The Relativistic Field of a Rotating Body

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Abstract

Based on the paths of signals emanating from a rotating point body, we find the equations and properties of the field that they form. Depending on the type of observer the field differs. The field is not central but varies in orientation and magnitude with both distance and angular velocity of rotation. The magnitude of the field in some cases forms a barrier away from the origin, which may be very strong depending on the angular velocity of rotation. The results apply both to microcosmos (sub-atomic level) and macrocosmos (cosmic level).

1. Introduction

This paper is a continuation of the paper [1] that studied the relativistic rotation of frames and the signals emanating from a rotating point body located at the origin. A field can be viewed as signals that do not cross each other. These signals are received by a body that is subject to the field and induce it to act in a certain way. We will assume that these signals behave like light signals and travel the same way. This will allow us to use the results found for the paths of the signals emanating from a rotating body [1] to deduce the form and behavior of a field, we call **G,** due to the mass of the body and its rotation. We examine two types of rotation: One is called rotation without slippage, where the angular velocity of the space around the rotating body is constant regardless of distance from the body and the other is rotation with slippage, when rotation of space has an exponentially decreasing angular velocity as the distance from the rotating body increases. In each case we will distinguish between two types of non rotating observers: One close to the body and one far away. The observers are assumed mass-less and not affecting or affected by the signals emanating from the rotating body. In the case of constant angular velocity the far away observer will notice that a cylindrical "barrier", (rapid increase of the field and sideway turn of its direction) is formed at radial distance, in cylindrical coordinates, c/w from the axis of rotation, while for the nearby observer no such barrier is formed. In the case of exponentially declining angular velocity, the far away observer will see a "barrier" being formed only when the angular velocity is very big and in that case, the barrier is formed at a radial distance approximately inverse to the angular velocity, leading us to the subatomic distances (microcosmos) and resembling the non slippage case. This barrier is stronger as the angular velocity of rotation increases. The characteristic of the barrier is, as in the previous case, a rapid increase in the magnitude of the field and turn of the direction of the field from the radial direction. Outside this barrier, the field gradually regains its radial direction and normal magnitude as the effect of rotation of the body on space declines, and returns back to the normal Newtonian gravitational field. On the other hand, for small angular velocity no barrier is formed as is the case for cosmic distances (macrocosmos). Simply the field direction starts radially (in cylindrical coordinates) from the body and turns gradually sideways with respect to the radial until it reaches a maximum deflection and then turns back gradually to the radial direction and again ends up looking like a normal Newtonian gravity field. The sideways turn is not accompanied by an increase in the magnitude of the field as in the microcosmos case.

This paper is organized as follows: In section 2 a short review of previous theory is presented. In section 3 the connection between signals and fields is exposed. In section 4 we find the field, $\mathbf{G}'$, for an observer, who does not rotate with the body, when the angular velocity is constant with respect to the distance from the body (no slippage). In section 5 we retain the no slippage assumption but change to the far away observer, and calculate the field $\mathbf{G}''$ that he sees. In section 6 we calculate the relativistic mass of the rotating body. In section 7 we assume that the angular velocity is decreasing exponentially with the distance from the body (slippage case) and we calculate and present the graph of the field $\mathbf{G}''$ that the far away observer sees. It is shown that a "barrier" is formed at the microcosmos level. In section 8 we continue with the slippage assumption and shortly discuss the $\mathbf{G}'$ field for this case. Conclusions follow in section 9.

2 A short review of formulas related to previous theory on rotating frames and the path of signals emanating from a rotating body at the origin.

We will summarize the results of the theory [1], on which this paper stands by presenting the transformation of cylindrical coordinates for each case.

A. Rotation without slippage (the angular velocity w of rotation of signals is constant with respect to the distance from the rotating body). Precession of the rotating body is assumed having a very small amplitude and is thus neglected.

