# Photon models are derived by solving a bug in Poynting and Maxwell theory

Shuang-ren Zhao

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#### Abstract

It is found that the Poynting theorem is conflict with the energy conservation principle. It is a bug of the Poynting theorem. The Poynting theorem is derived from Maxwell equations by using the superimposition principle of the fields. Hence, this bug also existed at ether in superimposition principle or in the Maxwell equations. Assume it is wrong with the field superimposition, even the Maxwell equations are correct with singular charge, we can still not prove that they are still correct for many charges. Hence, at least something are also wrong with Maxwell equations. The Poynting theorem is corrected in this article. After the correction the energy is not quadratic and hence the field is also not linear. The concept of the superposition of fields need also to be corrected. Hence the new definitions for the inner product and cross product are proposed. The corrected Poynting theorem is the mutual energy formula, it is strongly related to the mutual energy theorems. It is shown that starting from the mutual energy formula, the whole electromagnetic theory can be reconstructed. The Poynting theorem can be proved from the mutual energy formula by adding pseudo items. The Maxwell equations can be derived from Poynting theorem as sufficient conditions. Hence if the mutual energy formula is corrected, the Maxwell equations still can be applied with knowing its problem. Most the problems originally caused by Maxwell equations are solved. Examples of this problems are: (1) electric field infinity which need to be re-normalized in quantum physics; (2) collapse of the electromagnetic field, the waves has to be collapsed to its absorber, otherwise the energy is not conserved; (3) the emitter can send energy without absorber, this is conflict to the direct interaction principle and absorber theory; (4) if our universe is not completely opaque, the charge will continually send energy to the outside of our universe, our universe will have a continual loss of energy. However there is no testimony supporting that our universe is opaque. The new theory supports the existence of advanced wave, hence also strongly support the absorber theory and transactional interpretation of quantum physics. It can offer a equation for photon and a good explanation for the duality of the photon. If photon and electromagnetic field obeys the mutual energy formula, it is very possible that all other quanta also obey their similar mutual energy formula. Hence the mutual energy formula can be applied as a principle or axiom for the electromagnetic theory and quantum physics. According to this theory the asychronous retarded wave and the asychronous advanced wave of electromagnetic fields both are a ability or probability wave, which is also partly agree with Copenhagen interpretation.

### 1 Introduction

In this article when we speak about Maxwell equations, we will explicitly distinguish the two different situations, the first is the Maxwell equations for N (many) charges and the second situation Maxwell equations is only for a singular charge. The first is written as MEQN, the second is written as MEQS. If the electromagnetic fields can be superimposed, It is easy to prove MEQN from MEQS. Hence we do not need to distinguish these two concepts, however in this article we will question the superimposition principle, hence we have to distinguish these two situations.

We all know that there are some problems in electromagnetic field theory, especially when it is applied to quantum physics. The examples are (1) the field very close to the charge will be infinity which need to be re-normalized in quantum physics; (2) collapse of the electromagnetic wave, the wave has to be collapsed to the absorber so that the energy sends out by the emitter can be all received by the absorber; (3) the emitter can send energy without absorber. (4) A electron can continually send energy to the outside of our universe. According MEQN, the antenna can send wireless energy even without anything to receive this energy. These all have no problem for electronic engineering, but it is a big problem for physics. If wave can continuously send to outside of the system with all charges in our universe, our universe will continually loss the energy. This fact has been known for a long time. Schwarzschild, Tetrode and Fokker derived the theory of action at a distance, it is also referred as direct interaction [18, 7, 20]. From this principle all action and reaction can only take place between two charges. There is no field exist freely independent to the charge. Dirac has applied advanced wave to explain the force of a moving charge [6]. Wheeler and Feynman, designed the absorber theory in which the electron does not only sends retarded waves to the future but also sends advanced wave to the past [1]. Wheeler and Feynman also introduced the concept of the adjunct field [2]. Wheeler and Feynman has clearly point out there is problem in MEQN, but they did not point in details where the problem is and how to revises it. Instead they derived the MEQS from their adjunct field, it seems even there is problem in MEQS but no thing can be changed.

In the field of electromagnetic field theory, W.J. Welch has introduced time-domain reciprocity theorem[21] in 1960. In 1987 this author has introduced the mutual energy theorem [12, 23, 22]. This is strongly influenced by J. A. Kong's book[8, 9]. In 2014 this author wrote the online publication discussed the relationship between the reciprocity theorem, the mutual energy theorem and the Poynting theorem[13]. Among this work, this author noticed the book of Lawrence Stephenson[19] and read it with great interesting especially the topic

about the advanced potential. Afterwords this author begin search the publications about advanced potential or advanced waves, and noticed the absorber theory of Wheeler and Feynman[1, 2, 10] and John Cramer's transactional interpretation for quantum physics [3, 4]. After read this publications, this author begin to work at building a photo model with classical electromagnetic filed theory[16, 15].

Recently, this author compared the power of a system with N charges to the power calculate by Poynting theorem, found that the Poynting theory has offers more power than the system of N charges should be. If we take away all self energy items from Poynting theorem, it can obtained exactly the same as the power of the system with N charges. From this this author obtained the conclusion, there is a bug in Poynting theorem, which should be corrected. The correction can be done simply by taken away all self energy items. After this correction, the Poynting theorem become the mutual energy formula, which is related to [12, 23, 22, 13]. Since Poynting theorem is derived from MEQN. MEQN is derived through the superimposition principle of the field and MEQS, if Poynting theorem has bug, this bug also exists inside ether superimposition or the MEQS. The mutual energy formula can be also derived from MEQS. But even so, if MEQS have bugs inside, it is possible that the mutual energy formula is still correct.

When we take away of self energy items form the Poynting theorem, the energy expressed in the Poynting theorem is not quadratic any more, if the energy is not quadratic the field is also not linear. Hence the superimposition principle fails.

This means that even if MEQS is correct, we still cannot derive MEQN, because we have no the principle of the superposition of the fields. Actually MEQS is also very problematic, which can not be applied to interpret photon phenomenon, which is a quantum phenomenon. MEQS cannot interpret the double slits experiment, where wave is clear appear as probability character and doesn't like the result of MEQS which is a real physical wave. From MEQS the Poynting theorem for singular charge can be obtained, which claims that the singular charge can continually send wave to the infinity even their is no absorber. This is conflict with quantum physics in which singular charge can only send the probability wave, which is a mathematical wave. Hence this author reconstructed the whole electromagnetic theory from the mutual energy formula, which is related to the mutual energy theorem[12, 23, 22, 13]. And re-derive the Photon model.

The mutual energy formula agrees with direct interaction principle and absorber theory, hence the retarded wave and advanced wave of the photon, are not a real wave which can exist independent to the emitter and the absorber. It is adjunct field, hence in principle they can be seen as probability waves. However, the mutual energy flow is real in physics, which happens when the retarded wave and advanced wave just synchronized. This synchronized two waves can be seen as the transactional process described by John Crammer. If there is only one wave for example retarded wave, there is no any advanced wave to synchronize to it, it perhaps returns to it's source and hence in principle this

wave can been seen as probability wave which is also agree with Copenhagen interpretation in quantum physics.

# 2 Find bug in Poynting theorem and MEQN

### 2.1 Power of a system with N charges

If the charge move and has the speed  $\overrightarrow{v}_i$ , where i is the index of the charge, we know that,

$$\overrightarrow{J}_i = \rho_i \overrightarrow{v}_i \tag{1}$$

where  $\overrightarrow{J}_i$  is the current intensity.  $\rho_i$  is the charge intensity. There is,

$$\rho_i = q\delta(\overrightarrow{x} - \overrightarrow{x}_i) \tag{2}$$

and hence the current of the charge is,

$$I_{i} \equiv \iiint_{V} \overrightarrow{J}_{i} dV = \iiint_{V} q \delta(\overrightarrow{x} - \overrightarrow{x}_{i}) \overrightarrow{v}_{i} dV = q \overrightarrow{v}_{i}$$
(3)

we know the power which of singular charge is,

$$P(\overrightarrow{x}_i) = \overrightarrow{F}(\overrightarrow{x}_i) \cdot \overrightarrow{v}_i \tag{4}$$

 $\overrightarrow{x}_i$  is the position of the charge.  $\overrightarrow{F}(\overrightarrow{x}_i)$  is Coulomb's force on *i*-th charge, which can be given as following,

$$\overrightarrow{F}(\overrightarrow{x}_i) = \sum_{i=1}^{N} \frac{q_i q_j}{4\pi\epsilon_0} \frac{(\overrightarrow{x}_i - \overrightarrow{x}_j)}{||\overrightarrow{x}_i - \overrightarrow{x}_j||^3}$$
 (5)

where  $q_i$  or  $q_j$  is amount of charge at the place  $\overrightarrow{x}_i$  or  $\overrightarrow{x}_j$ , write,

$$E(\overrightarrow{x}_j, \overrightarrow{x}_i) = \frac{q_j}{4\pi\epsilon_0} \frac{(\overrightarrow{x}_i - \overrightarrow{x}_j)}{||\overrightarrow{x}_i - \overrightarrow{x}_j||^3}$$
 (6)

which is the electric field of charge  $q_i$  to  $q_i$ . Hence, we have

$$\overrightarrow{F}(\overrightarrow{x}_i) = q_i \overrightarrow{E}(\overrightarrow{x}_i) \tag{7}$$

Hence power of charge i is,

$$P_i = q_i \overrightarrow{E}(\overrightarrow{x}_i) \cdot \overrightarrow{v}_i \tag{8}$$

Hence the power of the whole system with N charges is,

$$P = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} \overrightarrow{E}(\overrightarrow{x}_i) \cdot (q_i \overrightarrow{v}_i)$$

$$= \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \overrightarrow{E}(\overrightarrow{x}_{j}, \overrightarrow{x}_{i}) \cdot \overrightarrow{I}_{i}$$

$$(9)$$

We find when we calculate the power of N charges, we have used the following summation.

$$\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \tag{10}$$

### 2.2 The Poynting theorem of N charges

According to the traditional electromagnetic field theory, The Poynting theorem is give as following,

$$- \oiint_{\Gamma} (\overrightarrow{E} \times \overrightarrow{H}) \cdot \hat{n} d\Gamma = \iiint_{V} (\overrightarrow{E} \cdot \overrightarrow{J} + \overrightarrow{E} \cdot \partial \overrightarrow{D} + \overrightarrow{H} \cdot \partial \overrightarrow{B}) dV$$
 (11)

where  $\zeta = [\overrightarrow{E}, \overrightarrow{H}, \overrightarrow{J}, \overrightarrow{K}, \overrightarrow{D}, \overrightarrow{B}]$ , is electromagnetic field system of N charges,  $\overrightarrow{K} = 0$  is the magnetic current intensity.  $\partial = \frac{\partial}{\partial t}$ , t is time.  $\overrightarrow{E}$  is electric field,  $\overrightarrow{H}$  are magnetic H-field. J is current,  $\overrightarrow{D}$  is electric displacement.  $\overrightarrow{B}$  is magnetic B-field. According to the traditional definition, the electromagnetic field of N charges is,

$$\overrightarrow{E}(\overrightarrow{x}) = \sum_{i=1}^{N} \overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x}) \tag{12}$$

$$\overrightarrow{H}(\overrightarrow{x}) = \sum_{i=1}^{N} \overrightarrow{H}(\overrightarrow{x}_i, \overrightarrow{x})$$
 (13)

It is same to,  $\overrightarrow{D}(x)$  and  $\overrightarrow{B}(x)$ , hence we have,

$$- \oiint\limits_{\Gamma} (\sum_{i=1}^{N} \sum_{j=1}^{N} \overrightarrow{E}(\overrightarrow{x}_{i}, \overrightarrow{x}) \times \overrightarrow{H}(\overrightarrow{x}_{j}, \overrightarrow{x})) \cdot \hat{n} d\Gamma = \sum_{i=1}^{N} \sum_{j=1}^{N} \overrightarrow{I}_{i} \cdot \overrightarrow{E}(\overrightarrow{x}_{j}, \overrightarrow{x}_{i})$$

$$+ \iiint\limits_{V} \sum_{i=1}^{N} \sum_{j=1}^{N} (\overrightarrow{E}(\overrightarrow{x}_{i}, \overrightarrow{x}) \cdot \partial \overrightarrow{D}(\overrightarrow{x}_{j}, \overrightarrow{x}) + \overrightarrow{H}(\overrightarrow{x}_{i}, \overrightarrow{x}) \cdot \partial \overrightarrow{B}(\overrightarrow{x}_{j}, \overrightarrow{x})) dV \quad (14)$$

In the above second item, we have considered that,

$$\iiint\limits_{V}(\overrightarrow{J}_{i}\cdot\overrightarrow{E}(\overrightarrow{x}_{i},\overrightarrow{x}))dV=\iiint\limits_{V}(\overrightarrow{I}_{i}\delta(\overrightarrow{x}-\overrightarrow{x}_{i})\cdot\overrightarrow{E}(\overrightarrow{x}_{j},\overrightarrow{x}))dV=\overrightarrow{I}_{i}\cdot\overrightarrow{E}(\overrightarrow{x}_{j},\overrightarrow{x}_{i})$$

In the above formula we have considered Eq.(1, 2, 3).

