The Dark side of Gravity vs MOND/DM

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New developments of the Dark Gravity Theory which foundations and some consequences we have detailed in two previous articles predict that the strength of gravity could be enhanced in some space-time domains. A MOND radius also arises naturally in this framework so that hopefully the MOND/DM phenomenology may finally be within reach for Dark Gravity.

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1. Introduction

In the presence of a flat non dynamical background $\eta_{\mu\nu}$, it turns out that the usual gravitational field $g_{\mu\nu}$ has a twin, the "inverse" metric $\tilde{g}_{\mu\nu}$. The two being linked by

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\left[g^{-1}\right]^{\rho\sigma} = \left[\eta^{\mu\rho}\eta^{\nu\sigma}g_{\rho\sigma}\right]^{-1} \tag{1}$$

are just the two faces of a single field (no new degrees of freedom) that we called a Janus field [2][3][6][13][14]. See also [4][7][8][5] [29][30][31][32][33][27] for alternative approaches to Anti-gravity with two metric fields.

The action treating our two faces of the Janus field on the same footing is achieved by simply adding to the usual GR and SM (standard model) action, the similar action with $\tilde{g}_{\mu\nu}$ in place of $g_{\mu\nu}$ everywhere.

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L}) \tag{2}$$

where R and $\tilde{R}$ are the familiar Ricci scalars built from g or $\tilde{g}$ as usual and L and $\tilde{L}$ the Lagrangians for respectively SM F type fields propagating along $g_{\mu\nu}$ geodesics and $\tilde{F}$ fields propagating along $\tilde{g}_{\mu\nu}$ geodesics. This is invariant under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. This theory symmetrizing the roles of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ is Dark Gravity (DG) and the field equation satisfied by the Janus field derived from the minimization of the action is:

$$\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}G^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{G}^{\rho\mu} + \mu \leftrightarrow \nu = -8\pi G(\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}T^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{T}^{\rho\mu} + \mu \leftrightarrow \nu) \tag{3}$$

with $T^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ the energy momentum tensors for F and $\tilde{F}$ fields respectively and $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ the Einstein tensors (e.g. $G^{\mu\nu} = R^{\mu\nu} - 1/2g^{\mu\nu}R$). The equation was reformatted in such a way as to maintain as explicit as possible the symmetrical roles played by the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ of the Janus field which then required to restore by hand the $\mu \leftrightarrow \nu$ symmetry of the lhs and rhs tensors which we had lost by the way doing so. Of course $\tilde{g}_{\mu\nu}$ could also be replaced by it's expression (1) to get an equation for the dynamical field $g_{\mu\nu}$ in presence of the non dynamical $\eta_{\mu\nu}$. The value of (3) is the manifest anti-symmetry of the lhs under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. Replacing $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$, this translates into the requirement that the lhs be odd to all orders in $\bar{h}_{\mu\nu}$.

The contracted form of the DG equation simply is :

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{4}$$

We already detailed in previous articles why it is the most natural way to rehabilitate and understand time reversal, negative energies and anti-gravity : we have preferred frames where $\eta = diag(-1, 1, 1, 1)$ and a preferred global cosmic conformal time to reverse.

In the seventies, theories with a flat non dynamical background metric and/or implying many kinds of preferred frame effects became momentarily fashionable and Clifford Will has reviewed some of them (Rosen theory, Rastall theory, BSLL theory ...) in his book [34]. Because those attempts were generically roughly conflicting with accurate tests of various versions of the equivalence principle, the flat non dynamical background metric was progressively given up. The dark gravity theory we support here is a remarkable exception as it can easily reproduce most predictions of general relativity up to Post Newtonian order (as we shall remind in the two following sections) and for this reason deserves much attention since it might call into question the assumption behind most modern theoretical avenues: background independence.

It is well known that GR is the unique theory of a massless spin 2 field. However DG is not the theory of one field but of two fields: $g_{\mu\nu}$ and $\eta_{\mu\nu}$. Then it is also well known that there is no viable (ghost free) theory of two interacting massless spin 2 fields. However, even though $\eta_{\mu\nu}$ is a genuine order two tensor field transforming as it should under general coordinate transformations[a], $\eta_{\mu\nu}$ actually propagates no degrees of freedom : it is really non dynamical, not in the sense that there is no kinetic (Einstein-Hilbert) term for it in the action, but in the sense that all it's degrees of freedom were frozen a priori before entering the action and need not extremize the action : we have the pre-action requirement that Riemm(η)=0 like in the BSLL, Rastall and Rosen theories [34]. So DG is also not the theory of two interacting spin 2 fields.

[a]in contrast to a background Minkowski metric $\hat{\eta}_{\mu\nu}$ such as when we write $g_{\mu\nu} = \hat{\eta}_{\mu\nu} + h_{\mu\nu}$, which by definition is invariant since only the transformation of $h_{\mu\nu}$ is supposed to reflect the effect of a general coordinate transformation applied to $g_{\mu\nu}$

We found that the theory trivially avoids any kind of instabilities, at least about a Minkowskian background common to the two faces of the Janus field, because :

- Fields minimally coupled to the two different sides of the Janus field never meet each other from the point of view of the other interactions (EM, weak, strong) so stability issues could only arise in the purely gravitational sector.
- The run away issue [9] [10] is avoided between two masses propagating on $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively, because those just repel each other, anti-gravitationally as in all other versions of DG theories [8][5] rather than one chasing the other ad infinitum.
- The energy of DG gravitational waves vanishes about a common Minkowski (we remind in a forthcoming section that DG has a vanishing energy momentum pseudo tensor $t_{\mu\nu} - \tilde{t}_{\mu\nu}$) avoiding for instance the instability of positive energy fields through the emission of negative energy gravitational waves.

These points are very attractive so we were not surprised discovering that recently the ideas of ghost free dRGT bimetric massive gravity [35] have led to a PN phenomenology identical to our though through an extremely heavy, unnatural and Ad Hoc collection of mass terms fine tuned just to avoid the so called BD ghost. Indeed the first order differential equation in [31] is exactly the same as our: see e.g eq (3.12) supplemented by (4.10) and for comparison our section devoted to the linearized DG equations. This is because the particular coupling through the mass term between the two dynamical metrics in dRGT eventually constrains them to satisfy a relation Eq (2.4) which for $\alpha = \beta$ [31] becomes very similar to our Eq (1) to first order in the perturbations which then turn out to be opposite (to first order) as Eq (4.10) makes it clear.[b]

In section 2 we remind and complement the results of previous articles as for the global homogeneous solution and in section 3 the local static isotropic asymptotically Minkowskian solutions of the DG equation. In section 4 we discuss the linearized theory about this common Minkowskian background for $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ and the prediction of the theory as for the emission of gravitational waves. It turns out that the theory of one single Janus field can't account for both the global gravity of section 2 and the local gravity of section 3 and 4. In sections 5 and 6 we are then led to propose a unification scheme for the global and local Janus field theories based on an original quantization postulate, resulting in a renewed understanding of global expansion effects and the Pioneer anomaly. Sections 7, 8, 9, 10 explore the phenomenological consequences of such naturally extended DG theory in which the

[b]By the way, all such kind of bimetric constructions seriously question the usual interpretation of the gravitational field as being the metric describing the geometry of space-time itself. There is indeed no reason why any of the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, which describe a different geometry should be preferred to represent the metric of space-time. At the contrary our non dynamical flat $\eta_{\mu\nu}$ is now the perfect candidate for this role.

two sides of the local Janus field are not any more asymptotically identical: MOND like phenomenology, gravitational waves and black holes. Section 11 emphasizes the need for a theory of gravity such as Dark Gravity which very principles being based on discrete as well as continuous symmetries, for the first time open a natural bridge to quantum mechanics and hopefully a royal road toward a genuine unification. Section 12 discusses all kind of stability issues within the extended theory to conclude that the theory can be safe if it is not quantized otherwise a less natural new extension is necessary to insure the stability of a quantized version of the DG theory. Before concluding, section 13 further discusses our currently preferred option : DG as a semi-classical theory of gravity.

2. Global gravity

2.1. *The scalar-tensor cosmological field*

We found that an homogeneous and isotropic solution is necessarily spatially flat because the two sides of the Janus field are required to satisfy the same isometries. However, it is also static so that the only way to save cosmology in the DG framework is to introduce a tensor-scalar Janus field built from a scalar Φ such that $g_{\mu\nu} = \Phi\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \frac{1}{\Phi}\eta_{\mu\nu}$. Then our fundamental cosmological single equation obtained by requiring the action to be extremal under any variation of $\Phi(t) = a^2(t)$ is:

$$a^2\frac{\ddot{a}}{a} - \tilde{a}^2\frac{\ddot{\tilde{a}}}{\tilde{a}} = \frac{4\pi G}{3}(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{5}$$

where $\tilde{a}(t) = \frac{1}{a(t)}$. With this scalar cosmology we avoid the other degrees of freedom and corresponding equations which for a spatial curvature k=0, could only be satisfied all together by a static solution for any equations of state[c]. That the two cosmological equations would be incompatible unless in the static case is most easily checked in the $a(t) = e^{h(t)}$, $\tilde{a}(t) = e^{-h(t)}$ domain of small h(t) in which one equation leads to $\dot{h}$ constant and the other one to $h\dot{h}^2$ constant.

Moreover this scalar field is understood to be genetically homogeneous e.g. the spatially independent $\Phi(t)$ at any scale and sourced by the mean expectation value of the usual sources averaged over space rather than the sources themselves. So there are no scalar waves associated to this field and there is also no scale related to a loss of homogeneity of the background effects as in GR. In the future, we might

[c] In GR cosmology there is for instance the first order equation $H^2(t) = \frac{8\pi G\rho}{3}$ for k=0. Here for $a >> \tilde{a}$ we can neglect $\tilde{a}$ terms in our equation to get an equation that is also valid within GR. For the scale factor in standard time coordinate, it's just: $\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 = \frac{4\pi G}{3}(\rho - 3p)$. Since we only have this second order equation, in principle the initial conditions e.g a(t) and $\dot{a}(t)$ could be chosen at will at some particular time yielding $H^2(t)$ very different from $\frac{8\pi G\rho}{3}$ at this time, however this discrepancy can only be maintained on a small range of variation of the scale factor. Anywhere else we recover $H^2(t) \approx \frac{8\pi G\rho}{3}$ just as in GR for $p \approx 0$ with the same deduction that the baryonic matter is cosmologically not abundant enough to account for the measured Hubble rate: in other words we again have a missing mass issue at the cosmological scale.

relax this hypothesis [d] to allow a new complete scalar sector, because its G coupling constant could be different and actually much smaller than the gravitational coupling constant G of the separate spin 2 theory. This weakness of the new scalar coupling constant would of course be necessary to satisfy all known observational constraints.

But as of now an independent Janus field is of course required to describe all other (other than cosmological) aspects of gravity with all it's usual degrees of freedom, but then a field forced to remain asymptotically static to satisfy all the equations. Thus in DG we have two different fields to separately describe the homogeneous evolution and fluctuations respectively. So for instance the source densities and pressures are $\bar{\rho}(t)$ and $\bar{p}(t)$ for the background and $\delta\rho = \rho(\mathbf{x}, t) - \bar{\rho}(t)$, $\delta p = p(\mathbf{x}, t) - \bar{p}(t)$, $\rho(\mathbf{x}, t)\mathbf{v}(\mathbf{x}, t)$ ... for the fluctuations, where "bar" denotes spatial averaging.

2.2. *Cosmology*

This section is mainly a review of the results already obtained in [13][14]. At the end a new cosmological alternative is also considered.