A.I Observer O' at the origin but not rotating with the body. (The transformation holds for $|z| \le \frac{c}{w}$).

$$°' = c \sin ¶\, I(¶, t) \tag{1}$$

$$\Theta' = \Theta + vt \tag{2}$$

$$z' = z \tag{3}$$

$$t' = t \tag{4}$$

$$®' = ® \frac{°}{°'} \frac{c}{\sqrt{c^2 + w^2 °^2}} = ® \frac{t}{I(¶, t)} \frac{c}{\sqrt{c^2 + w^2 °^2}} \tag{5}$$

$$v' = v \tag{6}$$

where $°, \Theta, z, t, ®, v$ are the radial distance in cylindrical coordinates, the angle of rotation as fraction of a circle (for example degrees), the z direction that coincides with the axis of rotation, time, the number pi, and the frequency of rotation respectively for observer O , who is located at the origin and rotates with the body. And where $°', \Theta', z', t', ®', v'$ are the same quantities for observer O', who is located at the origin but not rotating with the body. The speed of light is c for both observers.

Further, where,

$$I(\P,t)=\int_0^t \cos\text{¨}\,dt \tag{7}$$

$$\cos\text{¨} = \sqrt{\frac{1-w^2t^2\cos^2\P}{1+w^2t^2\sin^2\P}} = \sqrt{\frac{c^2-w^2z^2}{c^2+w^2\circ^2}} \tag{8}$$

with $\cos\P=\frac{z}{\sqrt{\circ^2+z^2}}$, $\sin\P=\frac{\circ}{\sqrt{\circ^2+z^2}}$, $\circ = ct\sin\P$, $z=ct\cos\P$ where $\P$ is the angle of inclination of the signal with respect to the z axis. Angle ¨ is the angle of deflection of the signal from the radial as observer O' sees it.
From the above we can find the transformation of the angular velocity w using the formula ($w=2\circledR v$ and $w'=2\circledR' v$) and the angle of rotation ¯ measured in radians (using $\text{¯}=2\circledR\Theta$ and $\text{¯}'=2\circledR'\Theta'$) as,

$$\frac{w'}{w}=\frac{\circledR'}{\circledR} \tag{9}$$

$$\text{¯}'=(\text{¯}+wt)\frac{\circledR'}{\circledR} \tag{10}$$

Relation (5) is obtained by requiring that the special relativistic Lorentz contraction of the perimeter holds for all light rays

$$2\circledR'\circ'=2\circledR\circ\sqrt{1-\frac{w'^2\circ'^2}{c^2}} \tag{11}$$

and using the fact that $w=2\circledR v$ and $w'=2\circledR' v$ we find $w'^2\circ'^2=\frac{w^2\circ^2c^2}{c^2+w^2\circ^2}$. Then we may express (11) as

$$\circledR'\circ'=\circledR\circ\frac{c}{\sqrt{c^2+w^2\circ^2}} \tag{12}$$

From this (5) is obtained.

Note that when $z=0$ ($\P=90^\circ$) (1) becomes, $\circ'=\frac{c}{w}\operatorname{arcsinh}(\frac{w\circ}{c})$

A.II. Observer O'' is the far away observer outside the cylindrical volume defined by $\circ''\le\frac{c}{w}$) for which the transformation below holds.

$$\circ''=\circ\frac{c}{\sqrt{c^2+w^2\circ^2}} \tag{13}$$

$$\Theta''=\Theta+vt \tag{14}$$

$$z''=z \tag{15}$$

$$t''=t \tag{16}$$

$$\circledR''=\circledR \tag{17}$$

$$v''=v \tag{18}$$

$$\text{¯}''=\text{¯}+wt \tag{19}$$

Where the double primed quantities have the same meaning as the single primed above but refer to observer O''. The angle of deflection is $\text{¨}''$, and is given by

$$\tan\text{¨}''=wt(1+w^2t^2\sin^2\P) \tag{20}$$

And the angle of inclination of the signal with respect to the z axis is given by

$$\tan ¶'' = \tan ¶ \, \frac{\sqrt{1+w^2t^2(1+w^2t^2\sin^2 ¶)^2}}{(1+w^2t^2\sin^2 ¶)^2} \tag{21}$$