# 2.3 The bug in Poynting theorem and MEQN

We obtain Eq.(9) In the subsection 2.1, and we obtain Eq.(14) in last subsection. Inside the two formula all has a items,

$$\overrightarrow{E}(\overrightarrow{x}_j, \overrightarrow{x}_i) \cdot \overrightarrow{I}_i \tag{16}$$

But the summations before it are different. This item together with the summation all express the interaction power of all charges in the system. From this comparison, this author believe the Poynting theorem has overestimated the power of all charges in the system.

Using the summation in Eq.(10) to replace the original summation in Eq.(14) we obtain,

$$- \iint_{\Gamma} \left( \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \overrightarrow{E}_{i} \times \overrightarrow{H}_{j} \right) \cdot \hat{n} d\Gamma$$

$$= \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (\overrightarrow{E}_{i} \cdot \overrightarrow{J}_{j}) dV$$

$$\iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (\overrightarrow{E}_{i} \cdot \partial \overrightarrow{D}_{j} + \overrightarrow{H}_{i} \cdot \partial \overrightarrow{B}_{j}) dV$$

$$(17)$$

In the above formula we have written  $\overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x})$  as  $\overrightarrow{E}_i$  and we have considered Eq.(15) and replaced  $\overrightarrow{I}_i$  by  $\overrightarrow{J}_i$  with the integral. The corresponding differential formula is,

$$-\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \nabla \cdot \overrightarrow{E}_{i} \times \overrightarrow{H}_{j}$$

$$= \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (\overrightarrow{E}_{i} \cdot \overrightarrow{J}_{j} + \overrightarrow{E}_{i} \cdot \partial \overrightarrow{D}_{j} + \overrightarrow{H}_{i} \cdot \partial \overrightarrow{B}_{j})$$
(18)

The above formula Eq.(17) is the rest items of the Poynting theorem Eq.(14) if all self items is taken away. The all self items are as following,

$$- \iint_{\Gamma} (\sum_{i=1}^{N} \overrightarrow{E}_{i} \times \overrightarrow{H}_{i}) \cdot \hat{n} d\Gamma = \iiint_{V} (\sum_{i=1}^{N} (\overrightarrow{E}_{i} \cdot \overrightarrow{J}_{i}) dV$$

$$\iiint_{V} (\sum_{i=1}^{N} (\overrightarrow{E}_{i} \cdot \partial \overrightarrow{D}_{i} + \overrightarrow{H}_{i} \cdot \partial \overrightarrow{B}_{i}) dV$$
(19)

Eq.(18) can be referred as mutual energy formula, which is closed related the mutual energy theorems, [21],[12, 23, 22, 13]. and [5]. This mutual energy

formula is correct in two ways. (1), it can be derived from MEQN or from Poynting theorem. If MEQN is correct this formula is also correct, it is easy to prove this. Because we take away all self items which also satisfy Poynting theorem for a single charge. From the Poynting theorem of N charges take away all corresponding Poynting theorem for single charges, this guarantees the rest items still correct if Poynting theorem is correct. Since Poynting theorem can be derived from MEQN, the rest items also satisfy MEQN. (2) The second way to show this formula is correct because it satisfies also the "direct interaction" principle [7]. The direction interaction principle actually tells us the action and reaction can only happens between two charges, there is no any action or reaction in space sends by singular charge. The direct interaction principle has been further developed to as the adjunct field theory of Wheeler and Feynman[2]. The mutual energy formula Eq. (18) is agreed with the direct interaction theory and can be seen as a new definition of the so called adjunct field. Wheeler and Feynman did not point out this formula, they developed a new QED (theory quantum electrodynamics) from their adjunct field theory. Wheeler and Feynman try abandon the classical electromagnetic theory in quantum physics where only a few charges is involved (N is very small).

If someone claim he find a new theorem which is the above mutual energy formula, no any journals can accept it, because it just a a direct deduction of Poynting theorem. However we will show that it is so important, should be applied as axiom of the electromagnetic theory.

About the self energy formula Eq.(19) which is the Poynting theorem for singular charge. It need to be taken out that means this formula is problematic. If singular charge has a current change  $\overrightarrow{J}_i$ , according to the Maxwell's theory there is a real physical wave sent from this current change. According to quantum physics double slit experiment, this wave is not a real wave but a probability wave. Experiments shows that the photon is only randomly received by the absorbers which can receive the wave sent out from the emitter charge with current change  $\overrightarrow{J}_i$ . Traditionally, the people thought that Maxwell's theory is only suitable to the wireless wave which has lower frequency, it is not suitable to the high frequency phenomena like photon. Photons needs quantum theory, quantum electrodynamics or quantum field theory to solve. This author believe the suitable revises from electromagnetic field theory of Maxwell can still keep this theory alive even with the photon's frequency. The key of this is to take out the self-energy items Eq.(19) from the Poynting theorem 17.

# 2.4 Comparison of the Poynting theorem and the mutual energy formula

In the following we compare the Poynting theorem Eq.(14) and the mutual energy formula Eq.(17) and see which is more meaningful. The left side of Eq.(17) is,

$$\oint_{\Gamma} \left( \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \overrightarrow{E}_{i} \times \overrightarrow{H}_{j} \right) \cdot \hat{n} d\Gamma = 0$$
(20)

which is the power sends to outside of our space if  $\Gamma$  is big sphere contains our universe, it is the flux of the energy flow send to outside of the universe, it should vanish. If there is only N charge in a empty space, there should no energy flow go outside according to the direct interaction principle. We have known from the mutual energy theorem[13, 15] if photon's field either retarded field for the emitter or advanced field from absorber, the mutual energy flow vanishes on the big sphere  $\Gamma$ , hence the left side of Eq.(17) vanishes. The second term in the right side of Eq.(17) is the system energy in the space. If started from some time there is no action or reaction to a end time there is also no action and reaction. The integral of this energy vanishes, i.e.,

$$\int_{t=-\infty}^{\infty} \iiint\limits_{V} \left(\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (\overrightarrow{E}_{i} \cdot \partial \overrightarrow{D}_{j} + \overrightarrow{H}_{i} \cdot \partial \overrightarrow{B}_{j}) dV dt = 0$$
 (21)

Substitute Eq. (20 and 21) to Eq. (17), we have the last term,

$$\int_{t=-\infty}^{\infty} \iiint_{V} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (\overrightarrow{E}_{i} \cdot \overrightarrow{J}_{j}) dV dt = 0$$
 (22)

These terms also vanish. The above formula tell us for the whole system, all energy is conserved. There is no any energy sends to outside of our universe (if our universe is composed as N charges). Hence this a corrected formula. The above formula vanishes means that Eq.(17) satisfies the direct interaction theory. The whole power of the system with all charges same as the subsection 2.1. It is much meaningful comparing to the Poynting theorem Eq.(14) in which it has the items,

$$\iiint\limits_{V}(\overrightarrow{E}_{i}\cdot\overrightarrow{J}_{i})dV=\iiint\limits_{V}(\overrightarrow{E}(\overrightarrow{x}_{i},\overrightarrow{x})\cdot\overrightarrow{J}_{i}(\overrightarrow{x}))dV=\infty \tag{23}$$

Since if the charge is a point, there is  $\overrightarrow{E}_i = \overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x}) \to \infty$ , if  $\overrightarrow{x} \to \overrightarrow{x}_i$ . It also has the items

$$- \iint_{\Gamma} (\overrightarrow{E}_i \times \overrightarrow{H}_i) \cdot \hat{n} d\Gamma \neq 0$$
 (24)

The system always has some energy go to outside even where is empty space without other charges. If the system is our universe, it must be opaque to receive all energy, otherwise our universe will have a continuous loss of energy. Up to now there is no any testimony that our universe is opaque. It is very strange. The following items in Poynting theorem,

$$\sum_{j=1}^{N} \sum_{i=1}^{N} (\overrightarrow{E}_i \cdot \overrightarrow{J}_j) \tag{25}$$

is not the power of the whole system of N charges. It is over estimated the power of a system with N charges! The problem of the Poynting theorem is

the cause that a re-normalization process has to be done for quantum physics. This is a bug of the Poynting theorem with N charges. Poynting theorem is derived from MEQN. MEQN is derived from MEQS by apply the principle of superimposition principle. The bug in Poynting theorem is also a bug in either in superimposition principle or in MEQS. We have not found any problem with mutual energy formula Eq. (17).

# 2.5 The confusion of the definition of the electromagnetic fields

Last subsection we have said, it is possible the superimposition principle has the problem. Now let us to see the concept of the electric and magnetic field. Assume there are N charges in the system, we can calculate the electric field in the place  $\overrightarrow{x}$  by superimposition,

$$\overrightarrow{E}(\overrightarrow{x}) = \sum_{j=1}^{N} \overrightarrow{E}(\overrightarrow{x}_j, \overrightarrow{x})$$
 (26)

where  $\overrightarrow{x}_j$  is the position of the charge  $q_j$ ,  $\overrightarrow{E}(\overrightarrow{x}_j, \overrightarrow{x})$  is the charge  $q_j$  produced field in the position  $\overrightarrow{x}$ , this definition looks good. However if we need to know the field at a the position of any charges, we can write,

$$\overrightarrow{E}(\overrightarrow{x}_i) = \sum_{j=1, j \neq i}^{N} \overrightarrow{E}(\overrightarrow{x}_j, \overrightarrow{x}_i) + \overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x}_i)$$
 (27)

but

$$\overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x}_i) = \infty \tag{28}$$

if the charge is a point charge. Hence we have to change the definition of the field as following,

$$\overrightarrow{E}(\overrightarrow{x}) = \begin{cases} \sum_{j=1}^{N} \overrightarrow{E}(\overrightarrow{x}_{j}, \overrightarrow{x}) & \overrightarrow{x} \notin I\\ \sum_{j=1, j \neq i}^{N} \overrightarrow{E}(\overrightarrow{x}_{j}, \overrightarrow{x}) & \overrightarrow{x} \in I \end{cases}$$
(29)

 $I=1,\cdots i\cdots N$ , it is the set of the index of the charges. The above definition does also not very satisfy. Many people will ague that is this correct that the field is extended to the any position without a test charge? According to the principle of direct interaction, only the action and reaction force can be defined, hence the field can only be defined on the charge which is,

$$\overrightarrow{E}(\overrightarrow{x}) = \begin{cases} No \ difinition & \overrightarrow{x} \notin I \\ \sum_{j=1, j \neq i}^{N} \overrightarrow{E}(\overrightarrow{x}_{j}, \overrightarrow{x}) & \overrightarrow{x} \in I \end{cases}$$
(30)

Hence we have 3 version of the definition about the field, which is correct? The concept of field is very confused. The magnetic field has the same problem we do not discuss it here.