The expansion of our side implies that the dark side of the universe is in contraction. Provided dark side terms can be neglected, our cosmological equation reduces to a cosmological equation known to be valid within GR. For this reason it is straightforward for DG to reproduce the same scale factor expansion evolution as obtained within the standard LCDM Model at least up to the redshift of the LCDM Lambda dominated era when something new must have started to drive the evolution in case we want to avoid a cosmological constant term. The evolution of our side scale factor before the transition to the accelerated regime is depicted in blue on the top left quarter of Figure 1 as a function of the conformal time t and the corresponding evolution laws as a function of standard time t' are also given in the radiative and cold era. Notice however the new behaviour about t=0 meaning that the Big-Bang singularity is avoided.

A discontinuous transition is a natural possibility within a theory involving truly dynamical discrete symmetries as is time reversal in DG. The basic idea is that some of our beloved differential equations might only be valid piecewise, only valid within space-time domains at the frontier of which new discrete rules apply implying genuine field discontinuities. Here this will be the case for the scale factor.

We postulated that a transition occurred billion years ago as a genuine permutation of the conjugate scale factors, understood to be a discrete transition in time modifying all terms explicitly depending on $a(t)$ but not the densities and pressures themselves in our cosmological equation: in other words, the equations of free fall for our "average source field" did not apply at the discrete transition in time.

[d] We strongly support the idea that the homogeneity of the scalar field is fundamental just because we want to rehabilitate field discontinuities: in a sense the field will need to vary discontinuously just because it cannot vary continuously in space.

Let's be more specific. The equations of free fall for the perfect fluids on both sides of course apply as usual before and after the transition and for instance on our side in the cold era dominated by non relativistic matter with negligible pressure, we have $\frac{d}{dt}(\rho a^3) = 0$. Such conservation equation is valid just because it follows from our action for the matter fields on our side. But equations of motion and conservation equations are less fundamental than the symmetry principles of the action they are derived from. Here we not only have the usual invariance of our action under continuous space-time symmetries from which we can derive the corresponding field conservation equations closely related to the continuous field equations of motion. But we also have the invariance of the action under a permutation which is a discrete symmetry. In the same way as continuous symmetries generate continuous evolution and interactions of the fields we here take it for granted that our new permutation symmetry also allows a new kind of process to take place : the actual permutation of the conjugate a and $\tilde{a}$. The process is understood to modify all terms explicitly scale factor dependent in the cosmological equation whereas all density and pressure terms remain unchanged. Because such process is not at all related to the continuous symmetries that generate the continuous field equation there is indeed no reason why the discrete version $(\rho a^3)_{before} = (\rho a^3)_{after}$ of a conservation equation such as $\frac{d}{dt}(\rho a^3) = 0$ should be satisfied by this particular process. Again symmetry principles are more fundamental than conservation equations so we should not be disturbed by a process which violates the conservation of energy since this process is discontinuous and related to a new discrete symmetry for which we have no equivalent of the Noether theorem. Here the valid rule when the permutation of the scale factors occurs is rather $\rho_{before} = \rho_{after}$ and the same for the pressure densities.

This permutation (at the purple point depicted on figure 1) could trigger the recent acceleration of the universe. This was demonstrated in previous articles [13] and [14] assuming our side source $a^4(\rho - 3p)$ term has always been dominant and has driven the evolution up to the transition to acceleration. If this term is still dominant after the transition (this scenario needs densities on the conjugate side much smaller than on our side) we get an accelerated expansion regime $(t' - t'_0)^{-2}$ in standard time coordinate with a Big Rip at future time t'_0[14].

It is much more natural (also for our understanding of time reversal [13] and the continuity of the Hubble expansion rate at the transition) that, following the transition, the dark side source term have started to drive the evolution : $a^4(\rho - 3p) << \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ resulting from $a(t) << \tilde{a}(t)$ and $\rho - 3p \approx \tilde{\rho} - 3\tilde{p}$. Then, if the conjugate side is currently in a radiative regime, our cosmological equation simplifies in a different way:

$$\tilde{a}^2 \frac{\ddot{\tilde{a}}}{\tilde{a}} \approx \frac{4\pi G}{3} \tilde{a}^4(\tilde{\rho} - 3\tilde{p}) = K\tilde{a}^2 \tag{6}$$

That a quantity such as $\tilde{\rho} - 3\tilde{p}$ is expected to follow a $1/\tilde{a}^2$ evolution in the limit where all species are ultra-relativistic can be deduced from Eq (21)-(25) of [39] and the matter and radiation energy conservation equation rewritten as $\tilde{\rho}-3\tilde{p} = 4\tilde{\rho}+\tilde{a}\frac{d\tilde{\rho}}{d\tilde{a}}$ in our radiation dominated Dark Side of the universe when $\tilde{\rho}$ and $\tilde{p} \approx 1/\tilde{a}^4(t)$.

The solution $\tilde{a}(t) = C.sh(\sqrt{K}(t-t_0)) \approx C\sqrt{K}(t-t_0)$ for $1/C << \sqrt{K}(t-t_0) << 1$ so $a(t) \propto 1/(t-t_0)$ which translates into an exponentially accelerated expansion regime $e^{t'}$ in standard time coordinate. t_0 is determined by demanding the continuity of $H(t) = \frac{\dot{a}}{a} = \frac{-1}{t-t_0}$ after the transition which should match 2/t before the transition. This again is not in concordance with our understanding of time reversal[13] so our privileged case is still that of [13] in which the dark side is also in a cold era at the transition and satisfies $\tilde{\rho}-3\tilde{p} \approx \tilde{\rho} = \rho-3p \approx \rho$. Then the continuity of the Hubble rate is automatically satisfied without having to define a new t_0 after transition. This leads to a constantly accelerated universe in standard coordinate following the transition redshift predicted to be $z_{tr} = 0.78$ in good agreement with the measured $z_{tr} = 0.67 \pm 0.1$. In the Big Rip scenario, constraining the age of the universe to be the same as in LCDM the predicted transition redshift is $z_{tr} = 0.27$ (see [14] equation 6) in case it occurred everywhere simultaneously otherwise the mean transition redshift should be significantly increased by an expected dispersion of transition redshifts due to inhomogeneities smoothing the observed transition between decelerated and accelerated expansion after averaging over large regions and making the theory difficult to discriminate from the very progressive LCDM transition with observed $z_{tr} = 0.67 \pm 0.1$. The mean measured transition redshift is indeed very sensitive to a smoothing. For instance a fictitious LCDM discrete transition between a purely CDM and a purely Lambda driven expansion regime (the Hubble rate being continuous at the transition) would imply $z_{tr} \approx 0.4$ for the same constrained age of the universe instead of $z_{tr} \approx 0.7$ for the actual progressive LCDM transition. $z_{tr} \approx 0.4$ of course also corresponds to the transition to our exponentially accelerated expansion case if it occurred everywhere simultaneously while a smoothing effect would significantly increase z_{tr} again making this scenario even harder to discriminate from the real LCDM transition.

Whatever the actual scenario we believe that such alternative to the cosmological constant is more satisfactory as it follows from first principles of the theory and eventually should fit the data without any arbitrary parameter, everything being only determined by the actual baryonic and luminous content of the two conjugate universes, such content so far not being directly accessible for the Dark Side. Our interest in the Big Rip scenario is motivated by the anomaly of the best precision "recent" cosmological measurement of $\bar{H}_0 = 73.03 \pm 1.79 (km/sec)/Mpc$ [17][18] over the two last billion years (300 SNe Ia at $z < 0.15$ having a Cepheid-calibrated distance) appearing to be exceeding by three standard deviations the one predicted by LCDM from Planck data. This is noteworthy because an unexpectedly high recent acceleration could of course be the signature of our Big Rip scenario vs LCDM expectations.

3. Local gravity: the isotropic case about Minkowski

Another Janus field and it's own separate Einstein Hilbert action are required to describe local gravity with isotropic solution in vacuum of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$.

$$A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} \tag{7}$$

$$B = \frac{1}{A} = -e^{\frac{-2MG}{r}} \approx -1 + 2\frac{MG}{r} - 2\frac{M^2G^2}{r^2} + \frac{4}{3}\frac{M^3G^3}{r^3} \tag{8}$$

perfectly suited to represent the field generated outside an isotropic source mass M. This is different from the GR one, though in good agreement up to Post-Newtonian order. The detailed comparison will be carried out in section 10. It is straightforward to check that this Schwarzschild new solution involves no horizon. The solution also confirms that a positive mass M in the conjugate metric is seen as a negative mass -M from its gravitational effect felt on our side.

4. Local gravity : linear equations about Minkowski

The linearized equations about a common Minkowskian background look the same as in GR, the main differences being the additional dark side source term $\tilde{T}_{\mu\nu}$ and an additional factor 2 on the linear lhs:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu} + t_{\mu\nu} - \tilde{t}_{\mu\nu}) \tag{9}$$

however to second order in the perturbation $h_{\mu\nu}$ (plane wave expanded as usual) and given that $\tilde{h}_{\mu\nu} = -h_{\mu\nu} + h_{\mu\rho}h_{\nu\sigma}\eta^{\rho\sigma} + O(3)$ we found that the only non cancelling contributions to $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ on the rhs, vanish upon averaging over a region of space and time much larger than the wavelength and period (this is the way the energy and momentum of any wave are usually evaluated according [1] page 259). This $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ is standing as usual for the energy-momentum of the gravitational field itself because the Linearized Bianchi identities are still obeyed on the left hand side and it therefore follows the local conservation law:

$$\frac{\partial}{\partial x^\mu}(T^{\mu\nu} - \tilde{T}^{\mu\nu} + t^{\mu\nu} - \tilde{t}^{\mu\nu}) = 0 \tag{10}$$

We can come to the same conclusion that $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ vanishes but now to all orders if we remind ourselves that the geometrical part (lhs) of the DG equation (3) is odd to all orders in $\bar{h}_{\mu\nu}$ (not to be confused with $h_{\mu\nu}$ nor $\tilde{h}_{\mu\nu}$) after making the replacement $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$. Then we are free to use the plane wave expansion of this new $\bar{h}_{\mu\nu}$ instead of $h_{\mu\nu}$ and because each term of the perturbative series has an odd number of such $\bar{h}$ factors, such term will always exhibit a remaining $e^{i\mathbf{kx}}$ factor which average over regions much larger than wavelength and period vanishes (in contrast to [1] page 259 where the computation is carried on for quadratic terms for which we are left with some x^μ independent, hence non vanishing, cross-terms).

Our new interpretation is that any radiated wave will both carry away a positive energy in $t^{\mu\nu}$ as well as the same amount of energy with negative sign in $-\tilde{t}^{\mu\nu}$ about Minkowski resulting in a total vanishing radiated energy. Thus the DG theory, so far appears to be dramatically conflicting with both the indirect and direct observations of gravitational waves.

Actually, we shall show in a forthcoming section that, since the asymptotic behaviours of the two sides of the Janus field are not necessarily the same, we could both expect from the theory an isotropic solution approaching the GR Schwarzschild one with it's black hole horizon and the same gravitational wave solutions, including the production rate, as in GR but also, whenever some particular yet to be defined conditions are reached, the above DG solutions, with a vanishingly small production rate of gravitational waves and the B=1/A exponential DG Schwarzschild solution without horizon. Both will be limiting cases of a more general solution (see section 10).

5. The unified DG theory

5.1. *Actions and space-time domains*

Eventually the theory splits up into two parts, one with total action made of an Einstein Hilbert action for our scalar-tensor homogeneous and isotropic Janus field added to SM actions for F and $\tilde{F}$ type averaged fields respectively minimally coupled to $\Phi\eta_{\mu\nu}$ and $\Phi^{-1}\eta_{\mu\nu}$. The other part of the theory has an Einstein Hilbert (EH) action for the asymptotically Minkowskian Janus Field $g_{\mu\nu}$ for local gravity added to SM actions for F and $\tilde{F}$ type fields respectively minimally coupled to $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$.

