatives, to bring out key assumptions, to look for new phenomena, or to explain the present best model at a deeper level.

In that spirit, it would of course be interesting to specify and explore an L5 model, similar to L3 but with a (classical) Dirac spinor or two Dirac spinors, with V modified so that it still generates two topological charges.

It is also intriguing to consider trying to unify and explain the B and W fields themselves as aspects of something more fundamental. For example, consider the well known representation of 2 by 2 unitary matrices as:

\[
U = e^{iH} = e^{i\theta I} e^{i\phi \tau}
\]

(36),

the product of a matrix representing a U(1) gauge (like B) and another representing an SU(2) gauge (like W). Could there even be another way to unify EWT with gravity by somehow deriving U (in some form, perhaps real?) and g as two components of a single tensor, similar to the rotational and compressive components of a fluid flow? If Einstein were still alive, perhaps those are the kinds of possibilities he would be exploring most carefully, still without a need for additional dimensions of space-time. However, there is much more work to be done still in addressing issues within the scope of the standard model of physics, by building up step by step, starting with more analysis of what can be done with L3 (considering just modifications of the function V if needed, for now).

References