B. Rotation with slippage. (The angular velocity of rotation of signals decreases exponentially with respect to the distance from the rotating body). This case has more meaning physically than case **A** above, and we also avoid the unnatural boundaries that appear at $|z| = \frac{c}{w}$ and at $°'' = \frac{c}{w}$. The angular velocity is given by $w = w_0 e^{-(ª°+«z)} = w_0 e^{-ct(ª\sin ¶ + « \cos ¶)}$ with $ª \geq 0$, $« \geq 0$ and the frequency of rotation is $v = v_0 e^{-(ª°+«z)} = v_0 e^{-ct(ª\sin ¶ + « \cos ¶)}$. Precession is assumed to have very small amplitude and is neglected otherwise $¶$ must be replaced by $¶ + ¸$ where $\tan ¸ = \tan ¸_0 \cos \Omega t$, where Ω is the angular velocity of precession.

B.I. Observer O' at the origin but not rotating.

$$°' = c \sin ¶ \, I(¶, t, ª, «) \tag{22}$$

$$\Theta' = \Theta + \int_0^t v_0 e^{-ct(ª\sin ¶ + « \cos ¶)} dt = \Theta + \frac{v_0(1-e^{-ct(ª\sin ¶ + « \cos ¶)})}{c(ª \sin ¶ + « \cos ¶)} \tag{23}$$

$$z' = z \tag{24}$$

$$t' = t \tag{25}$$

$$®' = ® \, \frac{°}{°'} \frac{c}{\sqrt{c^2 + w_0^2 °^2 e^{-2(ª°+«z)}}} \tag{26}$$

where

$$I(¶, t, ª, «) = \int_0^t \cos ¨ \, dt \tag{27}$$

where $¨$ is the angle of deflection of the signal from the radial and

$$\cos ¨ = \sqrt{\frac{1 - w_0^2 t^2 e^{-2ct(ª\sin ¶ + « \cos ¶)} \cos^2 ¶}{1 + w_0^2 t^2 e^{-2ct(ª\sin ¶ + « \cos ¶)} \sin^2 ¶}} = \sqrt{\frac{c^2 - w_0^2 z^2 e^{-2(ª°+«z)}}{c^2 + w_0^2 °^2 e^{-2(ª°+«z)}}} \tag{28}$$

$$\frac{w'}{w} = \frac{®'}{®} \tag{29}$$

and using (23) with (26) and the fact that $¯ = 2®\Theta$, $¯' = 2®'\Theta'$, we find the transformation of the rotation angle in radians

$$¯' = \frac{®'}{®}\left(¯ + \int_0^t w_0 e^{-ct(ª\sin ¶ + « \cos ¶)} dt\right) = \frac{®'}{®}\left(¯ + \frac{w_0(1-e^{-ct(ª\sin ¶ + « \cos ¶)})}{c(ª \sin ¶ + « \cos ¶)}\right) \tag{30}$$

Also assume that $\frac{1}{«} \leq \frac{ce}{w_0}$, the condition needed for $\cos ¨$ to be real for all $¶$.

B.II Observer O'' (the far away not rotating observer)

$$°'' = ° \, \frac{c}{\sqrt{c^2 + w_0^2 °^2 e^{-2(ª°+«z)}}} \tag{31}$$

$$\Theta'' = \Theta + \int_0^t v_0 e^{-ct(ª\sin¶+«\cos¶)}dt = \Theta + \frac{v_0(1-e^{-ct(ª\sin¶+«\cos¶)})}{c(ª\sin¶+«\cos¶)} \tag{32}$$

$$z'' = z \tag{33}$$

$$t'' = t \tag{34}$$

$$®'' = ® \tag{35}$$

$$w_0'' = w_0 \tag{36}$$

$$-'' = - + \int_0^t w_0 e^{-ct(ª\sin¶+«\cos¶)}dt = - + \frac{w_0(1-e^{-ct(ª\sin¶+«\cos¶)})}{c(ª\sin¶+«\cos¶)} \tag{37}$$