The two theories must remain completely separate. Indeed, to remain asymptotically static, $g_{\mu\nu}$ must be isolated from the scale factor effect. But also as announced earlier the scalar field is spatially independent at all scales so admits only perfectly homogeneous sources. So a unified theory cannot be obtained by mixing the local and global gravity in a Lagrangian term. However it's still possible to add the following global and local actions, being understood that no dynamical field is shared between them.

$$\int_{Global} d^4x(\sqrt{g}(R+L)+\sqrt{\tilde{g}}(\tilde{R}+\tilde{L}))+ \tag{11}$$

$$\int_{Local} d^4x(\sqrt{g}(R+L)+\sqrt{\tilde{g}}(\tilde{R}+\tilde{L})) \tag{12}$$

This confirms that even for the sources, the average background and perturbations are different dynamical fields, the former in the global L and $\tilde{L}$, the latter in the local L and $\tilde{L}$ (yet this total action will be helpful to later establish a non trivial connection between global and local gravity in some particular areas). We could also have assumed that global and local gravity never apply at the same place and time so that only an alternating of the two would remain conceivable. But it would be difficult in this case to find out non arbitrary rules linking the unconnected successive time slots of both global and local evolution. Moreover, this would require the introduction of arbitrary parameters for the global and local slots duration.

On the other hand considering the global and local physics of those actions running in parallel totally decoupled and uninterrupted as implied by our above total action leads to another issue. We need to understand then how light, clocks and rods can both feel the effect of global expansion and local gravity being now understood that those light, clocks and rods do not even appear in the global Lagrangians L and $\tilde{L}$ above just because as we already noticed only the averaged perfectly homogeneous over the whole universe, perfect fluid densities and pressures are there.

Our proposal for solving this problem is that the asymptotic local static gravity is actually only a constant piecewise function of time rather than rigorously the stationary $\eta_{\mu\nu}$. In other words it is rather $C^2\eta_{\mu\nu}$ which asymptotic value C is piecewise constant, being periodically discontinuously updated to $a(t)$ in such a way that it closely follows the evolution of $a(t)$ through a series of fast discrete transitions on a regular basis. Eventually, clocks and rods coupling to local gravity only but never coupling directly to $a(t)$, can still feel the effects of the continuous global expansion indirectly thanks to this mechanism.

Here as in GR for the isotropic static case, C is a mere integration constant, and as such cannot depend on time, however it can take different values in successive time slots, the differential equations being only valid piece-wise. We shall soon understand better how relevant is this asymptotic value within DG which has no obvious peer within GR. But we can already point out a striking analogy with what Quantum Field Theory actually describes : the succession of continuous local and discontinuous non local processes respectively described by the propagation of fields according classical wave equations and the annihilation/creation of these fields wherever interactions take place, e.g respectively propagators and vertices in the Feynman language. We even feel tempted to name our discrete transition of C, a quantization rule even though it is quite an unusual one as it applies to a zero frequency component in contrast to what we learned from the Planck-Einstein relations predicting vanishing quanta in the zero frequency limit.

Another issue is that gravity in the inner part of the solar system as we know it from thorough studies during the last decades exclude that global gravity applied to clocks and rods without being strongly attenuated. Indeed, it would otherwise lead to strongly excluded expansion effects of orbital planetary periods relative to atomic periods: the gravitational constant G would seem to vary at a rate similar to H_0 which is not the case. GR solves this problem because it predicts that significant expansion effects only take place on scales beyond those of galaxy clusters. At the contrary, the theory involving the physics of the global action above would produce expansion effects with the same magnitude at all scales: in fact everywhere the scale factor $a(t)$ evolution takes place and therefore the C evolution according the mechanism explained above. Therefore the scale factor dynamics did not apply to the inner part of the solar system at least during the last decades. This is the only possible solution not to conflict with observational constraints: no evidence of expanding planet trajectories so far. This implies the existence of frontiers between space-time domains where the dynamics of the global $\Phi\eta_{\mu\nu}$ field takes place and the $g_{\mu\nu}$ asymptotic value C is step by step discontinuously driven by the scale factor and others where this dynamics is totally absent, for instance in the inner part of the solar system during the last decades.

5.2. *Field discontinuities*

Thus we have field discontinuities in time as well as at the frontier between spatial zones. Let's stress that those are not related at all to our permutation symmetry and the related discrete cosmological transition process that could trigger the acceleration of the universe but should rather be considered as a quantization rule for the asymptotic field. Now the usual conservation equations are not violated though in presence of genuine potential discontinuities. Indeed it's possible to describe the propagation of the wave function of any particle crossing this new kind of discontinuous gravitational potential frontier just as the Schrodinger equation can be solved exactly in presence of a squared potential well : we just need to require the continuity of the matter and radiation fields and continuity of their derivatives at the gravitational discontinuity. Since the differential equations are valid everywhere except at the discontinuity itself where they are just complemented by the former matching rules we obviously avoid the nuisance of any infinite potential gradients and eventually only potential differences between both sides of such discontinuity will physically matter. For instance we can now have have $(\rho a^3)_{beforecrossing} = (\rho a^3)_{aftercrossing}$ in contrast to what we had following the permutation transition ($\rho_{beforecrossing} = \rho_{aftercrossing}$). Of course clocks and rods must remain insensitive to the latter discrete permutation of the scale factor responsible for the cosmological transition to global acceleration. This is possible if clocks and rods are not sensitive to the corresponding transitions of the local field asymptotic value C: so the corresponding densities don't change in this case.

5.3. *Space-time domains and the Pioneer effect*

The following question therefore arises: suppose we have two identical clocks exchanging electromagnetic signals between one domain submitted to the expanding $a(t)$ in $\Phi\eta_{\mu\nu}$ (still through our indirect mechanism) and another without such effect. Electromagnetic periods and wavelengths are not impacted in any way during the propagation of electromagnetic waves in the conformal coordinate system where we wrote our cosmological equation even when crossing the inter-domain frontier. Through the exchange of electromagnetic signals, the period of the clock decreasing as $a(t)$ can then directly be tracked and compared to the static clock period and should be seen accelerated with respect to it at a rate equal to the Hubble rate H_0. Such clock acceleration effect indeed suddenly appeared in the radio-wave signal received from the Pioneer space-crafts but with the wrong magnitude by a factor two: $\frac{\dot{f}_P}{f_E} = 2H_0$ where f_P and f_E stand for Pioneer and earth clocks frequencies respectively. This is the so called Pioneer anomaly [11][12]. The reader is invited to visit a more detailed analysis in our previous publication [14]. The interpretation of the sudden onset of the Pioneer anomaly just after Saturn encounter would be straightforward if this is where the spacecraft crossed the frontier between the two regions. The region not submitted to global expansion (at least temporarily) would therefore be the inner part of the solar system where we find our earth clocks and where indeed various precision tests have shown that expansion or contraction effects on orbital periods are excluded during the last decades. Only the origin of the factor 2 discrepancy between theory and observation remains to be elucidated in the following sections as well as a PLL issue we need to clarify first.

5.3.1. *Back to PLL issues*

As we started to explain in our previous article [14] in principle a Pioneer spacecraft should behave as a mere mirror for radio waves even though it includes a frequency multiplier. This is because its re-emitted radio wave is phase locked to the received wave so one should not be sensitive to the own free speed of the Pioneer clock.

Our interpretation of the Pioneer effect thus requires that there was a failure of on board PLLs (Phase Lock Loop) to specifically "follow" a Pioneer like drift in time. We already pointed out that nobody knows how the scale factor actually varies on short time scales: in [14] we already imagined that it might only vary on very rare and short time slots but with a much bigger instantaneous Hubble factor than the average Hubble rate. This behaviour would produce high frequency components in the spectrum which might have not passed a low pass filter in the on board PLL system, resulting in the on board clocks not being able to follow those sudden drifts. The on board clocks would only efficiently follow the slow frequency variations allowing Doppler tracking of the spacecrafts. Only when the integrated total drift of the phase due to the cumulative effect of many successive clock fast accelerations would reach a too high level for the system, this system would "notice" that something went wrong, perhaps resulting in instabilities and loss of lock at regular intervals [14]. This view is now even better supported since our clocks and rods are understood not to be anymore directly sensitive to the scale factor, but rather indirectly, only through the local field asymptotic value C closely following by a succession of discontinuous steps rather than continuously the evolution of $a(t)$ as the latter is implied by our cosmological differential equation. The failure of the PLL system is then even better understood for discontinuous variations of the Pioneer clock frequency with respect to the earth clock frequency. As a result, the frequency of the re-emitted wave is impacted by the Pioneer clock successive drifts and the earth system could detect this as a Pioneer anomaly.

5.4. *Cyclic expanding and static regimes*

We are now ready to address the factor two discrepancy between our prediction and the observed Pioneer clock acceleration rate. We know from cosmology that, still in the same coordinate system, earth clocks must have been accelerating at a rate H_0 with respect to still standing electromagnetic periods of photons reaching us after travelling across cosmological distances: this is just the description of the so called cosmological redshift in conformal time rather than usual standard time coordinate. However, according our above analysis this was not locally the case at least during the last decades which did not manifest any cosmological effect (G did not vary) in the inner part of the solar system.

This necessarily implies that earth clocks must have been submitted to alternating static and expanding regimes. It just remains to assume (further justification will be provided in a forthcoming section) that through cosmological times, not only earth clocks but also all other clocks in the universe, spent exactly half of the time in the expanding regime and half of the time in the static regime, in a cyclic way. It follows that the instantaneous expansion rate $H_0 = 2\bar{H}_0$ of our global field as deduced from the Pioneer effect is twice bigger than the average expansion rate (the average of $2\bar{H}_0$ and zero respectively in the expanding and static halves of the cycle) as measured through a cumulative redshift over billions of years.

In our previous article we presented a very different more complicated and less natural explanation on how we could get the needed factor two which we do not support anymore. This article also discussed the possibility of field discontinuities at the frontier between regions with different expansion regimes, and likely related effects. Those discontinuities do not necessarily imply huge potential barriers even though the scale factors have varied by many orders of magnitude between BBN and now. At the contrary they could be so small to have remained unnoticed as far as our cycle is short enough to prevent some regions to accumulate a too much C drift relative to others.

6. Frontier dynamics

Our next purpose is to understand the physics that governs the location of frontier surfaces between regions identified in the previous sections. Consider the gravitational field total action in a space-time domain where global expansion takes place:

$$\int_{Global} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \tag{13}$$

$$\int_{Local} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) \tag{14}$$

where in the global (resp local) actions the gravitational field is $\Phi\eta_{\mu\nu}$ (resp C-asymptotic $g_{\mu\nu}$). We would like to determine the frontier surface of this domain at the time t the local field asymptotic value C is reset to the scale factor in our domain. Considering the frontier to be stationary between two such successive updates, the frontier position is determined at any time. If such surface is moving because of successive updates it will of course scan a space-time volume as time is running out. To determine this hypersurface we extend the extremum action principle. Not only the total action should be extremum under any infinitesimal field variations which as we all know allows to get the field equations but also the total action is required to be extremum i.e. stationary under any infinitesimal displacement of this hypersurface which is nothing but the frontier of the action validity domain. But the displaced hypersurface might only differ from the original one near some arbitrary point, so that requiring the action variation to vanish actually implies that the total integrand should vanish at this point and therefore anywhere on the hypersurface. Eventually, anywhere and at any time at the domain boundary we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} + (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{local} = 0 \tag{15}$$

This equation is merely a constraint relating local gravity (terms 3 and 4) to global gravity (terms 1 and 2) at the hyper surface.