The reason of this confusion is because that if we measure the field we need a test charge. But how can we know if the test charge is removed the measured field is still there? According to the direct interaction principle, if the test charge is removed, the field can not be defined as a real physics property. It is only an ability to give a force to the test charge, but it is not some thing real with energy in the space. It is also true for the radiation field, if the absorber received a photon, how can we know that the absorber is removed, the photon is still there? If the absorber is removed the retarded radiation field can only be a probability wave in quantum physics. It is not any wave with physical energy in the space. That is the reason many people will argue that after the removal of the test charge or the absorber, the field of the wave is not defined.

From this subsection we are clear that the concept of the field is very confused, actually this means the superimposition principle has problem. There not exist this kind of linear fields which can be simply added together.

In the last few subsections we have said there is a bug in Poynting theorem. If the superimposition principle has problem, we do not mean that the theory of the MEQS need to be thoroughly throw away. Instead the whole theory of electromagnetic fields need to be carefully reconstructed. Since without superimposition principle even if MEQS is correct (by the way last section we have to take away all the self-energy items which actually also means the MEQS has also the problem), the MEQN can still not be guaranteed to be correct. We still need to prove MEQN. In the proof we have to point where the approximation is made. This way to allow the MEQN survival with the bug.

Without the superimposition principle, we can still define fields as a collection of all fields of their charges,

$$\overrightarrow{E}(\overrightarrow{x}) = [\overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x}), \cdots E(\overrightarrow{x}_i, \overrightarrow{x}) \cdots]$$
(31)

or

$$\overrightarrow{E}(\overrightarrow{x}) = [\overrightarrow{E}_1 \cdots \overrightarrow{E}_j \cdots \overrightarrow{E}_N]$$
(32)

we have written  $\overrightarrow{E}_j = \overrightarrow{E}(\overrightarrow{x}_j, \overrightarrow{x})$  for simplicity.

# 3 Reconstruction the electromagnetic field theory

### 3.1 Define the point and cross multiplication of the fields

We have no the superimposition of the fields. We do not know how to "add" is really correct. However we still can re-define the cross and point multiplication "×", "·" by,

$$\overrightarrow{A} \cdot \overrightarrow{B} \equiv \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \overrightarrow{A}_{j} \cdot \overrightarrow{B}_{i}$$
(33)

$$\overrightarrow{A} \times \overrightarrow{B} \equiv \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \overrightarrow{A}_{j} \times \overrightarrow{B}_{i}$$
(34)

In the above formula the right side,  $\overrightarrow{A}_j \cdot \overrightarrow{B}_i \ \overrightarrow{A}_j \times \overrightarrow{B}_i$  still has the traditional definition of point or cross multiplications for only the fields of the two singular charges. The left sides are the point and cross multiplications for a system with N charges. In this way the mutual energy formula Eq.(18) can be written as

$$-\nabla \cdot (\overrightarrow{E} \times \overrightarrow{H}) = \overrightarrow{E} \cdot \overrightarrow{J} + \overrightarrow{E} \cdot \partial \overrightarrow{D} + \overrightarrow{H} \cdot \partial \overrightarrow{B}$$
 (35)

This looks like the Poynting theorem, according the new definitions of the point and cross multiplications, however, it is the mutual energy formula.

### 3.2 Linearization of electromagnetic fields

MEQN and Poynting theorem has a bug, we cannot apply it as a principle, which theorem can be a replacement of the MEQN and MEQS? We choose the mutual energy formula, the mutual energy formula can be derived from MEQN does not mean if MEQN is wrong, the mutual energy formula is also wrong. It only tells us if the mutual energy formula is wrong then the MEQN is also wrong. We have point out that the mutual energy formula agrees with the direct interaction principle and energy conservation principle, it should be a very good candidate to be upgraded as a principle or axiom instead as a formula or a theorem. The only thing left is to see whether or not we can reconstruct the whole electromagnetic theory base on the mutual energy formula alone.

Now we take the mutual energy formula as the axiom, hence started from it to build the whole electromagnetic theory. The field is not linear, that is not very convenient. In order to make things simple, we can add self energy items to the mutual energy theory.

Even we known that self energy flow formula for the singular charge (i-th charge)

$$-\nabla \cdot \overrightarrow{E}_{i} \times \overrightarrow{H}_{i} = \overrightarrow{E}_{i} \cdot \overrightarrow{J}_{i} + \overrightarrow{E}_{i} \cdot \partial \overrightarrow{D}_{i} + \overrightarrow{H}_{i} \cdot \partial \overrightarrow{B}_{i}$$
 (36)

is nonsense in physics according the discussion of last section, actually that  $\overrightarrow{E}_i \times \overrightarrow{H}_i$  is 0,  $\overrightarrow{E}_i \cdot \overrightarrow{J}_i$  is 0, but the mathematics calculation that is not 0. We also know that if the charge is point,  $\overrightarrow{E}_i = \overrightarrow{E}(\overrightarrow{x}_i, \overrightarrow{x}_i) = \infty$ , we can just assume the charge is not a point. But the charge is distribute inside a small sphere region with a radio R. We can choose a any small radio, for example  $R = 10^{-16}$  meter. Then the above formula can be a correct formula at least in the meaning of mathematics. It can be seen as a pseudo self energy flow formula. We can add this pseudo formula to the mutual energy formula Eq(18), we obtained,

$$-\nabla \cdot (\sum_{j=1}^{N} \sum_{i=1}^{N} \overrightarrow{E}_i \times \overrightarrow{H}_j) = \sum_{j=1}^{N} \sum_{i=1}^{N} (\overrightarrow{E}_i \cdot \overrightarrow{J}_j)$$

$$+\sum_{i=1}^{N}\sum_{i=1}^{N}(\overrightarrow{E}_{i}\cdot\partial\overrightarrow{D}_{j}+\overrightarrow{H}_{i}\cdot\partial\overrightarrow{B}_{j})$$
(37)

This formula is still correct as a equation, since we have add same amount values to both sides of the mutual energy formula (which now is assumed as an axiom). If the original equation is correct, the new equation is still correct in mathematics.

Keep in mind the above formula is only correct in the meaning of mathematics, not in physics. It is a mathematical formula not a physic formula.  $\sum_{j=1}^{N} \sum_{i=1}^{N} (\overrightarrow{E}_i \cdot \overrightarrow{J}_j) \text{ is lager than the total power of action and reaction of the whole system. } \sum_{j=1}^{N} \sum_{i=1}^{N} (\overrightarrow{E}_i \cdot \partial \overrightarrow{D}_j + \overrightarrow{H}_i \cdot \partial \overrightarrow{B}_j) \text{ is larger than the energy in the space. } -\nabla \cdot (\sum_{j=1}^{N} \sum_{i=1}^{N} \overrightarrow{E}_i \times \overrightarrow{H}_j) \text{ is also larger than the energy flows direct to the outside space. The formula can be further written as,}$ 

$$-\nabla \cdot (\sum_{i=1}^{N} \overrightarrow{E}_{i}) \times (\sum_{i=1}^{N} \overrightarrow{H}_{j}) = (\sum_{i=1}^{N} \overrightarrow{E}_{i}) \cdot (\sum_{i=1}^{N} \overrightarrow{J}_{j})$$
$$+(\sum_{i=1}^{N} \overrightarrow{E}_{i}) \cdot (\sum_{i=1}^{N} \partial \overrightarrow{D}_{j}) + (\sum_{i=1}^{N} \overrightarrow{H}_{i}) \cdot (\sum_{i=1}^{N} \partial \overrightarrow{B}_{j})$$
(38)

Write

$$\overrightarrow{E} = \sum_{i=1}^{N} \overrightarrow{E}_{i}, \quad \overrightarrow{H} = \sum_{i=1}^{N} \overrightarrow{H}_{i} \quad \overrightarrow{J} = \sum_{i=1}^{N} \overrightarrow{J}_{i}$$
 (39)

We obtain that,

$$-\nabla \cdot \overrightarrow{E} \times \overrightarrow{H} = \overrightarrow{E} \cdot \overrightarrow{J} + \overrightarrow{E} \cdot \partial \overrightarrow{D} + \overrightarrow{H} \cdot \partial \overrightarrow{B}$$
 (40)

Hence we obtained the Poynting theorem. The point and cross multiplications are in the traditional meaning. We also obtain the superimposition of the electromagnetic fields Eq. (39). Keep in mind the superimposition formula Eq. (39) and the Poynting theorem Eq. (40) are all mathematical results and are not results in physics. They are results when we add a pseudo self-energy items to the both sides of the mutual energy formula Eq. (18), which now is the axiom of the electromagnetic electromagnetic theory.

We can added a pseudo field only when N is very big, for example in the situation in wireless wave which is the low frequency electromagnetic fields. In that situation the difference of  $\sum_{j=1}^{N}\sum_{i=1}^{N}$  and  $\sum_{j=1}^{N}\sum_{i=1,j\neq i}^{N}$  is the difference  $N^2$  to N(N-1) which can be omitted. In light or photon situation there is only N=2, for example only a emitter and an absorber, the pseudo field is not small any more, it can not be applied. In that situation, Poynting theorem is wrong we can only apply the mutual energy formula. By the way, in the case of quantum entanglement situation the number of system can be not just N=2, it is possible N>2, but it is still a very small number, in which the pseudo self energy items cannot add to the mutual energy formula.

From this we know that why sometime we can assume the electron's charge distributes as a small sphere instead of point, still can obtains correct calculation result. The Poynting theorem can be written as,

$$-(\nabla \times \overrightarrow{E} \cdot \overrightarrow{H} - \nabla \times \overrightarrow{H} \cdot \overrightarrow{E}) = \overrightarrow{E} \cdot \overrightarrow{J} + \overrightarrow{E} \cdot \partial \overrightarrow{D} + \overrightarrow{H} \cdot \partial \overrightarrow{B}$$
 (41)

or

$$-(\nabla \times \overrightarrow{E} + \partial \overrightarrow{B}) \cdot \overrightarrow{H} + (\nabla \times \overrightarrow{H} - \overrightarrow{J} - \partial \overrightarrow{D}) \cdot \overrightarrow{E} = 0$$
 (42)

The sufficient conditions of the above formula is

$$\nabla \times \overrightarrow{E} + \partial \overrightarrow{B} = 0 \tag{43}$$

$$\nabla \times \overrightarrow{H} - \overrightarrow{J} - \partial \overrightarrow{D} = 0 \tag{44}$$

We got the MEQN. We did not get the MEQN as a necessarily conditions. But this is enough. In this article, MEQN are not need to be derived, it has a bug anyway! The above derivation of the MEQN is also by dint of the pseudo self energy items. We also obtained the superimposition of the field, but that is also dependent to the pseudo items. The pseudo items can be omitted only when the number of charges N is very large, which is only happens for low frequency electromagnetic fields or wireless wave but not for light or x-ray.

### 3.3 The other two equation of MEQN

The other two equations of MEQN are,

$$\nabla \cdot \overrightarrow{B} = 0 \tag{45}$$

$$\nabla \cdot \overrightarrow{D} = \rho \tag{46}$$

Here  $\overrightarrow{B}$  and  $\overrightarrow{D}$  are magnetic B-field and electric displacement. Or in the integral formula,

Take div operator or " $\nabla$ ·" to the two side of Eq.(43, 44), considering  $\nabla \cdot \nabla \times \overrightarrow{G} = 0$ , here  $\overrightarrow{G}$  is a arbitrary vector, we obtain,

$$-\nabla \cdot \overrightarrow{J} - \partial \nabla \cdot \overrightarrow{D} = 0 \tag{47}$$

considering charge continuous equation,

$$\nabla \cdot \overrightarrow{J} = -\partial \rho \tag{48}$$

we obtain,

$$\partial \rho - \partial \nabla \cdot \overrightarrow{D} = 0 \tag{49}$$

$$\nabla \cdot \overrightarrow{D} = \rho + Constant \tag{50}$$

similarly we have

$$\partial \nabla \cdot \overrightarrow{B} = 0 \tag{51}$$

or

$$\nabla \cdot \overrightarrow{B} = Constant \tag{52}$$

Because we thought the field any way is a problematic, especially in higher frequency we do not care a constant field. Hence for this equations we do not need to derive these constants from mutual energy formula. In the above we have applied the charge continuous equation. Hence we have to put the continuous equation also to our axioms. We do not know whether or not from mutual energy formula we can obtain Lorentz force formula

$$F = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}) \tag{53}$$

Hence, the 3 formula are all axioms of electromagnetic field theory our new electromagnetic field theory, (1) the mutual energy formula Eq.(17), (2) the continuous equation Eq.(48) of electric charges and there is no magnetic charges. (3) the Lorentz force formula Eq.(53).

