

On the recurrence $((((P^2-d)^2-d)^2-d)\dots)$ on Poulet numbers P having a prime factor d

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Abstract. In this paper I note two sequences of Poulet numbers: the terms of the first sequence are the Poulet numbers which can be written as $P^2 - d$; the terms of the second sequence are the Poulet numbers which can be written as $(P^2 - d)^2 - d$, where P is another Poulet number and d one of the prime factors of P . I also conjecture that the both sequences are infinite and I observe that the recurrent relation $((((P^2 - d)^2 - d)^2 - d)\dots)$ conducts sometimes to more than one Poulet number (for instance, starting with $P = 4369$ and $d = 257$, the first, the second and the third numbers obtained are 8481, 16705 and 33153, all three Poulet numbers).

Sequence I:

The terms of this sequence are the Poulet numbers which can be written as $P^2 - d$, where P is another Poulet number and d one of the prime factors of P . I conjecture that this sequence is infinite.

The terms of the sequence I:

: 1105 = 561*2 - 17;
: 2701 = 1387*2 - 73;
: 7957 = 4033*2 - 109;
: 8481 = 4369*2 - 257;
: 15841 = 7957*2 - 73;
: 16705 = 8481*2 - 257;
: 31609 = 15841*2 - 73;
: 33153 = 16705*2 - 257;
: 46657 = 23377*2 - 97;
: 62745 = 31417*2 - 89;
: 129889 = 65281*2 - 673;
: 181901 = 91001*2 - 101;
: 323713 = 162193*2 - 673;
(...)

Sequence II:

The terms of this sequence are the Poulet numbers which can be written as $(P^2 - d)^2 - d$, where P is another Poulet number and d one of the prime factors of P . I conjecture that this sequence is infinite.

The terms of the sequence II:

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:   4369 = (1105*2 - 17)*2 - 17;
:   10585 = (2701*2 - 73)*2 - 73;
:   16705 = (4369*2 - 257)*2 - 257;
:   31609 = (7957*2 - 73)*2 - 73;
:   33153 = (8481*2 - 257)*2 - 257;
:   60787 = (15709*2 - 683)*2 - 683;
:   126217 = (31609*2 - 73)*2 - 73;
:   164737 = (41665*2 - 641)*2 - 641;
:   196093 = (49141*2 - 157)*2 - 157;
:   241001 = (60701*2 - 601)*2 - 601;
:   256999 = (65077*2 - 1103)*2 - 1103;
:   271951 = (68101*2 - 151)*2 - 151;
:   318361 = (80581*2 - 1321)*2 - 1321;
:   452051 = (113201*2 - 251)*2 - 251;
:   481573 = (121465*2 - 1429)*2 - 1429;
:   486737 = (123251*2 - 2089)*2 - 2089;
:   745889 = (188057*2 - 2113)*2 - 2113;
:   (...)

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Observation:

The recurrent relation $((((P^2 - d)^2 - d)^2 - d) \dots)$ conducts sometimes to more than one Poulet number.

Examples:

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: for P = 4369 and d = 257, we have:

:   4369*2 - 257 = 8481, a Poulet number;
:   (4369*2 - 257)*2 - 257 = 16705, a Poulet number;
:   ((4369*2 - 257)*2 - 257)*2 - 257 = 33153, a Poulet
:   number;

: for P = 7957 and d = 73, we have:

:   7957*2 - 73 = 15841, a Poulet number;
:   (7957*2 - 73)*2 - 73 = 31609, a Poulet number;
:   (((7957*2 - 73)*2 - 73))*2 - 73)*2 - 73 = 126217, a
:   Poulet number.

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