Now provided one scale factor dominates the other side one we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} \approx \pm^{a>>\tilde{a}}_{a<<\tilde{a}}(\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R})_{global} \tag{16}$$

and then we can make use of the contracted equation 4 to replace:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} \approx \pm^{a>>\tilde{a}}_{a<<\tilde{a}} 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T})_{global} \tag{17}$$

in equation 15 and we can do the same for the local part provided one C asymptotic dominates the other side one. Then equation 15 becomes:

$$\pm^{a>>\tilde{a}}_{a<<\tilde{a}}(a^4 < \rho - 3p > - \tilde{a}^4 < \tilde{\rho} - 3\tilde{p} >) \pm^{C>>\tilde{C}}_{C<<\tilde{C}}(C^4 F(r)\Delta(\rho - 3p) - \tilde{C}^4 \tilde{F}(r)\Delta(\tilde{\rho} - 3\tilde{p})) = 0 \tag{18}$$

The F and $\tilde{F}$ here account for the effect a local assumed static isotropic gravitational field. The <> and Δ denote averages and fluctuations. First and third terms currently behave as $a(t)$ in our cold side of the universe while second and fourth terms are expected to follow a $\tilde{a}^2(t)$ evolution law.

The relative magnitudes of terms 1 and 2 can be very different and in the opposite way to the relative magnitudes of 3 and 4. This because the relative magnitudes of the fluctuations Δ can be very different from the relative magnitudes of the averages <>. Therefore, in the external gravity of a massive spherical body, planet or star on our side, which radial a-dimensional potential is $\Phi(r) = -GM/rc^2$ we are led to:

$$a^{\gamma}(t) \propto e^{\frac{2MG}{rc^2}} \tag{19}$$

with $\gamma \neq 0$ in case term 1 << term 2 and term 3 >> term 4. Our privileged constantly accelerated scenario has $\gamma = -2$ while the exponential acceleration scenario leads to $\gamma = -3$

This equation obtained here in the conformal time t coordinate system is also valid in standard time t' coordinate since the standard scale factor and the "conformal scale factor" are related by $a(t) = a'(t')$. It is valid to PN order being understood that the exponential metric is here used for simplicity as a weak field PN approximation of a GR Schwarzschild solution rather than really a DG Schwarzschild solution as we shall show in the next section. This equation I=J implies $\dot{I}/I = \dot{J}/J$ so that:

$$\gamma 2\bar{H}_0 = -2\frac{d\Phi}{dr}\frac{dr}{dt} \tag{20}$$

here taking into account that the instantaneous Hubble factor H_0 is actually $2\bar{H}_0$, e.g twice the average cosmological Hubble parameter that we know from cosmological probes as we explained earlier.

The latter equation tells us that in case $\gamma \neq 0$ the frontier between the two domains is drifting at speed

$$\frac{dr}{dt} = \gamma \frac{\bar{H}_0}{\frac{d\Phi(r)}{dr}} \tag{21}$$

and therefore could involve a characteristic period, the time needed for the scale factor to scan $e^{\frac{2MG}{r}}$ from the asymptotic value to the deepest level of the potential at which point a new scan cycle is started except that this time the two regions will need to exchange their roles about the moving frontier. In other words if for a given cycle the expanding region is the outer one and the static region the inner one, the next cycle will be with the inner part expanding and the outer part static. After two such complete cycles any area will have spent exactly the same total time static and expanding at $2\bar{H}_0$ resulting in the promised average $\bar{H}_0$. Thus the $\gamma \neq 0$ case must be the correct one if we want to understand both the Pioneer effect, the expansion of the universe, and an expansion dynamics which only takes place in some delimited space-time domains. $\gamma = 0$ at the contrary implies a static frontier leading to the unacceptable result that some regions would not expand at all over cosmological times. A Geogebra animation in [16] helps visualizing the evolution of the local potential over one complete cycle.

We may estimate an order of magnitude of the characteristic period of this cyclic drift assuming that the cycle is over when the frontier reaches the deepest potential levels. For collapsed stars such as white dwarfs or neutron stars this would give a far too long cycle exceeding billions of years because their surface potential is so deep and even much worse for black holes. But the majority of stars have very similar surface potentials even though there is a large variability in their masses and sizes. So we may take the value of our sun a-dimensional surface potential which is about 2.10^{-6} as indicative of a mean and common value. To that number we should add the potential in the gravitational field of the Milky Way and the potential to which the local cluster of galaxies is subjected. Knowing the velocities: 220 km/s of the sun about the center of the galaxy and 600km/s of the local cluster vs the CMB, the virial approximation formula $\frac{v^2}{c^2} \approx GM/rc^2$ may lead us to a crude estimation of each contribution and a total potential near 6.10^{-6}. Then the order of magnitude of the period cycle would be in between 10^4 and 10^5 years.

7. Unconventional asymptotic values

After many cycles of successive static and expanding phases, the local field asymptotic value is everywhere going to be very different from it's initial C=1 value. This also implies that the new asymptotic values of the local field and its conjugate will be very different.

Given that $g^{C^2\eta}_{\mu\nu} = C^2 g^{\eta}_{\mu\nu}$ and $\tilde{g}^{\eta/C^2}_{\mu\nu} = \frac{1}{C^2}\tilde{g}^{\eta}_{\mu\nu}$, where the $< g^{\eta}, \tilde{g}^{\eta} >$ Janus field is asymptotically η, it is straightforward to rewrite the local DG Janus Field equation now satisfied by this asymptotically Minkowskian Janus field after those replacements. Hereafter, we omit all labels specifying the asymptotic behaviour for better readability and only write the time-time equation satisfied by the asymptotically $\eta_{\mu\nu}$ Janus field.

$$C^2\sqrt{g}\frac{G_{tt}}{g_{tt}} - \frac{1}{C^2}\sqrt{\tilde{g}}\frac{\tilde{G_{tt}}}{\tilde{g}_{tt}} = -8\pi G(C^4\sqrt{g}\delta\rho - \frac{1}{C^4}\sqrt{\tilde{g}}\tilde{\delta\rho}) \tag{22}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\delta\rho$ is as usual the energy density for matter and radiation density fluctuations. The tilde terms again refer to the same tensors except that they are built from the corresponding tilde (dark side) fields. Notice that for no fluctuations, the solutions are Minkowskian as needed, being understood that the background plays it's dynamics in the global Janus field equation rather than in this local Janus field equation.

Then for $C >> 1$ we are back to $G_{tt} = -8\pi GC^2 g_{tt}\delta\rho$, a GR like equation for local gravity from sources on our side because all terms depending on the conjugate field become negligible on the left hand side of the equation while the local gravity from sources on the dark side is attenuated by the huge $1/C^8$ factor (in the weak field approximation, $G_{tt} = 8\pi G\frac{\delta\tilde{\rho}}{C^6}$. From $g^{\eta}_{\mu\nu}$ we can get back $g^{C^2\eta}_{\mu\nu}$ and then of course absorb the C constant by the adoption of a new coordinate system and redefinition of G, so for $C >> 1$ we tend to GR : we expect the same gravitational waves emission rate and the same weak field gravitational field. However on the dark side everything will feel the effect of the anti-gravitational field from bodies on our side amplified by the same huge factor relative to the gravity produced by bodies on their own side.

Of course the roles are exchanged in case $C << 1$. Then the GR equation $\tilde{G}_{tt} = -\frac{8\pi}{C^2}G\tilde{g}_{tt}\delta\tilde{\rho}$ is valid on the dark side while the anti-gravity we should feel from the dark side is enhanced by the huge $1/C^8$ factor relative to our own gravity (given in the weak field approximation by solving $\tilde{G}_{tt} = 8\pi GC^6\delta\rho$ for $\tilde{g}_{\mu\nu}$ from which we derive immediately our side $g_{\mu\nu}$ of the Janus field). Here is our promised justification for having assumed that the local gravitational field was the weak field PN approximation of the GR Schwarzschild solution rather than a DG Schwarzschild solution in the previous section.

Only in case C=1 do we recover our local Dark gravity, with no significant GW radiations and also a strength of gravity ($G_{tt} = -4\pi G\delta\rho$) reduced by a factor $2C^2$ relative to the above GR gravity ($G_{tt} = -8\pi GC^2\delta\rho$) as a consequence of two geometrical terms adding up on the lhs of the equations.

It's important to stress that the phenomenology following from different asymptotic behaviours of the two faces of the Janus field here has no peer within GR in which a mere coordinate transformation is always enough to put the gravitational field in an asymptotically Minkowskian form in which a redefinition of the gravitational constant G gives back the usual gravitational potentials. This would still be possible in DG for one face of the Janus field but not for both at the same time. The new physics emerges from their relative asymptotic behaviour which can't be absorbed by any choice of coordinate system.

Eventually, depending on the local C value in a given space-time domain, a departure from GR predictions could be expected or not both for the gravitational waves radiated power and the local static gravitational field e.g. depending on the context, we could get either the DG exponential elements or the GR Schwarzschild solution for the static isotropic gravity; and get either no gravitational waves at all or the same radiated power as in General Relativity.

8. Apparent variations of G

Because clocks and rods submitted to local gravity also indirectly felt the effects of global expansion through our quantized (discontinuous step by step) evolution of C, if we could test gravity over the past cycles we would necessarily detect that it's strength was different and has changed in the same proportion as the scale factor itself.

We come to this conclusion by deriving the equation of motion of a body of charge q and mass m orbiting with a quasi circular motion (so we can neglect radial speeds) in an isotropic electrostatic field described by the potential V(r) and gravitational field with metric

$$d\tau^2 = C^2(t)(B(r)dt^2 - A(r)(dx^2 + dy^2 + dz^2)) \tag{23}$$

Where C(t) stands for the evolution of the asymptotic value still following a(t) step by step. In the small speed approximation, following the method of [3] section 18 we get:

$$\ddot{r} = -\frac{1}{2}\frac{B'(r)}{A(r)} + \frac{q}{m}\frac{V'(r)}{C(t)} \tag{24}$$

$$\dot{\phi} = \frac{B(r)}{C(t)A(r)r^2} \tag{25}$$

for the radial acceleration and angular speed. This indeed means that, all else been equal, the scale factor impacts the relative strengths of the electrical and gravitational forces : in other words planet orbits should be seen expanding relative to atoms or any rods governed by atomic physics. Yet, current tests in the solar system and in some strong field binary systems constrain relative variations of G at levels much lower than H_0. On the other hand, a recent publication [24] claiming that galaxies 10 billion years ago were less dark matter dominated might support a long term variation of the strength of gravity in some areas all the more so if those effects are enhanced beyond a MOND radius as we shall argue in the next section. In the inner part of the solar system what we need is either an instantaneous test in the expanding regime (so far inaccessible because we are apparently currently in the stationary half cycle) or a test for multi-millennial variations hence necessarily over much longer time scales than the cycle period to exclude or not a mean variation at the Hubble rate. However, according to [28] "If G were to vary on a nuclear timescale (billions of years), then the rates of nuclear burning of hydrogen into helium on the main-sequence would also vary. This in turn would affect the current sun central abundances of hydrogen and helium. Because helio-seismology enables us to probe the structure of the solar interior, we can use the observed p-mode oscillation frequencies to constrain the rate of G variation." Again the relative variation of G at a rate similar to H_0 is completely excluded the precision being two orders of magnitude smaller.