### 3.4 Why we still need MEQN?

Perhaps the reader will ask why we need to derive MEQN? You have said the MEQN has bug and started the electromagnetic theory from mutual energy formula, why derive the MEQN again? We must make clear here. Since we have found the bug in Poynting theorem, Poynting theorem is derived from MEQN, that means the MEQN have the bug too. Actually the problem is at the superimposition of the field. We have said the field is a very confused concept. It cannot be superimposed. If we have no the concept of field for a system with N charges or if the field can not be superimposed, even MEQS is correct for a singular moving charge, it still can not prove the MEQN is correct. That is reason we need to derive the MEQN. MEQN still can be applied to most engineer problems. We have proved that MEQN are still correct in mathematics. The only wrong is at how to interpretation of physics meaning of the expressions in MEQN. This is because in the derivation of the MEQN we have add the the pseudo self energy items to the two sides of the Poynting theorem. For example we often say that Poynting vector,

$$\overrightarrow{S} = \overrightarrow{E} \times \overrightarrow{H} \tag{54}$$

expresses energy flow intensity, it is not true for a system with N charges, unless the cross multiplication change to the new definition Eq.(34). It is also not true for a system with only one charge. In one charge situation, Poynting theorem tell us there is a energy flow sends out from the charge. But according the direct interaction principle there is no this kind of energy flow which exist without a absorber. Even in quantum physics, the wave of this singular charge is a probability wave, which doesn't transfer energy.

Understand that we can now easily understand why in quantum physics need a re-normalization process, when it takes away all self energy terms in the quantum formula, they got correct result. In the correct physics these all self energy terms actually should be taken away. Hence we have two methods to solve the electromagnetic problems, (1) Applying the new definitions of the point and cross multiplications or (2) still apply MEQN, but keep in mind we have added a pseudo self energy items to Poynting theorem.

Perhaps some people will argue that, the Poynting theorem has bug, that is only the wrong of the superimposition, MEQS is still correct. That is also not true in general. Since the solution of MEQS produced a energy flow which can be expressed by a surface integral with Poynting vector. This energy does not vanish. A charge continually sends out the energy that violates the direct interaction principle. In the direct interaction principle there is no the field which can exist independent to the test charge or absorber. There is only the adjunct field which is depended to the action and reaction of two interacted charges. Hence one singular charge cannot send wave out. Or we can think a charge(emitter) can send the retarded wave (a real physics wave with energy) out, but since there is no any absorber to send an advanced and synchronized wave to receive this energy, this retarded wave cannot make the energy transaction. If we think it is a real wave send out from singular charge, we also can think it perhaps returned later to it's emitter.

In case wireless situation MEQN can still be applied, the error is only a very small part which is the pseudo self field items. In the high frequency like light or x-ray, we have to apply the mutual energy formula only.

### 3.5 What is accurate and what is not accurate?

All things related to the pseudo field are only approximately correct. The first is the electromagnetic field,

A few our often used concept is only approximately correct:

- (1) The energy flow related to Poynting vector. The replace should be the mutual energy flow.
- (2) The interaction energy of the whole system calculated from the Poynting theorem. The replace should be the calculation with the mutual energy formula.
- (3) the electric field and magnetic field, since the linearization process has applied the concept of pseudo items.

In electromagnetic field theory, there has lot of problems are not directly solved by using MEQN, but are solved by using Lorentz reciprocity theorem. We have shown that Lorentz reciprocity theorem is only a mathematical transformation of the mutual energy theorem[12, 23, 22, 13], it change the advanced wave in the mutual energy theorem to a retarded wave[13]. Many results obtained from Lorentz theorem is still correct. For example, the directivity diagram of the receiving antenna. We also know that the theory of the Green function can be derived from Lorentz reciprocity theory hence many results obtained from green function are still correct. All the problems can be solved by Lorentz reciprocity theorem can also be solved by the mutual energy theorem,

for example the directivity diagram of the receiving antenna. Apply mutual energy theorems will offer not only the correct directivity diagram, but also the current of the receiving antenna. As a contract the Lorentz reciprocity theory only offers the correct directivity diagram. We have shown in the lossy media, the Lorentz reciprocity theorem is not correct any more. If a antenna is put in salt water, even the directivity diagram can only be calculated with the mutual energy theorem[14].

# 4 Mutual energy theorems

We have found the bug of the Poynting theorem, however it is still can be used as approximate formula if the number of charges N is very large, for example wireless wave situation. In the one charge alone situation, the MEQS still can offers a retarded solution and also an advanced solution. But this is not true comparing to the direct interaction principle, in which there is no any field exist alone and can be separated from the interacted two charges, one is the emitter and the another is the absorber. Normally photon is a system with N=2, there is only an emitter and an absorber. All other charges does not involved. If there is quantum entanglement, the N will large than 2. The emitter sends retarded wave, the absorber send advanced wave the two waves must synchronized.

### 4.1 The mutual energy principle

The reason, the self energy flow is calculated as nonzero is because, the concept electromagnetic field is wrong. The electromagnetic field need a test charge in static field situation or an absorber in light wave situation to measure the field or absorb the field. However, this charge or absorber actually joined the creation of the action and reaction. We have measured the field and we cannot show whether or not if the test charge or the absorber is removed from the system, the electromagnetic field still exists.

Hence there is a continually debate between two group peoples, first group believe action and reaction is only happened between two charges, for example one is the charge its field need to be measured, one is the test charge. If the test charge exists, then the field to be measured is exist, if the test charge is removed, then the field to be measured does not defined any more. The people in this group claim direct interaction principle, this action or reaction is only exist in the case there are two electrons, the emitter and the absorber. The field cannot exist independent to the any one of emitter and the absorber. Direct interaction theory, absorber theory of Wheeler and Feynman and also QED (quantum electrodynamics) and QFT (quantum field theory) belong to this group.

Another group claim that the field can exist independent to its source for example charges. This group people are very success in classical electromagnetic theory. MEQN is also belong to this groups. Maxwell theory claims that wave can exist independent to its source. Our traditional electromagnetic theory

thought field is an independent concept which exist even without the charges. Without charges we can also obtain the field or waves by solving MEQN.

In the mainstream of quantum physics, they accept the second groups theory, hence the wave can exist without emitter and absorber. But this cause lot of problems in quantum physics, energy perhaps not conserved. Hence in quantum physics they create a concept called "wave function collapse". The retarded wave need to be collapsed to the absorber. After add the concept of wave function collapse, actually action and reaction can only happen between two charges. In quantum physic, in case there is no collapse, the wave is said to be a probability wave by Copenhagen interpretation. Hence quantum physics actually do not violate the direct interact principle. Even wave function collapse is very difficult to be accept because who can offers a equation of the process of collapse of wave? But the concept of wave function collapse is very useful to avoid the problem of the MEQN and MEQS, and also the Schrodinger equation. Schrodinger equation also offer a wave with divergence, hence also need a wave function collapse. In this article, we solved the photon problem without apply the concept of wave function collapse, this idea can also be applied to quantum physics.

In quantum physics by using the concept of the wave function collapse and the re-normalization process, the superimposition principle can still survival. this author cannot accept the concept of wave function collapse, and looking for other solution.

In order to solve this same problems, we notice that the problem is at the Poynting theorem for N charges. This problem cannot be solved through the concept field which can be superimposed. But we have successfully solved it through the energy. Started from the Poynting theorem, we remove all self items include self energy flow (Poynting vector), self energy increase  $(\overrightarrow{E}_i \partial \overrightarrow{D}_i + H_i \partial \overrightarrow{B}_i)$  and self reaction  $(\overrightarrow{J}_i \cdot \overrightarrow{E}_i)$ . After this removal, the Poynting theorem is changed to the mutual energy formula.

If self energy flow doesn't exist. MEQS is clear wrong, because from MEQS we can got a solution with self energy flow, which does not vanish. If self energy flow vanish, we can only got a zero solution with MEQS. Hence, up to now the only way is to amend the Maxwell theory especially in photon situation where N is small. Taking the self energy flow away and MEQN and MEQS away, the left is only the mutual energy formula and we can call it as the mutual energy principle, it is applied as axiom of electromagnetic field theory. In the above few sections we have shown that the mutual energy formula can be used to replace Poynting theorem, after this replacement, we got a new "Poynting theorem" which is actually the mutual energy formula. We also show that the Poynting theorem is equivalent in principle to the MEQN and MEQS. Hence, when we can replace the Poynting theorem with the mutual energy formula, the mutual energy formula actually can also replace the MEQN and MEQS. After we also show that the Gauss law can also merged to the mutual energy formula, Hence we can use mutual energy formula only one formula to replace all  $(8 = 2 \times 4, a \text{ photon is with } 2 \text{ charges the emitter and the absorber and need})$  2 groups of Maxwell equations) formulas of the MEQS. That is also correct in the philosophy that the principle should be simple.

We also show the problem of the MEQN, if we add a pseudo field to the two sides of the mutual energy formula, it becomes the Poynting theorem, and the Poynting theorem is still correct in mathematics. It also approximately correct in physics if N is larger. We have notice there are pseudo field items. However, most wireless problem which still can be solved with MEQN since the contribution of the pseudo field items is very small.

In the case of light, there is only two charges, one is emitter and one is absorber. This is the place we really need to deal the problem of self energy items. We have calculated that if it exist and does not return to its source, it will contribute half energy transfer from emitter to absorber[15]. In this situation the self energy items cannot be omitted. This author endorse the direct interaction principle, which leads us to denies all existent of the self energy items. After removal of all self energy items, we obtained the mutual energy formula or the mutual energy principle. Now we need to looking the solution from the mutual energy principle.

Now we are clear that we actually do not need to calculate the self energy flow, which does not exist, we have long trouble with it [15]. Hence, this means that we only need to find a solution which satisfy the mutual energy principle. For the photon situation there is only the emitter and absorber two electrons, we assume the index of the emitter is 1 and the index of the absorber is 2, the mutual energy principle Eq(18) is listed as following,

$$-\nabla \cdot (\overrightarrow{E}_{1} \times \overrightarrow{H}_{2} + \overrightarrow{E}_{2} \times \overrightarrow{H}_{1}) = \overrightarrow{E}_{2} \cdot \overrightarrow{J}_{1} + \overrightarrow{E}_{1} \cdot \overrightarrow{J}_{2}$$

$$+\overrightarrow{E}_{1} \cdot \partial \overrightarrow{D}_{2} + \overrightarrow{E}_{2} \cdot \partial \overrightarrow{D}_{1} + \overrightarrow{H}_{1} \cdot \partial \overrightarrow{B}_{2} + \overrightarrow{H}_{2} \cdot \partial \overrightarrow{B}_{1}$$

$$(55)$$

It is worth to say that the time domain reciprocity theorem[21], frequency domain mutual energy theorem[12, 23, 22], time domain correlation reciprocity theorem[5] and second reciprocity theorem [11] are very closed related to the mutual energy principle. They can all be referred as mutual energy theorems. They are not widely accept in electromagnetic field theory because the advanced wave is involved. The advanced wave is also not widely accept. But this author endorse the concept of the advanced wave. The mutual energy theorems originally is all derived from MEQS. Now the mutual energy principle has replaced MEQS become the axiom of the electromagnetic field theory. Hence all the mutual energy theorems must also be re-derived from the mutual energy principle. In next a few subsection we will derive all these theorems.

