On the Collatz Conjecture

By Clive Jones

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Abstract: "The Escape-Condition is a power of 2"

Some Observations:

Odd N produce a greater result

Even N produce a lesser result

The objective is to reach 1

You must necessarily visit a power of 2, typically 16

1	4	2	1									
2	1											
3	10	5	16	8	4	2	1					
4	2	1										
5	16	8	4	2	1							
6	3	...										
7	22	11	34	17	52	26	13	40	20	10	5	...
8	4	2	1									
9	28	14	7	...								

Fig 1: Low-Order N Collatz Sequence

The cycle of creating greater or lesser numbers is broken by landing on a power of 2 [at least for positive integers]

This is called an 'Escape-Condition' & is found in the computation of fractals, where an iteration count reaches a bail-out value causing the sequence to terminate

Q: Will 3N+1 always hit a power of 2?

A: Yes, using 'Proof by Inspection' [see Elsewhere]

Limit	Number	Step-Count	Peak
100	97	118	9,232
1,000	871	178	190,996
10,000	6171	261	975,400
100,000	77,031	350	21,933,016
1,000,000	837,799	524	2,974,984,576
10,000,000	8,400,511	685	159,424,614,880

Fig 2: Collatz Maximal Step-Count Data

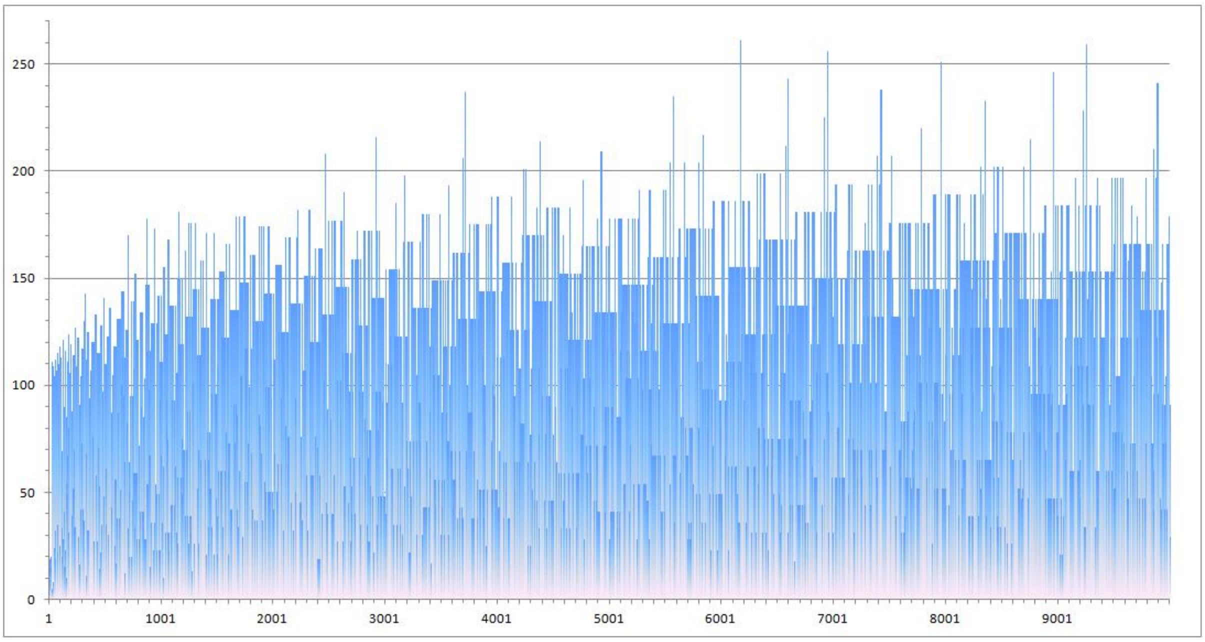

Fig 3: Collatz Step-Count plot for N up to 10,000

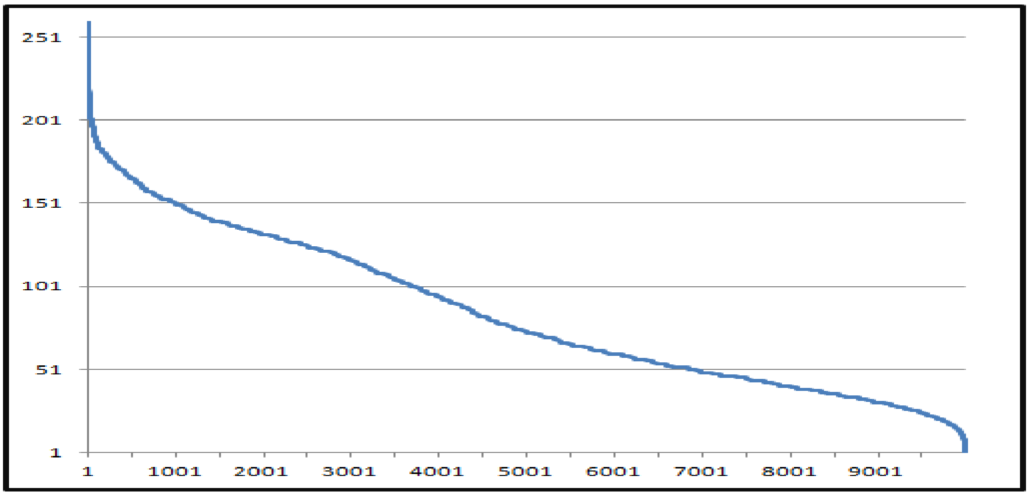

Fig 4: Sorted Step-Count plot for N up to 10,000

Which numbers hit a power of 2 the highest?

N	Pow2
21	64
42	64
75	256
84	64
85	256
113	256
128	64
150	256
151	1024
168	64
170	256
201	1024
226	256
227	1024
256	128
300	256
301	256
302	1024
336	64
340	256
341	1024
401	256
402	1024
403	1024
423	1024
452	256
453	256
454	1024
475	256
512	256

Fig 5: Power of 2 Escape-Values up to N=512

Up to 200 there are 51 numbers that hit 9232, they are:

27 31 41 47 54+55 62+63 71 73 82+83 91 94+95 97 103 105 107+108+109+110+111 121 124+125+126 129 137 142+143 145+146+147 155 159 161 164+165+166+167 171 175 188+189+190 193+194+195 199