To escape this new dead-end our understanding of the physics governing field discontinuities must again evolve in a new radical way: high density regions, for instance about stars, cut-out of the rest of the expanding universe, again by a discontinuity at their spherical surface defining a new volume for the global field dynamics which is not anymore submitted to the expanding:

$$d\tau^2 = C^2(t)(dt^2 - d\sigma^2) = dt'^2 - C'^2(t')d\sigma^2 \tag{26}$$

cosmological metric ($d\sigma^2 = dx^2 + dy^2 + dz^2$), but to the new Minkowski metric.

$$d\tau^2 = C^2(t)dt^2 - d\sigma^2 = dt'^2 - d\sigma^2 \tag{27}$$

Notice that this new Minkowski metric (27) is here attached to a locally free falling coordinate system with respect to the expanding (26). This new Minkowski metric is also not the same as that of the static half-cycle which remains:

$$d\tau^2 = dt^2 - d\sigma^2 \tag{28}$$

Both (27) and (28) share the crucial property that within a volume subjected to such metrics no expansion effect can be measured, which is what we need to avoid conflicts with the solar system constraints. However, as seen from the (28) metric region (on Earth) the (27) metric atoms (at Pioneer) are blueshifted as needed to get the Pioneer effect.

Eventually the transition from (28) to (27) can be seen as the succession of transition from (28) to (26) we already had in the previous sections but now supplemented by the additional from (26) to (27) but the latter implied discontinuity can only produce Shapiro delay or deflection of photons crossing it. With our new understanding the analysis of previous chapters thus remains valid provided the alternating of (28) and (26) being replaced by the alternating of (28) and (27), but valid except for genuine cosmological expansion effects impacting the relative periods of electromagnetic waves and atoms which of course need (26).

This necessarily implies that on the largest scales we still have the alternating (28) and (26) while near denser regions it's the alternating (28) and (27) that takes place so we have an additional frontier and we shall bet that this frontier is located at the MOND radius in the next section.

But now we need to reconsider equation (18) anywhere (27) is the actual metric rather than (26) in which case both local terms are now constant in time, so we now have $\gamma = 1$ in Equation (15) in the constantly accelerated scenario.[e]

9. The MOND phenomenology

As already pointed out DG crucially differs from GR in the way global expansion and local gravity work together. Any anomaly in the local physics of the solar system or galaxy seemingly pointing to effects related to the Hubble rate is completely puzzling in the context of GR while it may be naturally explained within Dark Gravity. Not only the Pioneer effect but also MOND phenomenology seem related to $\bar{H}_0$.

We derived in a former section the speed $\frac{dr}{dt} = -\gamma \frac{\bar{H}_0}{d\Phi(r)/dr}$ at which our local vs global frontier sitting at an isopotential between internal and external regions should radially propagate in the potential well of a given body. From this formula the speed of light $\frac{dr}{dt} = c$ is reached anywhere the acceleration of gravity equals $\gamma c \bar{H}_0$. For $\gamma = 1$, this appears to be the order of magnitude of the MOND acceleration and the corresponding radius even closer to the MOND radius beyond which gravity starts to be anomalous in galaxies [19][27]. We are therefore tempted to postulate that to prevent frontier discontinuities from propagating at relativistic speeds something must be happening near the MOND radius. Our best guess is that this is the radius beyond which the alternating (28) and (26) takes over the alternating (28) and (27) meaning true cosmological expansion responsible for photon redshifts as explained in the previous section. As a result, the new drift law beyond MOND radius would be with $\gamma = \pm 2$ and the drift speed could be retained below the speed of light provided the local Janus field asymptotic C^2 and $\frac{1}{C^2}$ exchange their roles beyond the MOND radius may be just because the cosmological permutation between $a(t)$ and $\tilde{a}(t)$ did not already take place in this region. This, as we explained in the previous section would result in the gravitational field from the dark side in the

[e]A curiosity is that for $\gamma = 1$ the total time to scan the sun potential well at the speed of Equation (15) is then about the same as the equinoxes precession period (26000 years). So it is possible to imagine an accurate resonant synchronization of the two phenomena (earth-moon system and cyclic path of a discontinuity through the solar system) along many cycles. This might even lead us to an accurate estimation of $\bar{H}_0$ from the precession of the equinoxes.

region beyond the MOND radius to be enhanced by a huge factor C^8 relative to the gravity due to our side matter in this region. Then because a galaxy on our side implies a slightly depleted region on the dark side by it's anti-gravitational effects, even a slightly under-dense fluctuation[f] fluid on the dark side would result in an anti-anti-gravitational effect on our side, significantly enhanced beyond the MOND radius in such a way that it would be difficult to discriminate from the effect of a Dark Matter hallow! Also the most spectacular features of Dark Matter and MOND Phenomenology in galaxies such as galaxies that seem to be dominated at more than 99 percent by Dark Matter [[20]] or unexpectedly high acceleration effects in the flyby of galaxies [[23]] are more naturally interpreted in a framework where the gravitational effects from the hidden side can be enhanced by huge factors beyond the MOND radius. At last, this would also mean that beyond the MOND frontiers the universe is still expanding according a decelerated expansion law.

10. Back to Black-Holes and gravitational waves

Let's consider the collapse of a massive star which according to GR should lead to the formation of a Black Hole. As the radius of the star approaches the Schwarzschild radius the metric becomes singular there so the process lasts an infinite time according to the exterior observer. If the local fields both outside and inside the star have huge asymptotic C values, we already demonstrated that the gravitational equations tend to GR. However this can't be the case when we approach the Schwarzschild radius because C is finite and the metric elements can grow in such a way that we could not anymore neglect the Dark side geometrical term. Therefore presumably the horizon singularity is avoided as well for C $\neq 1$. To check this we need the exact differential equations satisfied in vacuum by C-asymptotic isotropic static metrics of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$. With $A = C^2 e^a$ and $B = C^2 e^b$, we get the differential equations satisfied by a(r) and b(r):

$$a'' + 2a' + \frac{a'^2}{p} = 0 \tag{29}$$

$$b' = -a' \frac{1 + a'r/p}{1 + 2a'r/p} \tag{30}$$

where $p = 4\frac{e^{a+b}C^4+1}{e^{a+b}C^4-1}$. GR is recovered for C infinite thus p=4. Then the integration is straightforward leading as expected to

$$A = C(1 + U)^{p=4}; \tag{31}$$

[f] though the dark side is in contraction which can boost growing of fluctuations especially on the largest scales, if it's density is similar to our side density at redshifts about 0.8, it's density was for instance much smaller at the time of the CMB emission so it's fluctuations must have started to grow significantly at much lower redshifts and much larger scales than on our side

$$B = C(\frac{1-U}{1+U})^{(p=4)/2} \tag{32}$$

where $U = GM/2r$ and the infinite C can be absorbed by opting to a suitable coordinate system : then there is no dark side. DG C=1 corresponds to b=-a, p infinite and the integration, as expected, gives $A = e^U$, $B = e^{-U}$.

The integration is far less trivial for intermediary Cs because then p is not anymore a constant, however in the weak field approximation, treating p as the constant $4\frac{C^4+1}{C^4-1}$ the PPN development of the above solutions brings to light a possible departure from GR at the PostPostNewtonian level since:

$$A_{RG} \approx 1 + 4U + 6U^2 \tag{33}$$

$$B_{RG} \approx 1 - 4U + 8U^2 - 12U^3 \tag{34}$$

$$A_{p\neq 4} \approx 1 + pU + \frac{p(p-1)}{2}U^2 \tag{35}$$

$$B_{p\neq 4} \approx 1 - pU + \frac{p^2}{2}U^2 - p\frac{2+p^2}{6}U^3 \tag{36}$$

This makes clear that for $p \neq 4$ redefining the coupling constant to match GR at the Newtonian level, which amounts to replace U by 4U/p in the above expressions, a discrepancy would remain at the PPN level relative to GR predictions.

$$A_{p\neq 4} \approx 1 + 4U + 8(\frac{p-1}{p})U^2 \tag{37}$$

$$B_{p\neq 4} \approx 1 - 4U + 8U^2 - \frac{32}{3}(\frac{2+p^2}{p^2})U^3 \tag{38}$$

For $4 \leq p = 4\frac{1+1/C^4}{1-1/C^4} \leq \infty$ the departure from GR is the greatest for p infinite (C=1) :

$$A_{DG} \approx 1 + 4U + 8U^2 \tag{39}$$

$$B_{DG} \approx 1 - 4U + 8U^2 - \frac{32}{3}U^3 \tag{40}$$

but should hopefully soon become testable with the data from neutron stars or black holes mergers if C is not too big.

In the strong field regime we need to rely on numerical approximation methods to understand what's going on near the Schwarzschild radius. The numerical integration in Geogebra (using NRésolEquaDiff) was carried on and the resulting b(r) are shown in Figure 2 for various C values. It is found that as C increases b(r) will closely follow the GR solution near the Schwarzschild radius over an increasing range of b(r) which can be many orders of magnitude and perfectly mimic the GR black hole horizon, however at some point the solution deviates from GR and crosses the Schwarzschild radius without singularity. Therefore, as far as the numerical integration is reliable our theory appears to avoid horizon singularities (true Black Holes) for any finite C and not only C=1.

As for the central singularity of the metric it should also be avoided thanks to another mechanism within our framework : when the metric reaches some threshold, the inner region (the volume defined by the star itself) global and local fields might respectively be reset to Minkowski and C=1. This discrete transition would produce a huge discontinuity in the vicinity of the Schwarzschild Radius which phenomenological signature might already have been detected [22]. At the center of the star, the two faces of the Janus field will get very close to each other just because C=1 and because this is where the star potential vanishes. The crossing of the metrics is the required condition to allow the transfer of matter and radiation between the star and the conjugate side there. The lost of a significant part of its initial mass along with the strength of gravity being reduced by a factor 2C for DG relative to GR might eventually stop the collapse whenever the conditions are again reached for the stability of a star. The resulting object having no horizon is in principle still able to radiate extremely red-shifted and delayed light. Something new however is that the discontinuity itself might have a contribution to the total effective gravific mass and this might lead to pseudo BHs much more massive than we believed them to be.

Shocks and matter anti-matter annihilation at the discontinuity (an excess of gamma radiation from our Milky Way giant black hole has indeed been reported [21]) which we remember is also a bridge toward the Dark side and it's presumably anti-matter dominated fluid, could also produce further GWs radiation which would be much less natural from a regular GR Black Hole [22].

Eventually in the vicinity of stars as well as in "Black Holes" we can't exclude a transfer of matter and radiation through the discontinuity at crossing metrics that would proceed in the opposite way feeding them and increasing their total energy : a possible new mechanism to explain the unexpectedly high gravific masses of recently discovered BH mergers but also an attractive simple scenario to explain the six SN like enigmatic explosions of the single massive star iPTF14hls if they resulted from a succession of injections of antimatter from the Dark Side[40]. Such discontinuities in the vicinity of stars could also block matter accumulating in massive and opaque spherical shells around stars as well as the other kind of discontinuity that cuts out a star from the expanding universe and which we postulated at the frontier between (26) and (27) domains could block light directly : a possible scenario to explain the reduced light signal from the recently discovered neutron stars merger.

11. Discrete symmetries, discontinuities and quantum mechanics

We earlier explained that in a theory with discrete symmetries having a genuine dynamical role to play, here global time reversal relating the two faces of a Janus field [6][13][14], discontinuities are expected at the frontier of space-time domains. All along this article we started to postulate various possible new discrete physical laws assumed to apply there: we can have discontinuous transitions in time when the conjugate scale factors exchange their roles, other kind of discontinuities in space at the frontier between static and expanding spatial regions, and in the expanding regions we also postulated a succession of step by step discontinuous and fast periodic re-actualization of the local field piecewise constant asymptotic value allowing it to follow the evolution of the scale factor. We also already drew the reader attention to the harmlessness of discontinuous potentials as for the resolution of wave function equations in the presence of discontinuities. Of course the exploration of this new physics of discontinuities in relation to discrete symmetries is probably still at a very early and fragile stage and requires an open minded effort because it obviously questions habits and concepts we used to highly value as physicists.