### 4.2 The field of the emitter and the absorber

The mutual energy principle for two charges which is an emitter and an absorber can be changed as following,

$$-\nabla\times\overrightarrow{E}_{1}\cdot\overrightarrow{H}_{2}+\nabla\times\overrightarrow{H}_{2}\cdot\overrightarrow{E}_{1}-\nabla\times\overrightarrow{E}_{2}\cdot\overrightarrow{H}_{1}+\nabla\times\overrightarrow{H}_{1}\cdot\overrightarrow{E}_{2}$$

$$= \overrightarrow{E}_{2} \cdot \overrightarrow{J}_{1} + \overrightarrow{E}_{1} \cdot \overrightarrow{J}_{2}$$

$$+ \overrightarrow{E}_{1} \cdot \partial \overrightarrow{D}_{2} + \overrightarrow{E}_{2} \cdot \partial \overrightarrow{D}_{1} + \overrightarrow{H}_{1} \cdot \partial \overrightarrow{B}_{2} + \overrightarrow{H}_{2} \cdot \partial \overrightarrow{B}_{1}$$
(56)

Or

$$-(\nabla \times \overrightarrow{E}_{1} + \partial \overrightarrow{B}_{1}) \cdot \overrightarrow{H}_{2} + (\nabla \times \overrightarrow{H}_{1} - \overrightarrow{J}_{1} - \partial \overrightarrow{D}_{1}) \cdot \overrightarrow{E}_{2}$$

$$-(\nabla \times \overrightarrow{E}_{2} + \partial \overrightarrow{B}_{2}) \cdot \overrightarrow{H}_{1} + +(\nabla \times \overrightarrow{H}_{2} - \overrightarrow{J}_{2} - \partial \overrightarrow{D}_{2}) \cdot \overrightarrow{E}_{1}$$

$$= 0$$

$$(57)$$

From the above equation we can obtained the conclusions, that if  $\xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1] = 0$ , then  $\xi_2 = [\overrightarrow{E}_1, \overrightarrow{H}_1]$  can take,

$$\begin{cases} \nabla \times \overrightarrow{E}_2 + \partial \overrightarrow{B}_2 = arbitrary < \infty \\ \nabla \times \overrightarrow{H}_2 - \overrightarrow{J}_2 - \partial \overrightarrow{D}_2 = arbitrary < \infty \end{cases}$$
 (58)

This means  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2]$  can take arbitrary values. Vice versa, if  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2] = 0$ , then  $\xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1]$  can take arbitrary values. All these are not the physics solutions we are looking for. Hence  $\xi_1$  and  $\xi_2$  must nonzero in the simultaneously.

We know the MEQS are the sufficient conditions of the mutual energy principle, hence we can got the solution of the mutual energy principle by solving the MEQS. One of the solution of the above photon equation is MEQS solutions which is,

$$\begin{cases} \nabla \times \overrightarrow{E}_{1}(t) = -\partial \overrightarrow{B}_{1}(t) \\ \nabla \times \overrightarrow{H}_{1}(t) = +\overrightarrow{J}_{1}(t) + \partial \overrightarrow{D}_{1}(t) \end{cases}$$
 (59)

and

$$\begin{cases} \nabla \times \overrightarrow{E}_{2}(t) = -\partial \overrightarrow{B}_{2}(t) \\ \nabla \times \overrightarrow{H}_{2}(t) = +\overrightarrow{J}_{2}(t) + \partial \overrightarrow{D}_{2}(t) \end{cases}$$
(60)

It must notice that we are looking the solutions  $\zeta_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1, J_1], \ \zeta_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2, J_2]$  nonzero simultaneously. Here we use  $\zeta$  to express field together with the source and  $\xi$  to express only the field. In the above discussion, if  $\xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1] = 0, \ \xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2] \neq 0$ , this is not a physical solution of the mutual energy principle and which is not what we are looking for. However we if  $\xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1] = 0, \ \xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2] \neq 0$  and still satisfy the above MEQS Eq.(60), we will say that the  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2] \neq 0$  is a probability wave. The solution  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2] \neq 0$  is not exist as physics solution but it still can be a mathematical solution. Vice versa,  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2] = 0, \ \xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1] \neq 0$  and

satisfy MEQS Eq. (59) can be seen as a mathematical solution with the interpretation of probability. This way we have offers a very good explanation about the probability interpretation about MEQS. This means if we take the mutual energy principle as axiom of electromagnetic field theory, very naturally obtained that the solution MEQS is a probability wave. This also shows the advantage that take the mutual energy formula as the axiom of the electromagnetic theory than MEQS.

In the case the  $\xi_1$  and  $\xi_2$  are both nonzero for a very long time. We can see them as continuous waves. In the continuous wave situation, it easily see that the following formula is also the solution of the mutual energy principle,

$$\begin{cases} \nabla \times \overrightarrow{E}_{1}(t) = -\partial \overrightarrow{B}_{1}(t) \\ \nabla \times \overrightarrow{H}_{1}(t) = +\overrightarrow{J}_{1}(t) + \partial \overrightarrow{D}_{1}(t) \end{cases} \tag{61}$$

and

$$\begin{cases} \nabla \times \overrightarrow{E}_2(t+\tau) = -\partial \overrightarrow{B}_2(t+\tau) \\ \nabla \times \overrightarrow{H}_2(t+\tau) = +\overrightarrow{J}_2(t+\tau) + \partial \overrightarrow{D}_2(t+\tau) \end{cases} \tag{62}$$

These above formulas both satisfy MEQS and nonzero simultaneously. In the continuous wave situation, the two field do not need synchronized exactly.  $\xi_1$  and  $\xi_2$  can have a time difference  $\tau$  does not mean they do not need synchronized at all, in this situation it is better to consider the synchronization in frequency domain instead of time domain. We will discussion this in next section again.

However if the two waves  $\xi_1$  and  $\xi_2$  are very short-time impulse, the two waves nonzero simultaneously means that they must be synchronized. We will discuss the photon in next section which can be seen as short retarded wave sent from emitter and short advanced wave sent from absorber. Now the mutual energy principle require them must synchronized because they are short time waves. For example if the distance from the emitter to the absorber is 100 light years, the retarded wave and the advanced wave nonzero with a short time for example 1 nanosecond, the two impulses in the retarded wave and in the advanced wave will go 100 years in the space together synchronously. The mutual energy flow of the two synchronous waves is the photon which will be discussed in next section. It also need to say that if the two waves are both short-time retarded waves, or both short time advanced waves, then they cannot be synchronized, hence con not produce any energy package in the space or photon.

Hence only the above simultaneously nonzero solutions of the above two MEQS are the solution of the mutual energy principle. If the emitter and the absorber has some distance, the two wave can be synchronized only when one of which is retarded wave and another is advanced wave. We assume the  $\zeta_1$  is retarded wave and  $\zeta_2$  is advanced wave. Figure 1 shows the retarded wave and the advanced wave. If they are short time waves, they must synchronized, the energy for these waves can be transferred from the emitter to the absorber even it has been separated as a long distance for example 100 light years.

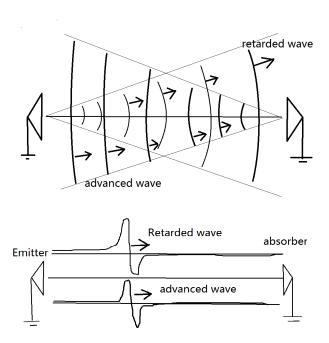


Figure 1: (a) Advanced wave and the retarded wave. (b) If the two waves are short time waves like photon, they have to be synchronized. This way even the are short-time waves, they still can send the energy from the emitter to the absorber, which perhaps separates a distance for example 100 light years.

# 4.3 W.J. Welch's time domain reciprocity theorem

Assume  $\xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1]$  is retarded wave,  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2]$  are advanced wave. Assume the electromagnetic field waves are only short time impulses.  $\xi_1$  and  $\xi_2$  satisfy Eq(59,60) and are synchronized.

In the beginning the electromagnetic energy in the space are 0. The energy in the space is increased in the time there is energy transfer between the emitter(transmitter) or the absorber(receiver). When the impulse finish, the energy in the space decrease to 0 again, hence there is,

$$\int_{-\infty}^{\infty} \iiint_{V} (\overrightarrow{E}_{1} \cdot \partial \overrightarrow{D}_{2} + \overrightarrow{E}_{2} \cdot \partial \overrightarrow{D}_{1} + \overrightarrow{H}_{1} \cdot \partial \overrightarrow{B}_{2} + \overrightarrow{H}_{2} \cdot \partial \overrightarrow{B}_{1}) dV dt = 0$$
 (63)

Considering the above formula in Eq.(55) we have,

$$-\int_{-\infty}^{\infty} \iint_{\Gamma} (\overrightarrow{E}_{1} \times \overrightarrow{H}_{2} + \overrightarrow{E}_{2} \times \overrightarrow{H}_{1}) \cdot d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V} (\overrightarrow{E}_{2} \cdot \overrightarrow{J}_{1} + \overrightarrow{E}_{1} \cdot \overrightarrow{J}_{2}) dV dt$$
(64)

When the radius of the surface  $\Gamma$  is much large the the distance of the emitter to the absorber, the retarded wave reach the surface  $\Gamma$  in a future time. The advanced wave reach the surface  $\Gamma$  in a past time. The two wave can not nonzero in the surface simultaneously. Hence there is,

$$\iint_{\Gamma} (\overrightarrow{E}_1 \times \overrightarrow{H}_2 + \overrightarrow{E}_2 \times \overrightarrow{H}_1) \cdot d\Gamma = 0$$
 (65)

Considering the above two formula, we have

$$\int_{-\infty}^{\infty} \iiint\limits_{V} (\overrightarrow{E}_{2} \cdot \overrightarrow{J}_{1} + \overrightarrow{E}_{1} \cdot \overrightarrow{J}_{2}) dV dt = 0$$
 (66)

Or

$$-\int_{-\infty}^{\infty} \iiint_{V} \overrightarrow{E}_{2} \cdot \overrightarrow{J}_{1} dV dt = \int_{-\infty}^{\infty} \iiint_{V} \overrightarrow{E}_{1} \cdot \overrightarrow{J}_{2} dV dt$$
 (67)

This is W.J. Welch's time domain reciprocity theorem[21]. The above time domain reciprocity theorem is suitable to the signal with very short time, for example the photon situation.

# 4.4 Adrianus T. de Hoop's time-correlation reciprocity theorem

In this situation we assume the waves are not short time impulse, but a long time waves. Assume  $\xi_1(t) = [\overrightarrow{E}_1(t), \overrightarrow{H}_1(t)]$  is retarded wave,  $\xi_2(t) = [\overrightarrow{E}_2(t), \overrightarrow{H}_2(t)]$  are advanced wave. In this time,  $\xi_1(t)$  and  $\xi_2(t)$  satisfy Eq(59,60). In this time  $\xi_1$  and  $\xi_2$  do not need to be synchronized exactly, they can still the solution of the mutual energy principle Eq.(55). Assume  $\xi_1(t), \xi_2(t)$  is the solution of mutual energy principle, then  $\xi_1(t), \xi_2(t+\tau)$  can also satisfy Eq.(59,60). From these two Maxwell equation we can obtained,

$$-\nabla \cdot (\overrightarrow{E}_{1}(t) \times \overrightarrow{H}_{2}(t+\tau) + \overrightarrow{E}_{2}(t+\tau) \times \overrightarrow{H}_{1}(t))$$

$$= \overrightarrow{E}_{2}(t+\tau) \cdot \overrightarrow{J}_{1}(t) + \overrightarrow{E}_{1}(t) \cdot \overrightarrow{J}_{2}(t+\tau)$$

$$+ \overrightarrow{E}_{1}(t) \cdot \partial \overrightarrow{D}_{2}(t+\tau) + \overrightarrow{E}_{2}(t+\tau) \cdot \partial \overrightarrow{D}_{1}(t)$$

$$+ \overrightarrow{H}_{1}(t) \cdot \partial \overrightarrow{B}_{2}(t+\tau) + \overrightarrow{H}_{2}(t+\tau) \cdot \partial \overrightarrow{B}_{1}(t)$$
(68)

Same reason with Eq.(63), we have

$$\begin{split} &\int\limits_{-\infty}^{\infty} \iiint\limits_{V} (\overrightarrow{E}_{1}(t) \cdot \partial \overrightarrow{D}_{2}(t+\tau) + \overrightarrow{E}_{2}(t+\tau) \cdot \partial \overrightarrow{D}_{1}(t) \\ + &\overrightarrow{H}_{1}(t) \cdot \partial \overrightarrow{B}_{2}(t+\tau) + \overrightarrow{H}_{2}(t+\tau) \cdot \partial \overrightarrow{B}_{1}(t)) dV dt = 0 \end{split}$$

Hence we have,

$$-\int_{-\infty}^{\infty} \iint_{\Gamma} (\overrightarrow{E}_{1}(t) \times \overrightarrow{H}_{2}(t+\tau) + \overrightarrow{E}_{2}(t+\tau) \times \overrightarrow{H}_{1}) \cdot d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V} (\overrightarrow{E}_{2}(t) \cdot \overrightarrow{J}_{1}(t+\tau) + \overrightarrow{E}_{1}(t) \cdot \overrightarrow{J}_{2}(t+\tau)) dV dt$$
(69)

The surface integral can be proved to be as 0, we will do that in the next subsection in the frequency domain. This is the time-correlation theorem of Adrianus T. de Hoop[5].