The angle of deflection $¨''$ is given by

$$\tan ¨'' = \frac{wt(1+w^2t^2\sin^2¶)}{1+c(ª\sin¶+«\cos¶)t^3w^2\sin^2¶} = \frac{w\sqrt{∘^2+z^2}}{c}\frac{1+\frac{w^2∘^2}{c^2}}{1+\frac{w^2∘^2(ª∘+«z)}{c^2}} \tag{38}$$

While the velocity of signals $³_c$ as observer O'' sees them will be

$$³_c = c\cos¶\sqrt{1+\tan^2¶\,\frac{w^2t^2}{1+w^2t^2\sin^2¶}\left(1+\frac{(1+c(ª\sin¶+«\cos¶)t^3w^2\sin^2¶)^2}{(1+w^2t^2\sin^2¶)^2}\right)} \tag{39}$$

The inclination of the path of the signal with respect to the z axis is given by

$$\tan¶'' = \tan¶\,\frac{wt}{\sqrt{1+w^2t^2\sin^2¶}}\sqrt{1+\frac{(1+c(ª\sin¶+«\cos¶)t^3w^2\sin^2¶)^2}{(1+w^2t^2\sin^2¶)^2}} \tag{40}$$

So that

$$³_c\cos¶'' = c\cos¶ \tag{41}$$

3 Signals and Fields

Consider a point in a field. At this point the field has a magnitude and direction. Let a small flat surface $\Delta\mathbf{a}$, whose normal is pointing in the same direction as the field. We will define the field strength n *as the number of signals per unit surface per unit time falling perpendicular to the surface*. Let also, $\mathbf{v}$ denote the velocity of the signals. We will assume that the direction of the field is given by the opposite direction to that of the velocity of the signals.
Then according to our definition,

$$\mathbf{G} = -n\hat{\mathbf{v}} \tag{42}$$

Where $\mathbf{G}$ is the vector field and $\hat{\mathbf{v}}$ is the unit vector in the direction of the velocity of the signals.
Letting now $\Delta\mathbf{a}$ become infinitesimal $d\mathbf{a}$ we say that the infinitesimal volume dV traversed by the signals in time dt is

$$dV = |\mathbf{v}||d\mathbf{a}|dt \tag{43}$$

Also the number of signals dN in the infinitesimal volume dV is,

$$dN = n|d\mathbf{a}|dt \tag{44}$$

dividing we obtain,

$$\frac{dN}{dV} = \frac{n}{|\mathbf{v}|} \tag{45}$$

Substituting n from (45) into (42) we find,

$$\mathbf{G} = -\frac{dN}{dV}\mathbf{v} \tag{46}$$

As an example, let us consider a field (like the Newtonian gravity field) that sends signals radially from a body of mass m. We assume that the total number of signals emitted from the body per unit time is proportional to m, say $k_G m$ for some constant k_G. Since the signals are emitted spherically they will cross the spherical surface at distance r homogeneously. In order to find the n (number of signals per unit time per unit surface crossing the surface of a sphere at distance r) we must divide the total number of signals per unit time by the surface area,

$$n = \frac{k_G m}{4\pi r^2} \tag{47}$$

It follows that this field may be represented using (42) by,

$$\mathbf{G} = -n\hat{\mathbf{v}} = -\frac{k_G m}{4\pi r^2}\hat{\mathbf{v}} \tag{48}$$

where $\hat{\mathbf{v}}$ is the unit vector in the direction of the velocity of the signals, which in this case coincides with the radius r.

4 The field $\mathbf{G}'$ created by a rotating body and no slippage (observer O' case A.I)

As discussed in [1], the signals emitted by a rotating point body look different to observer O'' (case A.II above), who is outside the volume defined by $\rho'' \leq \frac{c}{w}$ (which describes a cylinder of radius *c/w*) and to observer O' (case A.I) above), who is within this volume (as O'' defines it). We will examine the two phenomena separately starting with O'.