That’s 25.5%

Up to 1000, the only numbers that do not escape on 16 are 1 2 4 & 8

Up to 1000, the longer sequences have an 83rd term of 700 in 28.9% of cases

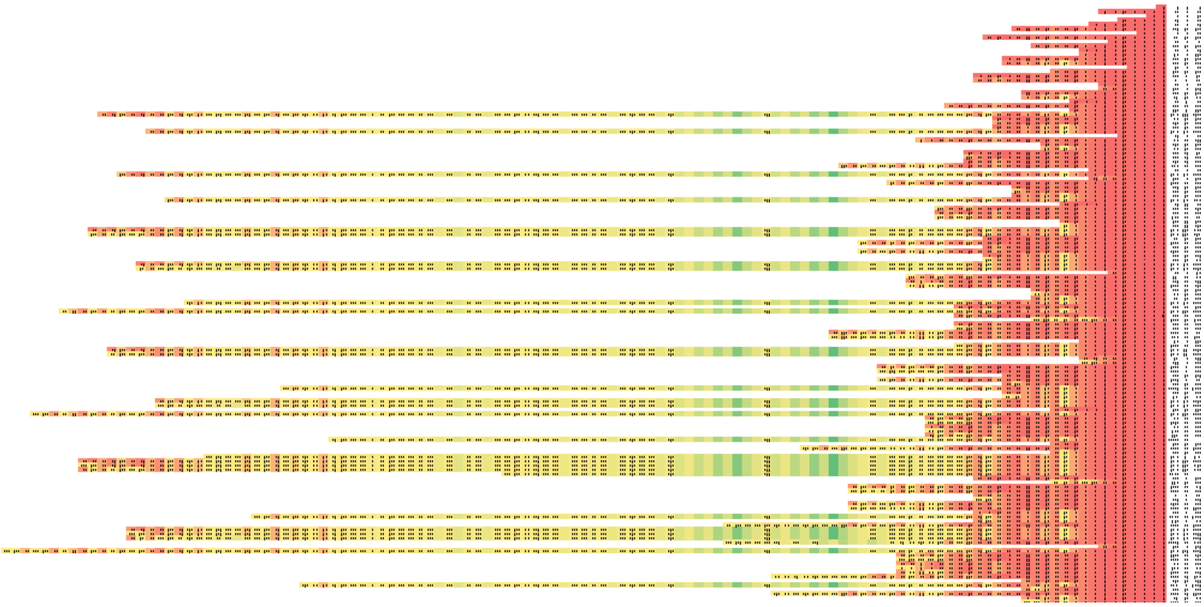

Fig 6: Right-Jusified Colourized Sequence Data for Low-Order N

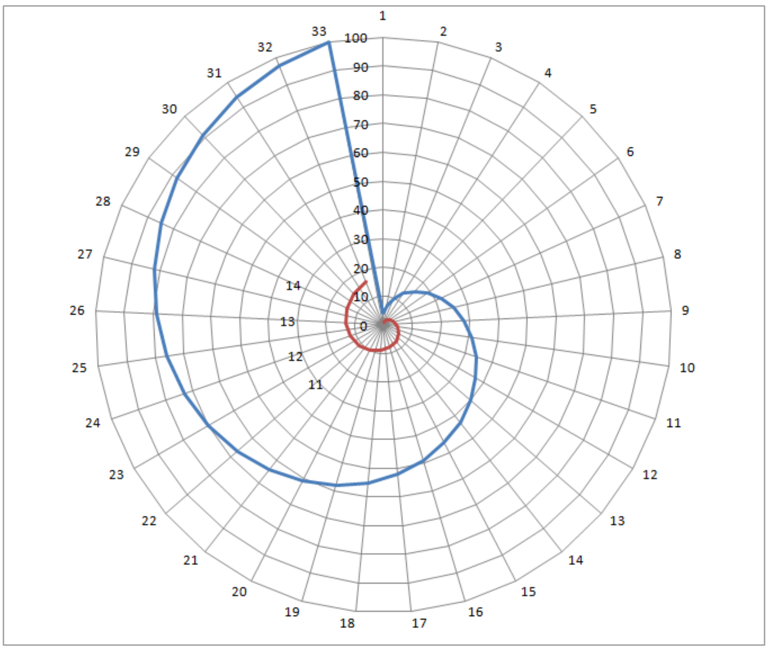

Fig 7: Polar plot of 3N+1 & N/2 for N<34

Collatz could be imagined as Snakes & Ladders traversal of upward spiral-staircase then.

Q: Which Even N when halved give Odd results?

A: 10 gives 5 30 gives 15 50 gives 25 70 gives 35

Looks like Odd multiples of 10 precede switch from Descent to Ascent

Ascent terminated by arrival at Even result

Q: Can you ascend to stair 32?

A: No, since no integer can multiply by 3 making 31

Whilst you can Start on any stair, you cannot Ascend to the following stairs in Red, only those in Green

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60

Fig 8: Collatz Ascension Table for Low-Order N

Ascent is more rapid at 3N+1 & guarantees Even result, as 3*Odd gives Odd then +1 gives Even result

Descent is slower as your altitude is only halved

Rate of Ascent > Rate of Descent

Odds of Ascending to a given stair: 1 in 3 due to Fig 8

Odds of Descending to a given stair: 1 in 2 as half of integers are positive

Multiple consecutive drops possible on power of 2 or on Even multiples of 10

92% of Odd N up to 200 escape through 40

Some exceptions:

3 5 thru 16

21 thru 64

75 85 113 151 thru 256

Average: 52 steps

79% of Even N up to 200 escape thru 40

Others thru 16 like 10 12 16 20 24 32 40 48 64 96 128 192

42 thru 64

150 170 thru 256

Average: 32.4 steps

85.5% of N up to 200 escape thru 40

91.9% of N up to 1000 escape thru 40

All N appear to resolve to 1 making conjecture true