### 4.5 Shuang-ren Zhao's mutual energy theorem

Since  $\xi_2(t+\tau)$  and  $\overrightarrow{J}_2^*(t+\tau)$  are real number, the above formula can be rewritten as

$$-\int_{-\infty}^{\infty} \iint_{\Gamma} (\overrightarrow{E}_{1}(t) \times \overrightarrow{H}_{2}^{*}(t+\tau) + \overrightarrow{E}_{2}^{*}(t+\tau) \times \overrightarrow{H}_{1}(t)) \cdot \hat{n} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V} (\overrightarrow{E}_{2}^{*}(t) \cdot \overrightarrow{J}_{1}(t+\tau) + \overrightarrow{E}_{1}(t) \cdot \overrightarrow{J}_{2}^{*}(t+\tau)) dV dt$$
 (70)

where "\*" is complex conjugate operator. Take a Fourier transform of the above formula, it becomes,

$$- \iint_{\Gamma} (\overrightarrow{E}_{1}(\omega) \times \overrightarrow{H}_{2}^{*}(\omega) + \overrightarrow{E}_{2}^{*}(\omega) \times \overrightarrow{H}_{1}(\omega)) \cdot d\Gamma$$

$$= \iiint_{V} (\overrightarrow{E}_{2}^{*}(\omega) \cdot \overrightarrow{J}_{1}(\omega) + \overrightarrow{E}_{1}(\omega) \cdot \overrightarrow{J}_{2}^{*}(\omega)) dV$$
(71)

This is the mutual energy theorem of Shuang-ren Zhao[12, 23, 22].

Since  $\xi_1$  is retarded wave, when  $R \to \infty$ ,  $\hat{n}$  is the direction of the wave, there is Sommerfeld radiation condition,

$$\overrightarrow{H}_{1}(\omega) = \frac{1}{\eta} \hat{n} \times \overrightarrow{E}_{2}(\omega) \tag{72}$$

Since  $\xi_2$  is advanced wave,  $(-\hat{n})$  is the direction of the wave, there is the corresponding Sommerfeld radiation condition,

$$\overrightarrow{H}_2(\omega) = \frac{1}{\eta}(-\hat{n}) \times \overrightarrow{E}_2(\omega) \tag{73}$$

where is  $\eta$  is resistant constant. According to the above two formula we have,

$$\iint_{\Gamma} (\overrightarrow{E}_{1}(\omega) \times \overrightarrow{H}_{2}^{*}(\omega) + \overrightarrow{E}_{2}^{*}(\omega) \times \overrightarrow{H}_{1}(\omega)) \cdot \hat{n} d\Gamma = 0$$
 (74)

Hence we have,

$$\iiint\limits_{V} (\overrightarrow{E}_{2}^{*}(\omega) \cdot \overrightarrow{J}_{1}(\omega) + \overrightarrow{E}_{1}(\omega) \cdot \overrightarrow{J}_{2}^{*}(\omega)) dV = 0$$
 (75)

Or

$$-\iiint\limits_{V} (\overrightarrow{E}_{2}^{*}(\omega) \cdot \overrightarrow{J}_{1}(\omega) = \overrightarrow{E}_{1}(\omega) \cdot \overrightarrow{J}_{2}^{*}(\omega))dV$$
 (76)

The mutual energy theorem[12, 23, 22] is suitable to wireless wave situation, in which the most wireless wave signal can be described in Frequency domain. The time correlation reciprocity theorem[5] is the Fourier transform form of the mutual energy theorem. The time domain reciprocity theorem[21] it a special situation where  $\tau=0$  of the time correlation reciprocity theorem. Considering these three formula are all strongly related to mutual energy principle, they can be all referred as mutual energy theorems. In time domain, the above formula can be written as,

$$-\int_{-\infty}^{\infty} \iiint_{V} \overrightarrow{E}_{2}(t) \cdot \overrightarrow{J}_{1}(t+\tau) dV dt = \int_{-\infty}^{\infty} \iiint_{V} \overrightarrow{E}_{1}(t) \cdot \overrightarrow{J}_{2}(t+\tau) dV dt$$
 (77)

### 4.6 Why we say that Lorentz reciprocity theorem is wrong

30 years ago this author obtained the mutual energy theorem was from Lorentz reciprocity theorem by a time reverse transform. Assume the electromagnetic field are  $\zeta_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1, \overrightarrow{J}_1, \overrightarrow{K}_1, \epsilon_1, \mu_1]$  and  $\zeta_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2, \overrightarrow{J}_2, \overrightarrow{K}_2, \epsilon_2, \mu_2]$ , where  $\overrightarrow{K}_1$  and  $\overrightarrow{K}_2$  are magnetic current intensity. The time-reverse transform h is defined as,

$$h \zeta_2 = [\overrightarrow{E}_2^*, -\overrightarrow{H}_2^*, -\overrightarrow{J}_2^*, \overrightarrow{K}_2^*, \epsilon_2^*, \mu_2^*]$$
 (78)

Now because we have proved the mutual energy theorem from mutual energy principle, if we need the Lorentz reciprocity theorem we can also make the time-reverse transform to obtain the Lorentz reciprocity theorem as following,

$$\iiint\limits_{V} \overrightarrow{E}_{2}(\omega) \cdot \overrightarrow{J}_{1}(\omega) dV dt = \iiint\limits_{V} \overrightarrow{E}_{1}(\omega) \cdot \overrightarrow{J}_{2}(\omega) dV$$
 (79)

We have shown that the the mutual energy theorem is the real physical theorem, that Lorentz theorem is only a mathematical theorem. It is very difficult to show what is wrong with the Lorentz theorem in the empty space where the Lorenz theorem and the mutual energy theorem can both obtained correct directivity diagram of a reciving antenna. Hence we have to check these two theory in lossy media, in which we have shown that the mutual energy theorem can still extended to lossy media. The Lorentz theorem cannot make the extension. Hence the mutual energy theorem is the winner. Lorentz theorem is a wrong theorem, but it sometimes still can obtained correct results. We have claim that even the Lorentz reciprocity theorem can obtain correct directivity diagram of a reciving antenna, it cannot obtained correct current distribution of the receiving antenna, which only can be obtained from the mutual energy theorem.

We also know that the time reverse transform satisfy MEQS, that means if a electromagnetic field  $\zeta = [\overrightarrow{E}, \overrightarrow{H}, \overrightarrow{J}, \overrightarrow{K}, \epsilon, \mu]$  satisfies MEQS after the time reverse transform,  $h\zeta = [\overrightarrow{E}^*, -\overrightarrow{H}^*, -\overrightarrow{J}^*, \overrightarrow{K}^*, \epsilon^*, \mu^*]$  still satisfies MEQS. However now the axiom of the electromagnetic theory is change to the mutual energy principle, we cannot must prove whether or not the time reverse transform can satisfy the mutual energy principle instead of MEQS. Assume  $\zeta_1$  is retarded wave  $\zeta_2$  is advanced wave, after the time reverse transform h acting to  $\zeta_2$ , both wave  $\zeta_1$  and  $\zeta_2$  become the retarded waves. Two retarded wave con not satisfy the mutual energy principle, especially the two sources  $\overrightarrow{J}_1$  and  $\overrightarrow{J}_2$  separated as a large distance. Hence we can not prove the Lorentz reciprocity theorem. Since that Lorenz theorem can still be proved from MEQS, it still can be seen as mathematical theorem.

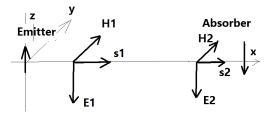


Figure 2: photon model, in this model the field  $\zeta_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1, \overrightarrow{J}_1], \zeta_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2, \overrightarrow{J}_2]$  all satisfy MEQS.

This also shows that the advantage to use the mutual energy theorem as a axiom vs MEQS. The mutual energy principle offers a real physical solution with energy or energy flow. MEQS offers a mathematical solution which is a probability wave or ability wave not physical wave. Lorentz theorem can only be obtained from MEQS, hence it is only a mathematical solution instead of a physical solution.

# 5 Photon equations and models

We will discuss two situations. First the advanced wave and the retarded waves both satisfy MEQS. Since MEQS is not the necessary condition of the mutual energy principle. It is still has the possibility that the solution of the mutual energy principle is not the solution of MEQS. Hence, to the second situation, we discuss that they do not satisfy MEQS, but still satisfy the mutual energy principle.

### 5.1 The solution 1 for photon

Figure 2 shows the photon model of this kind solution. The emitter and absorber can be think as small antenna inside a atom. They also has their currents  $\overrightarrow{J}_1$  and  $\overrightarrow{J}_2$ .

Assume we have put a metal place between the emitter and the absorber. We make a hole to allow the light can go through it from the emitter to the absorber. The mutual energy flow (will be defined in Eq.(81)) is exist only on the overlap of the two fields  $\zeta_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1]$  and  $\zeta_1 = [\overrightarrow{E}_2, \overrightarrow{H}_2]$ , see Figure 3 (it is possible there is still a little bit mutual energy flow outside the overlap region, but it become very very weak). This overlap region create a perfect wave guide for light wave. Inside this wave guide the normal TE and TM wave can be supported and they are perpendicular to each other and hence the polarization include linear and circle polarization of the waves can be supported.

The disadvantage of this photon model is that it can only send the wave with linear polarization. If we need the photon as circular polarized field, we have to make the current  $\overrightarrow{J}_1$  and  $\overrightarrow{J}_2$  all have two components for example along y

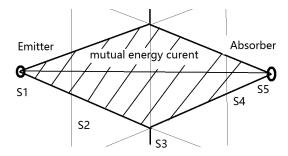


Figure 3: The mutual energy flow only exists at the overlap place of the two solutions of the MEQS. The field of the emitter is retarded wave. The wave of the absorber is advanced wave.

and z, or to make the currents rotating along x axis. This is perhaps possible, because the electron is at spin, their current is also possible to have spin. In this way the radiate wave becomes circular rotated.