How will the field **G** look to observer O', when the body m rotates with angular velocity w ? There will be two effects that determine what O' observes. One is that the rotating mass will look bigger due to relativity, (denote it m'). This effect, which appears if we let the rotating body have dimensions instead of being a point mass, will be examined in section 6. The second effect is that the signals emanating from this body will not follow a radial path. Let's start with the second effect that is the signals that are emitted from such a body and look at Figure 1.

The signals according to observer O, who rotates with the body, will expand spherically and the field **G** will follow equation (48), the classical Newtonian field. The radially traveling signal OAC for observer O will be mapped to a helical signal shown as OA'C' for observer O'. A small volume $dV = \rho\, d\rho\, dz d\varphi$ (in cylindrical coordinates as observer O perceives space) around point such as A or C will be mapped to a small volume $dV' = \rho' d\rho' dz' d\varphi'$ in (cylindrical contracted coordinates as observer O' perceives space) around point A' or C' respectively. Still both volumes will contain the same number of signals dN . It follows, therefore, that O' will observe a field $\mathbf{G}'$ according to, (46)

$$\mathbf{G}' = -\frac{dN}{dV'}\mathbf{v}' \tag{49}$$

where $\mathbf{v}'$ is the velocity vector for signals as seen by O' . Using the chain rule of differentiation (49) can be written as

$$\mathbf{G}' = -\frac{dN}{dV}\frac{dV}{dV'}\mathbf{v}' \tag{50}$$

Assuming that signals travel with the same speed c for our observers $|\mathbf{v}'| = |\mathbf{v}| = c$, we may rewrite (50) as

$$|\mathbf{G}'| = \frac{dN}{dV}|\mathbf{v}'|\hat{\mathbf{v}}'\frac{dV}{dV'} = \frac{dN}{dV}|\mathbf{v}|\hat{\mathbf{v}}'\frac{dV}{dV'} = |\mathbf{G}|\frac{dV}{dV'}\hat{\mathbf{v}}' \tag{51}$$

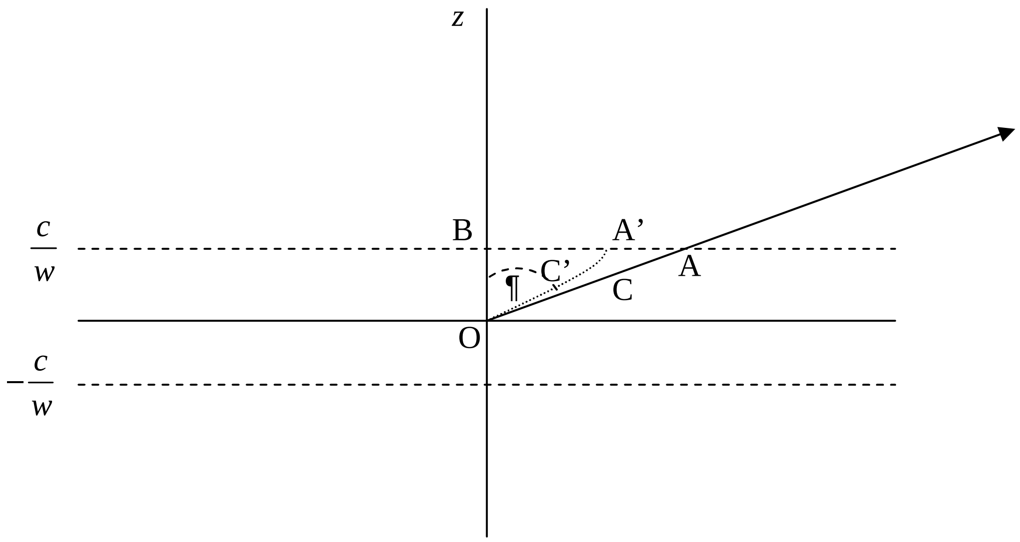


Figure 1 The signal as observer O, who rotates with the body, sees it is the straight line OCA. For observer O', OCA is mapped to a helical path shown as O C’A’ because of contraction of radial (in cylindrical coordinates) distance due to rotation.

