A condition on a non-collatz number at the boundary of a successive collatz numbers set

Abdelghaffar Slimane
email: abdelghaffar.slimane@polymtl.ca
Ecole polytechnique de Montreal
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Abstract

We give a condition that an odd number in the neighborhood of a successive collatz numbers set must verify to be a non-collatz number, and we use the result for odd numbers of the form $6k - 1$ at the boundary of a successive collatz numbers set.

1 Introduction

Since 1937, mathematicians tried to prove the conjecture of Collatz also known as the $3x+1$ problem and Syracuse conjecture. In this paper, we prove a theorem that give another criteria on odd numbers to verify the conjecture under some conditions.

2 Definitions

We will use the modified form of Collatz’s sequence defined by : $U : \mathbb{N}^* \rightarrow \mathbb{N}^*$

$$U(n) = \begin{cases} 
\frac{n}{2} & \text{if } n \text{ is even} \\
\frac{3n+1}{2} & \text{if } n \text{ is odd}
\end{cases}$$

The conjecture assumes that for every (non-zero) integer $n$ there exists $d$ such that $U_d(n) = 1$. In this paper, we mean by:
An odd (respectively even) collatz number: an odd (respectively even) number of \( N^* \) verifying the Collatz conjecture.

A non-collatz number (assumption): a number of \( N^* \) that does not verify the Collatz conjecture.

3 Lemma

Let \( \mathcal{A}_n = \{1, 2, 3, ..., E\left(\frac{2n-1}{3}\right)\} \) be the set of all successive collatz numbers \( \leq E\left(\frac{2n-1}{3}\right) \) where \( n \) is an odd number \( \geq 3 \).

\[ \mathcal{A}_n = \{t \in \mathbb{N}^* : t \text{ a collatz number } \leq E\left(\frac{2n-1}{3}\right)\}. \]

Let \( \mathcal{B}_n \) be the set of all successive collatz numbers inferior or equal to \( E\left(\frac{n+1}{6}\right) \).

\[ \mathcal{B}_n = \{t \in \mathbb{N}^* : t \text{ a collatz number } \leq E\left(\frac{n+1}{6}\right)\}. \]

Lemma 1 Let \( n = 2p + 1 \) be an odd number \( (p \in \mathbb{N}^*) \) with \( \mathcal{A}_n \neq \emptyset \)

\( n \) is an odd non-collatz number \( \Rightarrow \forall \ t \text{ odd } \in \mathcal{A}_n, \frac{3t+1}{2} \neq n \)

Proof 1 Let \( t \in \mathcal{A}_n \), where \( t \) is odd \( \Rightarrow t \) is an odd collatz number \( \Rightarrow U_1(t) = \frac{3t+1}{2} \) is a collatz number \( \Rightarrow \text{if } n = \frac{3t+1}{2} \text{ then } n = U_1(t) \) is also an odd collatz number.

Then we can now state the theorem:

4 Theorem

Theorem 1 Let \( n = 2p + 1 \) be an odd number \( (p \in \mathbb{N}^*) \) with \( \mathcal{B}_n \neq \emptyset \)

If \( n \) is an odd non-collatz number then \( n \neq 6k - 1 \) (where \( k = 1, 2, 3, \cdots \in \mathcal{B}_n \))

Proof 2 We consider \( n = 2p + 1 \) (where \( p = 1, 2, 3, \cdots \in \mathbb{N}^* \)) as an odd non-collatz number.

From Lemma 1, we can write that,

\[ \forall t \in \mathcal{A}_n : \frac{3n+1}{2} \neq n \rightarrow \frac{3t+1}{2} \neq 2p + 1 \rightarrow t \neq \frac{4p+1}{3} \]

The resolution of the equation \( t = \frac{4p+1}{3} \) giving odd numbers \( t \), shows that it is satisfied by values of \( p = 2, 5, 8, 11, 14, \cdots \)

Therefore \( p \) is of the form \( p = 2 + 3\alpha \) (where \( \alpha = 0, 1, 2, 3, \cdots \))

\( t \) will have values like \( t = 3, 7, 11, 15, 19 \cdots \), and \( n \) will have the form:
\[ n = 2p + 1 = 2(2 + 3\alpha) + 1 = 6\alpha + 5 = 6k - 1 \quad (where \ k = 1, 2, 3, \cdots \in \mathbb{B}_n) \]

5 Conclusion

We can generalize this result on a given set of successive collatz numbers already verified \( C_n = \{1, 2, 3, \ldots, n = 2p\} \). Therefore if the next odd element \( n + 1 \) is of the form \( 6k - 1 \) then it is a collatz number.