Discontinuous and global fields as our scalar-tensor field also put into question the validity of the Noether theorem implying the violation of local conservation laws wherever the new physics rules apply. However, we should remind ourselves that the most fundamental postulates of quantum physics remain today as enigmatic as they appeared to physicists one century ago: with the Planck-Einstein quantization rules, discontinuous processes came on to the scene of physics as well as the collapse of a wave function taken at face value obviously implies a violation of almost all local conservation laws. Based on these facts, a new theoretical framework involving a new set of discrete and non local rules which, being implied by symmetry principles are no more arbitrary at the contrary to the as well discontinuous and non-local quantum mechanics postulates, might actually be a chance. A real chance indeed as they open for the first time a concrete way to hopefully derive the so arbitrary looking quantum rules from symmetry principles and may be eventually relate the value of the Planck constant to the electrical charge, in other words compute the fine structure constant. We are certain that only our ability to compute the fine structure constant would demonstrate that at last we understand where quantum physics comes from rather than being only able to use it's rules like a toolbox.

In this perspective, it may be meaningful to notice that the Black Hole postulated discontinuity of the previous section, which would lie at the frontier between approximate GR and DG domains, behaves as a wave annihilator for incoming GW waves and a wave creator for outgoing waves. In the DG domain the waves if any, carry no energy while in the GR domain they carry energy and momentum as usual. This is a fascinating remark because this would make it the only known concrete mechanism for creating or annihilating waves à la QFT or even a step toward a real understanding of the wave function collapse e.g in line with a realistic view of quantum mechanics. Such collapse is indeed known to be completely irreducible to classical wave physics because it is non local, and in fact just as non local as would be a transition from GR $C >> 1$ to DG, C=1 in the inside domain. The latter transition is indeed non local because it is first of all driven by a transition of our global scalar-tensor field which by definition ignores distances.

12. Stability issues about distinct backgrounds: C $\neq$ 1

12.1. *Stability issues in the purely gravitational sector*

Our action for gravity being built out of two Einstein Hilbert terms, each single one is obviously free of ghost. This means that all degrees of freedom have the same sign of their kinetic term in each action.

There might still remain issues in the purely gravitational sector when we add the two actions and express everything in terms of a single dynamical field $g_{\mu\nu}$: everything is all right as we could demonstrate for C=1, but otherwise what we need to insure stability is that in the field equation resulting from the total action, all degrees of freedom will have their kinetic term tilting to the same sign. Again adopting $\bar{h}_{\mu\nu}$ from $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ and $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$ as the dynamical field puts forward that we have exactly the same quadratic (dominant) terms in $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ except that for $C > 1$ (resp $C < 1$) all terms in $t_{\mu\nu}$ are enhanced (resp attenuated) by a C-dependent factor while all terms in $\tilde{t}_{\mu\nu}$ are attenuated (resp enhanced) by a 1/C dependent factor, so that we will find in $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ all such quadratic terms tilting to the same sign, ensuring that the theory is still free of ghost in the purely gravitational sector.

Of course there remains an instability menace whenever $C \neq 1$ in the interactions between matters and gravity which we shall inspect now.

12.2. *Stability issues in the interactions between matter and gravity*

Generic instability issues arise again when C is not anymore strictly equal to one. This is because the positive and negative energy gravitational terms $t^{\mu\nu}$ and $\tilde{t}^{\mu\nu}$ do not anymore cancel each other as in the DG C=1 solution. Gravitational waves are emitted either of positive or negative (depending on C being less or greater than 1) energy whereas on the source side of the equation we have both positive and negative energy source terms. Whenever two interacting fields (here the gravitational field and some of the matter and radiation fields) carry energies with opposite sign, instabilities would seem unavoidable (see [25] section IV and V for a simple description of the problem and [26] for a much more advanced one) and the problem is even worsen by the massless property of the gravitational field.

Yet, the most obvious kind of instability, the runaway of a couple of particles with opposite sign of the energy, is trivially avoided in DG theories [4][7][8][5] [29][30][31][32][33][27] where particles from the two sides of the Janus field just repel each other.

It is also straightforward to extend the theory of small gravitational fluctuations to DG in the Newtonian approximation and neglecting expansion (C=1): the equations governing the decay or grow of compressional fluctuations are :

$$\ddot{\delta\rho} = v_s^2 \Delta\delta\rho + 4\pi G < \rho > (\delta\rho - \delta\tilde{\rho}) \quad (41)$$

$$\ddot{\delta\tilde{\rho}} = \tilde{v}_s^2 \Delta\delta\tilde{\rho} + 4\pi G < \tilde{\rho} > (\delta\tilde{\rho} - \delta\rho) \quad (42)$$

which in case the speeds of sound v_s and $\tilde{v}_s$ would be the same on both sides allows to subtract and add the two equations with appropriate weights resulting in two new equations governing the evolution of modes $\delta^- = \delta\rho - \delta\tilde{\rho}$ and $\delta^+ = \delta\rho + \frac{<\rho>}{<\tilde{\rho}>}\delta\tilde{\rho}$.

$$\Box_s \delta^- = 4\pi G(<\rho> + <\tilde{\rho}>)\delta^- \tag{43}$$

$$\Box_s \delta^+ = 0 \tag{44}$$

Where $\Box_s$ is a fake Dalembertian in which the speed of sound replaces the speed of light. Because δ^+ does not grow we know that $\delta\rho \approx -\frac{<\rho>}{<\tilde{\rho}>}\delta\tilde{\rho}$ and the two can grow according the growing mode of δ^-. The complete study, involving different sound speeds, attenuation of gravity between the two sides and the effect of expansion (here represented by the evolution of C following the scale factor) will be the subject of the next section. It is already clear that in the linear domain anti-gravity by itself does not lead to a more pathological growing of fluctuations than in standard only attractive gravity: eventually we would expect the growth of a gravitational condensate on one side to proceed along with the corresponding growth of a void in the conjugate side and vice versa if the speeds of sound were the same. The situation is less dramatic than Ref [25] section IV might have led us to think mainly because our leading order terms are linear in a gravitational field perturbation h whereas the leading order coupling term is quadratic in the lagrangian (22) of [25] leading to equations of motion of the form $\ddot{\Psi} \propto \Psi^3$. In other words our "instabilities" in the linear domain are nothing but the usual instabilities of gravity which fortunately arise since we need them to account for the growing of matter structures in the universe. These instabilities could be classified as tachyonic (the harmless and necessary ones for the formation of structures), non gradient (fortunately because those instabilities are catastrophic even at the classical level), and ghost (energy unbounded from below which is only catastrophic for a quantum theory) in the terminology of [36] reviewing various kind of NEC violations in scalar tensor theories which confirms that these are acceptable for a classical theory.

From this it appears that DG is not less viable than GR in the linear domain as a classical theory and that the real concern with all DG models proposed to this date will actually arise for the quantized DG theories for which ghost instabilities are of course prohibitive, and may be in the strong field regime for the classical theories. Only then the real energy exchange between the gravitational field itself (it's kinetic energy quadratic terms) and other fields kinetic energies should start to become significant relative to the Newtonian like energy exchange between kinetic energy of the fields and their gravitational potential energy that drives the evolution of the compressional modes according Eq 41 and 42.

In the strong field regime the problem is thus related to the radiation of gravitational waves when they are carrying non zero energy (for C$\neq$1) while they can couple to matter sources with both positive and negative energies. This remains true even when great care is being taken to avoid the so-called BD ghost in the massive gravity approach particularly when the perturbations of the two metrics about a common background have different magnitudes i.e. when one parameter of the couple α, β dominates the other in [31]. [g]

The next step is therefore to consider an hypothetical quantized DG theory to try to understand how we might solve stability issues in the quantum case. In the quantized theory the problematic couplings would produce divergent decay rates by opening an infinite space-phase for for instance the radiation of an arbitrary number of negative energy gravitons by normal matter (positive energy) particles. This is indeed instructive as it shows that we certainly would not try to quantize the Janus field as any other field! To avoid instabilities the most natural way would be to build the quantum Janus field operator also as a double-faced object, coupling it's positive energy face to usual positive energy particles and it's negative one to the negative energy particles of the Dark side thereby avoiding any kind of instabilities. However the picture described by our classical Janus field equation which in principle really allows the direct exchange of energy between GW (with a definite sign of the energy depending on C > 1 or $C < 1$) and matter fields with different signs of the energy does not really fit into such quantization idea. This gives us a key on how the classical theory would need to be extended or modified to avoid any stability issues even in the strong field regime and pave the way toward quantization.

We are led to consider the possibility that whenever C becomes different from 1, the Local Janus field $< g^{C^2}, \tilde{g}^{1/C^2} >$ needs to split in two independent Janus fields $< f^{C^2}, \tilde{f}^{1/C^2} >$ and $< h^{C^2}, \tilde{h}^{1/C^2} >$ (superscripts C^2 and $1/C^2$ still denote asymptotic values). It is considered a well established result that a theory with two interacting massless spin 2 fields is not viable. However, the no-go demonstration was carried on only in case the theory is derived from a single action. Here we shall consider two separate actions where the two spin 2 fields respectively play their dynamics separately in the sense that in one action where one field plays it's dynamics the other field is non dynamical (not varied). The equations following from the extremization process will then be quite different from the equations we would have derived from a single total action, sum of the two actions. As a starting point we are tempted to consider the following actions running in parallel and decoupled in which we omit asymptotic behaviour superscripts for better readability.

$$\int_{Local} d^4x\sqrt{f}R_f + \sqrt{f}L_f \tag{45}$$

[g]By the way there is a much worse problem in models having two independent differential equations instead of one to describe the dynamics of two fields assumed independent, i.e. not related from the beginning by a relation such as Eq (1). Then the energy losses through the generation of gravitational waves predicted by each equation are different so that such models are inconsistent [4][7][8][5] [29] as shown in [15].

$$\int_{Local} d^4x\sqrt{\tilde{h}}R_{\tilde{h}} + \sqrt{\tilde{h}}\tilde{L}_{\tilde{h}} \tag{46}$$

to avoid $\tilde{L}_{\tilde{f}}$ and L_h terms in the first and second action respectively which ensures that we will not end up with source terms carrying an energy opposite to the energy of gravitational waves in any of the two actions. The permutation symmetry is now between f and $\tilde{h}$. This is a bit silly however because we lost $\tilde{f}$ and h and anti-gravity in that new game. But this is just an intermediary step because we can actually recover easily the conjugates of the Janus fields along with anti-gravity if matter and radiation are actually coupled to a combination of f and h instead of f alone in the first action, and equivalently to a combination of $\tilde{f}$ and $\tilde{h}$ rather than $\tilde{h}$ alone in the second action. The composite metrics being denoted and defined by $[fh]_{\mu\nu} = \eta^{\rho\sigma}f_{\mu\rho}h_{\nu\sigma}$ and $[\tilde{f}\tilde{h}]_{\mu\nu} = \eta^{\rho\sigma}\tilde{f}_{\mu\rho}\tilde{h}_{\nu\sigma}$ let's now consider the two actions.

$$\int_{Local} d^4x\sqrt{f}R_f + \sqrt{[fh]}L_{[fh]} \tag{47}$$

$$\int_{Local} d^4x\sqrt{\tilde{h}}R_{\tilde{h}} + \sqrt{[\tilde{f}\tilde{h}]}\tilde{L}_{[\tilde{f}\tilde{h}]} \tag{48}$$

Being understood that the Janus field $f, \tilde{f}$ is only dynamical in the first action and $\tilde{h}$,h only dynamical in the second action, stability is still granted because even though our side matter and radiation fields in L feel the anti gravitational effect of matter and radiation fields from $\tilde{L}$ through h and reciprocally through $\tilde{f}$, the gravitational field f is only sourced by matter and radiation fields coupled to f (and not $\tilde{f}$) and spectator h in the first action and equivalently the gravitational field $\tilde{h}$ is only sourced by matter and radiation fields coupled to $\tilde{h}$ (and not h) and spectator $\tilde{f}$ in the second action.