For the case of the gravitational field using (51) on (48) it becomes,

$$\mathbf{G}' = -\frac{k_G m'}{4\text{®} r^2}\hat{\mathbf{v}}'\frac{dV}{dV'} \tag{52}$$

where we have replaced m by m' to indicate that the mass will look bigger, when it is rotating.

Using cylindrical coordinates, $r^2 = \circ^2 + z^2$, we obtain,

$$\mathbf{G}' = -\frac{k_G m'}{4\text{®}\left(\circ^2 + z^2\right)}\hat{\mathbf{v}}'\frac{dV}{dV'} \tag{53}$$

where $\hat{\mathbf{v}}'$ is the unit vector in the direction of the signals as seen by O' .

What remains now is to calculate $\frac{dV}{dV'}$. But $\frac{dV}{dV'}$ is given by the Jacobian of the transformation, $\frac{\partial(°,¯,z)}{\partial(°',¯',z')}$.

For case A.I equations (1) through (12) apply, which hold for $|z| < \frac{c}{w}$, (equivalently $t < \frac{1}{w\cos¶}$), the transformation is,

$$°' = c\sin¶ \int_0^t \cos¨\, dt = c\sin¶\, I(¶,t) \tag{54}$$

$$z' = z \tag{55}$$

$$¯' = (¯ + wt)\frac{®'}{®} \tag{56}$$

where $w'°' = \frac{w°c}{\sqrt{(c^2 + °^2 w^2)}}$ and $\cos¨ = \sqrt{\frac{1 - w^2t^2\cos^2¶}{1 + w^2t^2\sin^2¶}} = \sqrt{\frac{c^2 - w^2z^2}{c^2 + w^2°^2}}$.

The Jacobian of the above transformation is $\frac{\partial(°',¯',z')}{\partial(°,¯,z)}$

$$J' = \frac{\partial(°',¯',z')}{\partial(°,¯,z)} = \begin{vmatrix} \frac{\partial°'}{\partial°} & \frac{\partial°'}{\partial¯} & \frac{\partial°'}{\partial z} \\ \frac{\partial¯'}{\partial°} & \frac{\partial¯'}{\partial¯} & \frac{\partial¯'}{\partial z} \\ \frac{\partial z'}{\partial°} & \frac{\partial z'}{\partial¯} & \frac{\partial z'}{\partial z} \end{vmatrix} = \begin{vmatrix} \frac{\partial°'}{\partial°} & 0 & \frac{\partial°'}{\partial z} \\ \frac{\partial¯'}{\partial°} & \frac{\partial¯'}{\partial¯} & \frac{\partial¯'}{\partial z} \\ 0 & 0 & 1 \end{vmatrix} = \frac{\partial°'}{\partial°}\frac{\partial¯'}{\partial¯} \tag{57}$$

Observe that $\frac{\partial°'}{\partial¯} = 0$ since from (54) $°'$ does not depend on $¯$, which was anyway expected because of symmetry around the z axis as we take into account all signals emanating in all directions from the rotating body.

Now

$$\frac{\partial¯'}{\partial¯} = \frac{®'}{®} \tag{58}$$

while from (5)

$$®' = ®\frac{°}{°'}\frac{c}{\sqrt{c^2 + w^2°^2}} = ®\frac{t}{I(¶,t)}\frac{c}{\sqrt{c^2 + w^2°^2}} \tag{59}$$