We can take the volume V only includes the emitter or only includes only the absorber, this way we can prove that the mutual energy flow go through each surface  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$  and  $S_5$  are equal that is[17, 15]

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} (\overrightarrow{E}_2 \cdot \overrightarrow{J}_1)$$

$$= Q_1 = Q_2 = Q_3 = Q_4 = Q_5$$

$$\int_{t=-\infty}^{\infty} \iiint_{V_2} (\overrightarrow{E}_1 \cdot \overrightarrow{J}_2) dV$$
(80)

where  $Q_i$  is the mutual energy flow integral with time,

$$Q_{i} = \int_{t=-\infty}^{\infty} \iint_{S_{i}} \cdot (\overrightarrow{E}_{1} \times \overrightarrow{H}_{2} + \overrightarrow{E}_{2} \times \overrightarrow{H}_{1}) \cdot \hat{n} d\Gamma dt \qquad i = 1, 2, 3, 4, 5$$
 (81)

where  $\hat{n}$  is normal vector of the surface  $S_i$ , the direction the normal vector is from the emitter to the absorber. This formula clear tells us the photon's energy flow is just the mutual energy flow. The mutual energy flow integral with time is equal at the 5 different surfaces or any other surfaces. We know that the surface  $S_1$  and  $S_5$  are very near to the emitter or absorber. This surface becomes so small, hence the wave beam is concentrated to a very small point. It looks very like a particle. In the middle, the wave beam is very thick. We can put other kind plate for example the metal plate with two slits. In this case the wave will produce interference patterns. This can explain the duality character of the

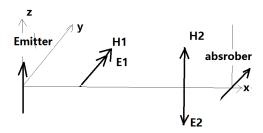


Figure 4: In this photon model, the fields  $\zeta_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1, \overrightarrow{J}_1], \zeta_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2, \overrightarrow{J}_2]$  do not satisfy the MEQS. However, they still satisfy the mutual energy principle.

photon. In the two slits situation the above formula Eq. (80) is still established. The above formula can be referred as the mutual energy flow theorem.

The left of the formula Eq.(80) can be seen as the energy sucked by the advanced wave  $\xi_2 = [\overrightarrow{E}_2, \overrightarrow{H}_2]$  from the emitter's current  $\overrightarrow{J}_1$ . The right of the formula Eq.(80) can be seen as the current of the absorber  $\overrightarrow{J}_2$  received the energy from the retarded wave  $\xi_1 = [\overrightarrow{E}_1, \overrightarrow{H}_1]$ . Integral of this energy with time is equal to each other and all equal to the integral of mutual energy flow in each surface  $S_i$ . The mutual energy flow is produced with retarded wave and advanced wave together. The two waves must synchronized. The retarded wave can be referred as emitting wave of the emitter. The advanced wave can be referred as receiving wave of the absorber.

For the above solution, we have use the sufficient condition of the mutual energy principle. It is not the necessary condition. The mutual energy principle perhaps has some other solution which does not satisfy MEQS, which will be discussed in next subsection.

#### 5.2 The solution 2 for photon

In this situation perhaps we can find a solution which is not the solution of MEQS, but it still satisfies the mutual energy principle.

If it is true this solution is also possible the solution for electromagnetic fields, that means we have found another photon model.

We can easily prove that the above Figure 4 satisfy mutual energy principle even it does not satisfy MEQS. We can see that this model is just a rotation from the Figure 2. We know that the photon model in Figure 2 satisfy MEQS, that guarantees it also satisfies the mutual energy principle. Now we have rotated  $\overrightarrow{J}_2$ ,  $\overrightarrow{H}_2$  and  $\overrightarrow{E}_1$  along the x axis 90 degree. Hence  $\overrightarrow{E}_1 \times \overrightarrow{H}_2$  and  $\overrightarrow{E}_1 \cdot \overrightarrow{J}_2$  don't change. Hence  $\overrightarrow{E}_2 \times \overrightarrow{H}_1$  and  $\overrightarrow{E}_2 \cdot \overrightarrow{J}_1$  is also not change, because  $\overrightarrow{E}_2$ ,  $\overrightarrow{H}_2$   $\overrightarrow{J}_1$  doesn't rotate. This guaranties the values of all items in the mutual energy principle does not change, if it is satisfied before the rotation, after the rotation the mutual energy flow theorem Eq.(80) should still be satisfied.

Even this model do not satisfy the MEQS, it has 3 advantages.

(1) Since field  $\overrightarrow{E}_1||\overrightarrow{H}_1|$  and  $\overrightarrow{E}_2||\overrightarrow{H}_2|$  the self energy flow vanish automatically.

We do not to tell the reader the self mutual energy flow is 0, even when we calculated it, it is nonzero.

$$\oint_{\Gamma} \cdot (\overrightarrow{E}_1 \times \overrightarrow{H}_1) \cdot d\Gamma = 0, \qquad \oint_{\Gamma} \cdot (\overrightarrow{E}_2 \times \overrightarrow{H}_2) \cdot d\Gamma = 0$$
(82)

(2) we can see that,  $\overrightarrow{E}_1 \perp \overrightarrow{J}_1$  and  $\overrightarrow{E}_2 \perp \overrightarrow{J}_2$ , we have clear that the self energy action vanish automatically.

$$\iiint\limits_{V} (\overrightarrow{E}_1 \cdot \overrightarrow{J}_1) dV = 0 \tag{83}$$

$$\iiint\limits_{V} (\overrightarrow{E}_{2} \cdot \overrightarrow{J}_{2}) dV = 0 \tag{84}$$

(3) this model is easy to support circular polarization. Because not  $\overrightarrow{E}_1$  and  $\overrightarrow{E}_2$  also perpendicular. We know it is difficult to explain that why electron is so small it still can send circular polarization waves. In our antenna experience we need very complicated antenna to send circular polarized waves. If two electric fields  $\overrightarrow{E}_1$  and  $\overrightarrow{E}_2$  are perpendicular and they have 90 degree phase difference, we have obtained a circular polarization.

Even this solution does not satisfy MEQS, it has 0 self energy flow and 0 self action and can offer a simple current to support the circular polarization. It looks as a miracle.

Even this is not a physic solution of photon, using the mutual energy principle as axiom is still much better than to take the MEQS as the axiom, considering the economy principle. The mutual energy principle is only one equation, but the MEQS for two charges perhaps need  $8=2\times 4$  equations. If MEQS is axiom, we have to add other conditions which ask the solutions exist simultaneously for two charges. And also need to explain why one need to be the retarded wave and another need to be the advanced wave. And have to explain why the self energy flow should vanish (or like quantum physics it is a probability wave) even it is calculated as nonzero. Started from the mutual energy principle all this kind additional conditions are avoid. Only when a retarded wave and an advanced wave synchronized together, it can be a solution of the mutual energy principle for two charges separating with a long distance.

### 5.3 A guess about the return of the self energy flow

In above our electron magnetic theory only the mutual energy flow is involved. What about the self energy current? It does not carry any energy. It is a probability wave like in quantum physics. This is often difficult to be understand. The engineer would like to think the wave is a real wave instead a probability wave. We found that a wave it exist but do not carry energy perhaps can explained as the wave is actually send out with energy but later it is returned to its source the pure effect is that the energy flow vanishes.

Most electromagnetic engineering believes that the electromagnetic wave especially the retarded wave is a real wave and does not like the wave in quantum physics which is probability wave. In this article we have shown that the electromagnetic wave, the retarded wave and the advanced wave do not carry any energy if it is alone. The energy is carried through the mutual energy flow which needs the retarded wave and advanced wave together and have been synchronized. Hence in principle, if there is only one wave for example the retarded wave, it can only offer a ability to transfer the energy. The real transfer energy need an advanced wave to react to it. Hence the retarded wave still not a real wave with energy on itself, it can be interpreted as ability wave or probability wave. If you do not satisfy this result, perhaps you can think the retarded wave is still a real wave carries the energy and transferred the energy in the space, but if there is no advanced wave to receive it, this energy is returned to its source, i.e the emitter. Hence the retarded wave returns to emitter and the advanced wave returns to the absorber if they are not synchronized. This is similar to the transactional process in the bank, if some thing wrong, the money can not be transferred from A bank to B bank, then the money must return to A. Energy is same as money if it can not be transferred from A to B, the energy conserved law do not allow it disappear in the space. Hence this energy no way to go and must return to its source. This way it can guarantee for the whole system the energy is still conserved.

We can assume the emitters and absorbers all can randomly send the retarded waves and advanced waves. Now this wave is real physical wave. If in the time of the retarded wave send out just has a advanced wave match it. The energy is transferred through the mutual energy flow from the emitter to the absorber. Otherwise the energy in the retarded wave or in the advanced wave just returned. The wave return is a time reverse process, which cannot satisfy by Maxwell equations, but can perhaps satisfy time-reversed Maxwell equations, which are,

$$\nabla \times \overrightarrow{E} = \partial \overrightarrow{B} \tag{85}$$

$$\nabla \times \overrightarrow{H} = -\overrightarrow{J} - \partial \overrightarrow{D} \tag{86}$$

The energy return process can be described with equations, it can be seen as a collapse process, the wave collapsed to it's source, ether emitter or the absorber. This collapsed process is different to the wave function collapse in quantum physics, in which the retarded wave collapse to the absorber, none offers a equation to describe that kind of collapse process. The wave is returned has not been proved by the experiment, just like the wave function collapse has not been proved in experiment, but it still can applied to interpret of the quantum physics.

Hence wave is returned with the above time reverse wave is still better than the wave function collapse process which cannot be described with any equations.

It should be noticed that if the retarded wave is a real wave with energy in space, it must be returned if there is no the transactional advanced wave. Actually even there is a photon sends out, that is only the mutual energy flow, the self energy is still in the space and it also need be returned. The self energy flow help the mutual energy transfer the energy in the space. After the transfer energy process, either the energy is transferred or not, the self energy flow of the emitter has to be returned.

In the above discussion about the mutual energy principle, we do not care the self energy flow which dos not carry energy in physics. But we can also assume it carry energy but it is returned hence no energy is transferred through self energy flow.

The self energy return can be divided two possibility which are (1) a immediately return and (2) return a little bit later.

#### 5.3.1 Immediately returns

In case the immediately return, the return wave is synchronized with the self energy wave. The self energy for the retarded wave is  $\xi_1' = [E_1', H_1']$  and the returned wave is  $\xi_1'' = [E_1', H_1'']$ . And the total wave satisfy,

$$\xi_1 = \xi_1' + \xi_1''$$

 $\xi_1'$  satisfy,

$$\begin{cases} \nabla \times \overrightarrow{E}'_{1}(t) = -\partial \overrightarrow{B}'_{1}(t) \\ \nabla \times \overrightarrow{H}'_{1}(t) = +\overrightarrow{J}_{1}(t) + \partial \overrightarrow{D}'_{1}(t) \end{cases}$$
(87)

 $\xi$ " 1 satisfy,

$$\begin{cases} \nabla \times \overrightarrow{E}_{1}(t) = \partial \overrightarrow{B}_{1}(t) \\ \nabla \times \overrightarrow{H}_{1}(t) = -\overrightarrow{J}_{1}(t) - \partial \overrightarrow{D}_{1}(t) \end{cases}$$
(88)

It can be proven that the self energy flow corresponding to Poynting vector of the two waves have the following relation,

$$\iint_{\Gamma} \cdot (\overrightarrow{E}_{1}' \times \overrightarrow{H}_{1}') \cdot \hat{n} d\Gamma = - \iint_{\Gamma} \cdot (\overrightarrow{E}_{1} \times \overrightarrow{H}_{1}') \cdot \hat{n} d\Gamma$$
 (89)

Hence that,

$$\iint\limits_{\Gamma}\cdot(\overrightarrow{E}_1'+\overrightarrow{E}_1"_1)\times(\overrightarrow{H}_1'+\overrightarrow{H}_1"_1)\cdot\hat{n}d\Gamma=0$$

or

$$\iint\limits_{\Gamma} \cdot \overrightarrow{E}_1 \times \overrightarrow{H}_1 \cdot \hat{n} d\Gamma = 0$$

This means that the returned filed  $\xi$ "<sub>1</sub> is also joined to the retarded wave  $\xi'_1$ . The superimposition of  $\xi$ "<sub>1</sub> and  $\xi'_1$  has obtain the self energy flow as 0. It should

be notice that in the above formula the self energy flow vanishes doesn't mean the field also vanish. It only means that there is,

$$\overrightarrow{E}_1 || \overrightarrow{H}_1$$

and we have,

$$\xi_1 \neq 0$$

This situation is a alternative explanation corresponding to the situation in subsection 5.2.

### 5.3.2 Doesn't immediately return

In this situation, the returned field  $\xi'_1$  doesn't join the retarded field  $\xi'_1$ 

$$\xi_1 = \xi_1'$$

This is a alternative explanation corresponding to the situation in subsection 5.1. After the photon has sent from emitter to the absorber, the field  $\xi$ "<sub>1</sub> bring the self energy back to the emitter.