We can gain more insight about what's new by varying the first action with respect to $f_{\mu\nu}$ to get :

$$\sqrt{f}G_f^{\mu\nu}\delta f_{\mu\nu} + 8\pi G\sqrt{[fh]}T_{[fh]}^{\mu\sigma}\eta^{\nu\rho}h_{\rho\sigma}\delta f_{\mu\nu} = 0 \tag{49}$$

In the perfect fluid case, after some replacements this yields :

$$G_f^{\mu\nu} = -8\pi G\sqrt{h}T_f^{\mu\nu} \tag{50}$$

and equally we could get

$$G_{\tilde{h}}^{\mu\nu} = -8\pi G\sqrt{\tilde{f}}\tilde{T}_{\tilde{h}}^{\mu\nu} \tag{51}$$

By the way, there is not any issue with the Bianchi identities in such equations because for instance $T_f^{\mu\nu}$ is not here a covariantly conserved energy momentum tensor with respect to the f metric. It is rather $T_{fh}^{\mu\nu}$ that is covariantly conserved with respect to metric fh and here we just denoted $T_f^{\mu\nu}$ the tensor obtained through replacing fh by f in $T_{fh}^{\mu\nu}$ which is straightforward in the case of a perfect fluid energy-momentum tensor.

The conservation of $T_{fh}^{\mu\nu}$ with respect to metric fh merely means that our side matter and radiation fields follow the kind of geodesics sourced by a positive mass through f as well as the kind of geodesics sourced by a negative mass through h even though on the other hand these matter and radiation fields can only exchange energy with the f gravitational field almost in the GR way and exactly as in GR to second order in perturbations as could be read in a straightforward way from the perturbative equations. This is why we are confident that this framework is completely free of instabilities even when the backgrounds are not the same on the two faces of the Janus field.

A very striking feature of those equations is that assuming the same cosmological constant source term in both equations as expected e.g. from the same vacuum energy presumably huge contributions, the resulting f and $\tilde{h}$ fields would be the same but the composite field fh that matter and radiation fields would feel would remain Minkowski as the result of a perfect compensation. However the question remains whether quantum vacuum energy terms are really expected in our fluctuations local equations rather than in our background global equation where the cancellation would not take place...

Now we can repeat the reasoning of a former section with the time-time equation for C^2 asymptotic f and h fields to recover a previous section conclusion that the gravity from the dark side is damped with respect to gravity from our side. And of course the situation again gets reversed for $C < 1$. Thus, eventually all our results as for the MOND/DM phenomenology would remain valid. Moreover we have paved the way toward the quantization of DG.

As another option, we might want to go even further by promoting the complete Janus field actions on the geometrical side:

$$\int_{Local} d^4x\sqrt{f}R_f + \sqrt{\tilde{f}}R_{\tilde{f}} + \sqrt{[fh]}L_{[fh]} \tag{52}$$

$$\int_{Local} d^4x\sqrt{\tilde{h}}R_{\tilde{h}} + \sqrt{h}R_h + \sqrt{[\tilde{f}\tilde{h}]}\tilde{L}_{[\tilde{f}\tilde{h}]} \tag{53}$$

This is tempting because we would thereby avoid the Horizon singularity at the Schwarzschild radius as already mentioned. But then we would need to supplement the actions by a selection rule : the local gravitational f and $\tilde{h}$ fields must always select the $C^2 > 1$ integration constant (the alternative is between C^2 and $1/C^2$) to make sure that we get the same sign of the energy in the source and in $t^{\mu\nu} - \tilde{t}^{\mu\nu}$ otherwise catastrophic instabilities would be back again in the strong field regime and in the quantized theory. The other side of the coin is that, as $C < 1$ is now a forbidden option with these new actions, we would lose the enhancement mechanism for the Dark side Gravity beyond the MOND radius. May be this new option also implies that on microscopic scales there is a spatial separation of "elementary gravitational sources" belonging to either conjugate metrics in the sense that there could not be any superposition of both kind of sources in the same microscopic area.

Eventually our initial DG action and equations remain the more appealing both for their simplicity and phenomenology and still viable in the linear domain. Moreover the transition to C=1 or the avoidance of black-hole singularities even for $C \neq 1$ might also help to avoid instabilities in the strong field regime. We are therefore led to seriously question the necessity for a Janus field quantization.

13. Evolution of fluctuations... In progress

To summarize the lessons learned from our investigation of actions involving the dark side of the metrics, we have:

- An Einstein-Hilbert action for our scalar tensor Janus field added to one for its homogenous source $\bar{\rho}, \bar{p}, \tilde{\bar{\rho}}, \tilde{\bar{p}}$. What we did not specify earlier is that $\bar{\rho}, \bar{p}, \tilde{\bar{\rho}}, \tilde{\bar{p}}$ are actually not understood to be dynamical here so their dynamics needs to have been played and their conservation equations established elsewhere. So for instance the covariant conservation involving the homogeneous energy momentum components $\bar{\rho}, \bar{p}$ in the geodesics of $a^2(t)\eta_{\mu\nu}$ are actually not deduced from these actions. Only $a(t)$ is here dynamical so far.
- An Einstein-Hilbert action for the local Janus gravitational field added to an action for its perturbation sources $\delta\rho, \delta p, \bar{\rho}\mathbf{v}$ and dark side equivalents. Here again the source fields are not understood to be dynamical so again their dynamics needs to have been played elsewhere. So there is actually no deducible covariant conservation of the energy momentum components for those perturbations alone in the geodesics of the local gravitational field. Only the local $g_{\mu\nu}$ Janus is dynamical at this level.
- So we now need to introduce the actual Actions for matter and radiation fields and their dark side equivalents to play their own dynamics on the geodesics of the total gravitational field $g_{\mu\nu}(\mathbf{x}, t)$ (which asymptotic value is driven step by step by $a(t)$) and it's conjugate respectively which are now no more dynamical, their components having already played their dynamics in the previous E-H actions. So these new actions are completely stand alone actions as these are not added to any Einstein-Hilbert type actions. This is where at last the covariant conservation equations for the total energy momentum tensors (including background and fluctuations) for the matter and radiation fields can be derived as usual (the derivation requires that these matter-radiation fields are "on shell" e.g satisfy the dynamical equations derived form those actions) from the requirement that these actions be scalars under general coordinate transformations. Thus we are led to exactly the same total Euler and continuity equations as in GR but now along with their Dark Side equivalents. These follow from: $\tilde{\partial}_{;\mu}\tilde{T}^{\mu}_{\nu} = 0$ and $\partial_{;\mu}T^{\mu}_{\nu} = 0$ in total metrics $d\tau^2 = C^2(t)((1+2\Psi)dt^2 - (1-2\Psi)d\sigma^2)$ and

$d\tilde{\tau}^2 = \tilde{C}^2(t)((1+2\tilde{\Psi})dt^2-(1-2\tilde{\Psi})d\sigma^2)$ respectively where the C(t) and $\tilde{C}(t)$ follow and can be replaced by $a[t]$ and $\tilde{a}(t)$ on the mean since the actual step by step evolution does not matter when integrated along cosmological times.

In the dynamical equations for the matter and radiation fields the evolution of the scale factor was thus plainly taken into account while in dynamical equations such as ... the scale factor is just an integration constant so it can't depend on time and the equation itself is only valid on a short time slot in between two discrete transitions. In case we could neglect all dark side terms it would be straightforward to actually derive the whole set of equations: these are just the same as in GR but for fluctuations about a k=0 static hence empty universe background. Let's remind the first order cosmological perturbation GR equations (4.4.169;4.4170;4.4.171 from[41]):

$$\nabla^2\Psi - 3H(\dot{\Psi} + H\Psi) = 4\pi G a^2 \delta\rho \tag{54}$$

$$\dot{\Psi} + H\Psi = -4\pi G a^2(\bar{\rho} + \bar{p}v) \tag{55}$$

$$\ddot{\Psi} + 3H\dot{\Psi} + (2\dot{H} + H^2)\Psi = 4\pi G a^2 \delta p \tag{56}$$

In the static case (H=0) we get our equations during the static time slot:

$$\nabla^2\Psi = 4\pi G a^2 \delta\rho \tag{57}$$

$$\dot{\Psi} = -4\pi G a^2(\bar{\rho} + \bar{p}v) \tag{58}$$

$$\ddot{\Psi} = 4\pi G a^2 \delta p \tag{59}$$

which would need to be complemented by discrete rules specifying how the potential Ψ should be affected at the transition between two slots. All effects being integrated over a long time duration we expect effective equations similar but different from the above GR ones to be valid with new terms involving the Hubble parameter.

We might actually not always need to determine those equations because for instance for all the physics below the Hubble scale even in GR we just need the Poisson equation which in this case simplifies to

$$\nabla^2\Psi = 4\pi G a^2 \delta\rho \tag{60}$$

which should also be valid in our framework whenever we can neglect Hubble terms (e.g. below the Hubble scale).

This as usual allows to eliminate the potential from the Euler equation (same as in GR) and then using the continuity equations (same as in GR) get the same differential equations satisfied by the fluctuations as in GR cosmology. Again this is only valid when the dark side terms can be neglected which is surely the case in the radiative era and in the early stage of the matter dominated era as we shall see. Of course we can also as usual determine the evolution of the potentials directly from the same Poisson equation once we know the evolution of the perturbations.

New behaviours of the fluctuations might occur at superhorizon scale withing our framework: it depends on how closely our equations for the potential resembles the first order perturbation equations of GR.

Eventually, as for the dynamics of background and perturbation densities, the new game follows the same rules as in GR and for instance our continuity equations at zeroth order give us the energy-momentum conservation equations for the background densities which we have used in our cosmological section.

The only difference is that the background and perturbations of the gravitational field itself here don't mix non linearly as in GR but play their dynamics as independent fields having their own E-H actions. This of course is necessary to avoid for instance the extinction of background effects in the non linear domain of matter perturbations and get Hubble dependent effects in the solar system or within galaxies. Only in the non linear domain (locally) do the effects of expansion (or contraction) differ completely from those within GR as we outlined earlier.

We already pointed out that the evolution of the background (our homogeneous scalar field) before the transition to acceleration seems to require Dark Matter just as in the standard model to reach the cosmological critical density implied by k=0 and the measured value of the Hubble expansion rate. However this might alternatively be avoided if the coupling constant G_s of our scalar-tensor field to averaged densities of matter is higher than the coupling constant G of local gravity (our asymptotically static spin 2 theory) and just large enough to insure that $H^2 = \frac{8\pi G_s}{3}\rho_{baryons}$ with baryons alone.

Thus it remains to investigate whether the theory could successfully also explain the growing of baryonic matter fluctuations without dark matter but with the help of the conjoint fluctuations from the dark side which is also in a cold state with the same density (again baryonic only) as on our side at the transition redshift, but in contraction and therefore having started from a very low and presumably highly homogeneous mean density at z=1000 at least on the scale of our side fluctuations.

The radiative era is essentially the same as in LCDM (we naturally can get almost the same sound horizon even though a true singularity is avoided at t=0) except that we don't have dark matter fluctuations so that when the baryonic matter dominated era starts, the baryons will not take advantage of already formed cold dark matter potentials to accelerate their own clustering.

This is where the dark side should hopefully come into the game to essentially mimic and play the same role as dark matter in LCDM. The dark side fluctuations could of course be boosted by the contracting scale factor especially on the largest scales but since the mean density was then extremely small with $\tilde{\rho} \approx z^{-6}\rho\ = 10^{-18}\rho$ at $z \approx 1000$, it is obvious that the grow of our side fluctuations starting from $\frac{\delta\rho}{\rho} \approx 10^{-5}$ of the CMB, could not be helped at high z. At low z, on the other hand, it is the weakness of the source term $\tilde{a^4}\tilde{\rho} \propto 1/a$ relative to $a^4\rho \propto a$ which makes the gravity from the dark side negligible.