In the above we only discussed the return wave for the retarded wave from emitter, actually there is also the return wave corresponding to the advanced wave from the absorber. The discussion should be similar, hence will not put here.

### 5.4 Is light satisfy MEQN or MEQS?

Many people ask in the internet the question whether or not the field of light satisfy MEQN? We need to distinguish the problem in wireless wave and light wave.

Light wave deals the problem of photon, the photon is a small system only with two electrons one is the emitter, the other is the absorber. From above discussion we know that photon satisfies the mutual energy principle. We also know that the MEQS is the sufficient condition of the mutual energy principle, hence, it is possible that the retarded wave and advanced wave of the photon satisfies the MEQS. But it is also possible that the photon satisfies the mutual energy principle but does not satisfy the MEQS.

In light wave, assume there are infinite photos, do they satisfy the MEQN? We speak no! The reason is that the photon is energy package, here, the energy is linear. For example, one photon the energy is 1, 2 photons have the energy 2. If energy is linear, its field can not be linear (because if the field is linear the energy should be quadratic). Hence in light wave situation, many photons also do not satisfy the MEQN. This result is different with wireless wave. In case of wireless wave, the wireless waves are linear (assume here that the number N is very large, and hence we can omit the effect of pseudo fields) and hence still satisfy MEQN.

According the discussion of lost subsection that the field of emitter and absorber of one photon is possible does not satisfy MEQS. There is the possibility it does not satisfy MEQS but it still satisfies the mutual energy principle.

However, to make things simple we still assume for each photon, the emitter and the absorber still satisfy the MEQS. For a system with big number of photon N, there is 2N emitters and absorbers. If the field of the light can be superimposed, then it is clear the 2N system also satisfy MEQN.

If

$$\begin{cases} \nabla \times \overrightarrow{E}_i = -\partial \overrightarrow{B}_i \\ \nabla \times \overrightarrow{H}_i = +\overrightarrow{J}_i + \partial \overrightarrow{D}_i \end{cases}$$
(90)

is satisfied it is clear that

$$\begin{cases}
\nabla \times \sum_{i=1}^{2N} \overrightarrow{E}_i = -\partial \sum_{i=1}^{2N} \overrightarrow{B}_i \\
\nabla \times \sum_{i=1}^{2N} \overrightarrow{H}_i = + \sum_{i=1}^{2N} \overrightarrow{J}_i + \partial \sum_{i=1}^{2N} \overrightarrow{D}_i
\end{cases}$$
(91)

However, the field of photon is not linear and cannot superimposed. There are two reasons we speak the field of light is not linear. The first reason is we have mentioned before in last few sections because of the pseudo self field. The second reason is photon as energy package and hence its energy is linear. N photons energy has N times of the energy of one photon. Energy is linear tell us that the field is not linear. if the field is linear the energy must quadratic.

Hence in general that the field of light does not satisfy MEQN.

#### 5.5 Is the wireless wave are composed as photons?

Some people ask in the internet that is the wireless wave also composed as many photons or there is a frequency limit beyond that frequency all energy become packages i.e. photons. We thought this is a very good question and would like make it clear here.

In the antenna system one transmitting antenna can send the energy to many receiving antennas. It is not send a package of the energy to only one antenna in a time and randomly send the the energy to another antenna. It sends the energy to all space and any antenna can receiving this energy.

This is a system with charges more than two. All these receiving antennas can be synchronized with the transmitting antenna with frequency and orientation. This kind of energy is not a energy package. Hence wireless wave is not composed as photons.

# 5.6 Are all current changes happened on the emitter or absorber will send photons out?

Assume in the emitter it has a current change for example a electron has from a higher energy level jump to a lower level, does this current change send a photon? We think the answer is no. This current change will send the retarded potential but if the retarded potential can not meet a synchronized advanced wave of any absorber, there will be no energy transfer. The photon does not send out. The electron charge has energy perhaps it will return to its original higher level. And next time it will jump to low level again try to find a matched advance wave.

In the case there is only an emitter and an absorber, the current change of the emitter produced a retarded wave, the absorber produced an advanced wave. If the retarded wave of the emitter has synchronized to the advanced wave of the absorber, there is a photon which is sent from the emitter to the absorber.

MEQS cannot tell us that should an advanced wave or a retarded wave be associated with a current change. Hence Wheeler and Feynman assume there always half advanced wave and half retarded wave associated to the current change.

From my experience I would thought only one kind wave can associate to the emitter and absorber in a time. That means the electron charge can only be ether an emitter or an absorber but both. This is from the experience of antenna system, if the absorber also send half retarded wave, then there should not have black body. The absorber become very bright. We never seen this phenomenon.

We do not know the emitter first sends the retarded wave or the absorber first send the advanced wave i.e, we do not know which is the cause, the retarded wave or the advanced wave. If the electron charge jump from the high energy to lower energy, in the same lime there is an advanced wave reaching at the place of the emitter, if the emitter run against that advanced wave, it will send a retarded wave with energy and that is a photon. When the retarded wave reached to the absorber, the electron charge in the absorber runs along the retarded wave, it will send the advanced wave out and receive the photon. We can assume that the emitter can self excitated to send retarded wave and the absorber also can self excitated to send advanced wave. These waves are all randomly send out. If by coincidence, these waves synchronized, a event of the photon happens. If there is no this kind of synchronization, there is no any energy transfer. The both retarded wave and the advanced wave can be seen as probability waves, they do not contain any energy.

Hence, the retarded wave is not real wave, it is only offers the ability or possibility to support a absorber to receive it. If this absorber appears by producing a current synchronized with the emitter, the energy will send from the emitter to the absorber. If there is no absorber to receive this energy, the energy of the emitter can not be sent out. There is no any energy is lost to the space and can move in the space independent to both their emitter and the absorber. For the advanced wave, this is also the same.

### 5.7 The mutual energy flow interpretation of the photon

We know that the mutual energy principle endorses the direct interact theory[18, 7, 20], hence the retarded wave is not a real thing. If there is no absorber or

the advance wave, the retarded wave cannot send the energy out. This tell us the retarded wave is only offers the ability to send out the energy but doesn't really offers the energy. This is real reason of the probability interpretation of Copenhagen. However, now we know that the probability interpretation of Copenhagen is at least correct at about the retarded wave. The retarded wave is only offers the ability to do some thing. It does not send real energy. The real energy is sent only in case there is a advanced wave to match it, which is the transaction process described by John Cramer.

In this way we actually endorse both the Copenhagen interpenetration of the quantum physics and also the transactional interpretation of John Cramer.

When we speak that the retarded wave is not real, actually we also mean the advanced wave is also not real, no energy is send by advanced wave, only if the retarded wave and the advanced wave meet together, the energy flow is sent from emitter to the absorber which is the photo. Hence photon is nothing else, it is only the discretized mutual energy flow. This can be referred as the interpretation of the mutual energy flow for quantum physics. In this interpretation the transactional interpretation of John Cramer and the interpretation of Copenhagen are combined together.

### 5.8 Is half photon possible?

The self energy is returned when the transactional process does not happen is only a good explanation for the wave is a ability wave or probability wave. Important thing to us, the energy is only transferred by the mutual energy flow. Photon is nothing else just the mutual energy flow. Wireless wave is not composed by many photons. Wireless wave obeys MEQN, photon obey MEQS. If wireless wave is composed with many many photons, its energy become linear, and its field can not be linear any more, that is not true.

The amount of photon energy  $\omega\hbar$  is decided by emission and absorption process. It is only because the antenna inside the atom can only send that amount of energy. The macrocosm wireless antenna sends energy is not a energy package. It is still a continuous wave. Only continuous wave can obey the MEQN.

Hence there should be a frequency limit  $f_c$ , if the frequency f of the radiation larger than  $f_c$  the energy is transferred by photon. If the frequency f of the radiation less than  $f_c$  the energy is transferred by continuous wave. If the the frequency of the radiation is close to  $f_c$ , it is possible that there are half photon, or  $\frac{1}{3}$  photon and so on.

### 6 Conclusion

A bug is found in Poynting theorem, which is also inside the MEQN and MEQS. When the bug is removed from the Poynting theorem, it become the mutual energy formula. The whole electromagnetic theory is reconstructed through the mutual energy formula which is treated as a axiom of the electromagnetic field

theory. It is referred as the mutual energy principle. The Poynting theorem is derived by add pseudo self field items. We find that the Poynting theorem is still correct as mathematical equations. However, the superimposition principle of the electric fields, the energy flow of based on Poynting vector, the whole energy of the system shown in the Poynting vector all depended to the concept of the pseudo field, and hence is only approximately established when the number of charge in a electromagnetic system is very large, which is the wireless wave situation. In the light wave situation, the number of charge in a electromagnetic system is very small normally is only 2 which is the emitter and the absorber, in this situation, the electromagnetic field calculation can only be done with the mutual energy principle. In the light wave situation if considering the quantum entanglement the number of charge in a electromagnetic system can larger than 2. The solutions of the mutual energy principle is required that a retarded wave sent from the emitter and a advanced wave sent from the absorber must exist simultaneously. That means only retarded wave and only advance wave is not a solution of mutual energy principle, hence they are not a physic solutions. However this solutions are the solutions of MEQS, they can be seen as a mathematics wave, a probability wave or ability wave but not the real physic wave carried with some energy.

MEQN can be derived from Poynting theorem as sufficient conditions of Poynting theorem. The whole electromagnetic theory still can be further derived from MEQN as a approximate theory when the number charges N in a system is very large which is the wireless wave situation.

The mutual energy principle is applied to photon to derived two kinds of photon models. For both models, the emitter sends retarded wave, the absorber sends the advanced wave, the two wave must exist or synchronized. For the first kind model, the wave of the emitter and the absorber satisfy MEQS. For the second kind model, it does not satisfy MEQS, instead it satisfies the mutual energy principle alone. Both kind models support circular polarization waves. The first kind model need a little bit complicate currents inside the emitter or absorber to support the circular polarization. For the second model even a simple charge moves along a line, it is still possible to produce circular polarized wave. The mutual energy flow is defined and calculated. The time integral of the mutual energy flow are equal on any surface between the emitter and the absorber, which is the energy flow of the photon or photon itself. Photon is nothing else, it just the mutual energy flow. Since the introduction of the mutual energy principle, the electromagnetic field theory can be extended from wireless wave to light wave. Originally it need to be discussed in quantum physics or QED (quantum electrodynamics) or QFT (quantum field theory).

Electromagnetic field is a confused concept, because if we measure it we have to put a test charge or a absorber in the place, however after we removed the test charge or absorber, we cannot prove the field we have measured is still there. Field is the ability if there is a test charge or absorber. In this situation the field can offer a force on the test charge or absorber. When the test charge or absorber is removed the field is not defined any more. Actually the field is also partly produced by this test charge or absorber. Hence if we still think

the field is some thing real in physics and assume after the removal of the test charge or absorber it is still there, we actually over estimate it. Even the field cannot be properly defined, the power or energy of the system still can be well defined. In this article we have shown that the energy correct of the system should be calculated with mutual energy principle instead of Poynting theorem. The energy flow should be calculated with mutual energy flow instead of the energy flow related to Poynting vector.

According to the mutual energy principle the electromagnetic field, the retarded wave alone and the advanced wave alone does not transfer energy. The energy is transferred by the mutual energy flow which exist only when both waves exist simultaneously and be synchronized. That means the wave for example the retarded wave need a current to drive, after the drive, the retarded wave propagates in the whole space, this can be called as the action. If there is no any advanced wave appear to react it, there is no any energy emitting from the emitter. Hence the retarded wave can be seen as a ability or probability wave. This agrees with the Copenhagen interpretation of quantum physics. It is same for the absorber it sends the advanced wave is also the ability or probability wave. If this advanced wave has not meet the retarded wave. Since we also applied both retarded wave and advanced wave, this photon model also support the absorber theory and the transactional interpretation of John Cramer.

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