So we entirely need to rely on the extremely efficient new mechanism we introduced in the previous section to see the gravitational effect of dark side fluctuations (voids) starting to play a significant role compensating for the lack of Dark Matter both in the linear and non linear regime.

What's indeed really new is that in accordance with what we also explained earlier each fluctuation has two regions : one central region where the gravity from our side $\delta\rho_{in}$ is hugely enhanced over the gravity from the dark side $\delta\tilde{\rho}_{in}$ and a peripheral one where at the contrary it is the gravity from the dark side $\delta\tilde{\rho}_{out}$ that hugely dominates that from $\delta\rho_{out}$. Moreover the permutation of the scale factors results in the same strength for $\delta\rho_{in}$ and $\delta\tilde{\rho}_{out}$ gravity in each equation.

As in LCDM, for the evolution of our side fluctuations, our side background evolution only becomes important in the matter dominated era arising as usual as an additional friction term $H\dot{\delta\rho}$ where H is the Hubble rate. So we can readily rewrite Eq (41) and (42) taking into account all non negligible effects depending on the scale factor[h].

$$\ddot{\delta\rho} + H\dot{\delta\rho} = 4\pi G < \rho > a^2(\delta\rho_{in} - \delta\tilde{\rho}_{out}) \tag{61}$$

$$\ddot{\delta\tilde{\rho}} - H\dot{\delta\tilde{\rho}} = 4\pi G < \tilde{\rho} > a^2(\delta\tilde{\rho}_{out} - \delta\rho_{in}) \tag{62}$$

We see that the interaction between the dark side and our side fluctuations can only be significant when $\tilde{\rho}$ is not too much smaller than ρ. Let's hope that this was already the case before re-ionization or that the physics of discontinuities alone can efficiently accelerate clusterization.

After the transition on the other hand, even small fluctuations $\tilde{\delta\rho}$ in the dark side distribution relative to the dark side average density can lead to gravific effects much larger than what our side fluctuations $\delta\rho$ are able to do (still baryonic matter only) and all the effects of their dominant gravity is probably attributed to Dark Matter within LCDM. Work still in progress...news coming soon.

14. The Janus field and the quantum

We already pointed out that none of the faces of our gravitational Janus field could be seriously considered as a candidate for the spacetime metric. Yet, though the gravitational field loses this very special status (be the spacetime metric) it had within GR, it acquires another one which again makes it an exceptional field : it is the unic field that makes the connection between the positive and negative energy worlds (this definition is relative: for any observer the negative field is the one that lives on the other side), the only one able to couple to both the dark side SM fields and our side SM fields.

This special status alone implies that the gravitational interaction might need a special understanding and treatment avoiding it to be quantized as the other

h

interactions. Avoiding ghost instabilities related to the infinite phase space opened by any interaction between quantum fields that do not carry energies with the same sign, is a requirement which also confirms that the gravitational Janus field in 1 and 2 can't be a quantum field. So the old question whether it is possible to build a theory with a classical gravitational field interacting with all other fields being quantum, is back to the front of the stage just because the usual answer "gravity must be quantized because everything else is quantum" fails for the Janus theory of the gravitational field.

We are therefore tempted to consider semiclassical gravity favourably since it indeed treats matter fields as being quantum and the gravitational field as being classical, which is not problematic as far as we just want to describe quantum fields propagating and interacting with each others in the gravity of a curved space-time considered as a spectator background. To describe the other way of the bidirectional dialog between matter and gravity i.e how matter fields source gravity, semi-classical gravity promotes the expectation value of the energy momentum tensor of quantum fields as the source of the Einstein equation. One often raised issue is that this is incompatible with the Multi Worlds Interpretation (MWI) of QM since within the MWI the other terms of quantum superpositions which are still alive and represent as many parallel worlds would still be gravific as they contribute to the energy momentum tensor expectation value and should therefore produce large observational effects in our world. The MWI, considered as a natural outcome of decoherence is adopted by a large and growing fraction of physicists mainly because is considered the only alternative to avoid the physical wavefunction collapse. For this reason incompatibility with the MWI is often deemed prohibitive for a theory. Since we have nothing against a physically real wave function collapse (our theory even has opened new ways to hopefully understand it; discontinuity and non locality are closely related) we are not very sensitive to such argument. The wave function collapse might eventually be triggered at the gravitational level: a simple achievement of something similar to the Penrose idea (gravitationally triggered collapse) seems within reach in our framework, thanks to a transition to C=1 which is tantamount to a gravitational wave collapse. We are all the more supported in considering semi-classical gravity and the Schrodinger-Newton equation it implies [38] as the correct answers, as the usual arguments based on the measurement theory often believed to imply that gravity must be quantized have recently been re-investigated in [37] and the authors to conclude that "Despite the many physical arguments which speak in favor of a quantum theory of gravity, it appears that the justification for such a theory must be based on empirical tests and does not follow from logical arguments alone." This has even reactivated an ongoing research which has led to experiment proposals to test predictions of semiclassical gravity, for instance the possibility for different parts of the wave functions of a particle to interact with each other non linearly according classical gravity laws. However "together with the standard collapse postulate, fundamentally semi-classical gravity gives rise to superluminal signalling" [37] so the theoretical effort is toward suitable models of the wavefunction collapse that would avoid this superluminal signalling. From the point of view of the Dark Gravity theory this effort is probably unnecessary because superluminal signalling would not lead to inconsistencies as long as there exists a unic privileged frame for any collapse and any instantaneous transmission exploiting it. We indeed have such a natural privileged frame since we have a global privileged time to reverse, so it is natural in our framework to postulate that this frame is the unic frame of instantaneity. Then the usual gedanken experiments producing CTCs (closed timelike curves) do not work any more : the total round trip duration is usually found to be possibly negative only because these gedanken experiments exploit two or more different frames of instantaneous signaling. Let's be more specific : Does instantaneous hence faster than light signalling unavoidably lead to causality issues ? : apparently not if there is a single unic privileged frame where all collapses are instantaneous. Then i (A) can send a message to my colleague (B) far away from me instantaneously and he can send it back to me also instantaneously still in this same privileged frame using QM collapses (whatever the relative motions and speeds of A and B and relative to the global privileged frame): the round trip duration is then zero in this frame so it is zero in any other frames according special relativity because the spatial coordinates of the two end events are the same: so there is no causality issue since there is actually no possible backward in time signalling with those instantaneous transmissions... in case there is some amount of time elapsed between B reception and re-emission, eventually A still receives it's message in it's future: no CTC here.

15. Last remarks and Outlooks

Eventually what deserves attention is that to get rid of stability issues we repeatedly made use of the following helpful unusual argument : not all field degrees of freedom should be considered a priori completely dynamical in an action.

First we had the most extreme case of our $\eta_{\mu\nu}$ metric which was completely fixed before the action so there is obviously no ghost menace from such metric.

We also encountered in the previous section the case of a metric which has already played it's dynamics in one action and could enter a new action as a completely non dynamical metric, which needs not extremize the action, hence again avoiding ghost issues for this metric in the most trivial way (we could even stuckelbergize the field [35], there would be no Action extremization hence no field equation hence no propagator associated to the scalar kinetic terms generated this way).

In the meantime, an even more interesting case was the action for our global scalar field when we demanded that the field should be spatially maximally symmetric not to reflect the fact that the source is homogeneous and isotropic on the largest scales as in usual GR cosmology but already before entering the action which then forced the source to be a purely homogeneous one. This pre-action requirement for the field could of course be expressed in a fully covariant way using the language of killing vectors. This is why such field actually does not admit any non homogeneous perturbation.

The game we are playing now may appear very unnatural if one did not completely figure out that the permutation symmetry linking the two faces of a Janus field has an interpretation in terms of a discrete global privileged time reversal symmetry and that such global symmetry also constrains the metrics in a non trivial way: they should share the same isometries, the form B=A of our scalar tensor field and B=1/A for asymptotically Minkowskian C=1 isotropic metrics are necessary ones, and once we have a global privileged coordinate system, other unusual symmetry properties linking space and time coordinates become meaningful and so on ... (see [3]).

16. Conclusion

New developments of DG not only seem to be able to solve the tension between the theory and gravitational waves observations but also provide a renewed and reinforced understanding of the Pioneer effect as well as a recent cosmological acceleration greater than expected. An amazing unifying explanation of MOND/Dark Matter phenomenology seems also at hand. The most important theoretical result is the avoidance of both the Big-Bang singularity and Black Hole horizon. The outlook for a wave-function collapse new mechanism also appears promising on an unprecedented scale.

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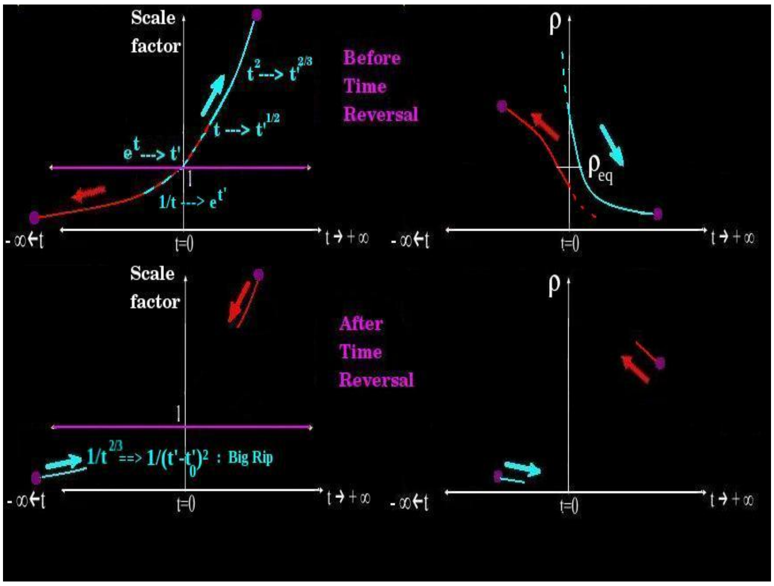

Fig. 1. Evolution laws and time reversal of the conjugate universes, our side in blue

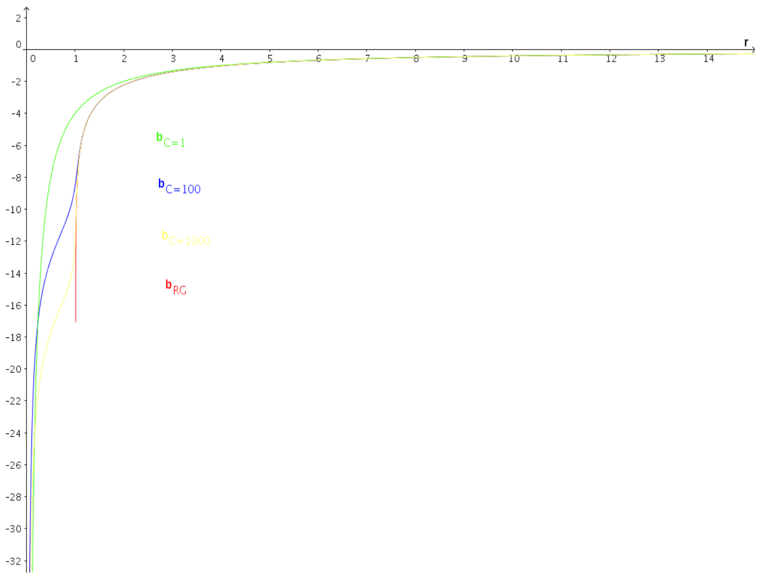

Fig. 2. b(r) near the Schwarzschild radius (r=1) for various C values