Hence, substituting (58), (59), (54) in (57) it becomes,

$$\frac{\partial°'}{\partial°}\frac{\partial¯'}{\partial¯} = \frac{\partial}{\partial°}\left(cI(¶,t)\sin¶\right)\frac{\partial¯'}{\partial¯} = c\left[I(¶,t)\frac{\partial}{\partial°}\sin¶ + \sin¶\frac{\partial}{\partial°}I(¶,t)\right]\frac{°}{c\sin¶\, I(¶,t)}\frac{c}{\sqrt{c^2 + w^2°^2}}$$

(60)

or

$$J' = \left(\frac{°}{\sin¶}\frac{\partial}{\partial°}\frac{°}{\sqrt{°^2 + z^2}} + \frac{°}{I(¶,t)}\frac{\partial I(¶,t)}{\partial°}\right)\frac{c}{\sqrt{c^2 + w^2°^2}} \tag{61}$$

where we have used $\sin ¶ = \frac{\circ}{\sqrt{\circ^2 + z^2}}$

Focus on the term $\frac{\partial I(¶,t)}{\partial \circ}$ in (61). and observe that we may write $I(¶,t) = I^*(\cos ¶, t)$ since $I(¶,t)$ can be regarded as a function of $\cos ¶$ instead of ¶ . Hence, we may write,

$$\frac{\partial}{\partial \circ} I(¶,t) = \frac{\partial}{\partial \circ} I^*(\cos ¶, t) = \frac{\partial}{\partial \cos ¶} I^*(\cos ¶, t) \frac{\partial \cos ¶}{\partial \circ} + \frac{\partial}{\partial t} I^*(\cos ¶, t) \frac{\partial t}{\partial \circ} \qquad (62)$$

Now calculate terms one by one,

$$\frac{\partial}{\partial \cos ¶} I^*(\cos ¶, t) = \int_0^t \frac{\partial}{\partial \cos ¶} \sqrt{\frac{1 - w^2 t^2 \cos^2 ¶}{1 + w^2 t^2 \sin^2 ¶}} dt = -w^4 \cos ¶ \int_0^t \frac{t^4}{\sqrt{1 - w^2 t^2 \cos^2 ¶}\,(1 + w^2 t^2 \sin^2 ¶)^{\frac{3}{2}}} dt$$

(63)

and letting

$$U(¶,t) = \int_0^t \frac{t^4}{\sqrt{1 - w^2 t^2 \cos^2 ¶}\,(1 + w^2 t^2 \sin^2 ¶)^{\frac{3}{2}}} dt \qquad (64)$$

$$\frac{\partial}{\partial \cos ¶} I^*(\cos ¶, t) = -w^4 \cos ¶ \, U(¶,t) \qquad (65)$$

Also

$$\frac{\partial \cos ¶}{\partial \circ} = \frac{\partial}{\partial \circ} \frac{z}{\sqrt{\circ^2 + z^2}} = -\frac{\sin ¶ \cos ¶}{\sqrt{\circ^2 + z^2}} \qquad (66)$$

$$\frac{\partial}{\partial \circ} \sin ¶ = \frac{\partial}{\partial \circ} \frac{\circ}{\sqrt{\circ^2 + z^2}} = \frac{\cos^2 ¶}{\sqrt{\circ^2 + z^2}} \qquad (67)$$

$$\frac{\partial t}{\partial \circ} = \frac{\sin ¶}{c} \qquad (68)$$

Substituting (64), (65), (66), (67), (68) back in (62) and then in (60) we find,

$$\frac{\partial I(¶,t)}{\partial \circ} = w^4 U(¶,t) \frac{\cos^2 ¶ \sin ¶}{\sqrt{\circ^2 + z^2}} + \frac{\cos ¨ \sin ¶}{c} \qquad (69)$$

where we have used the fact that $\frac{\partial I(¶,t)}{\partial t} = \cos ¨$

Finally, substituting in (61) and using (67) it becomes,

$$J' = \left(\cos^2 ¶ + \frac{w^4 \cos^2 ¶ \sin^2 ¶ \, U(¶,t)}{I(¶,t)} + \frac{\circ \sin ¶ \cos ¨}{c I(¶,t)} \right) \frac{c}{\sqrt{c^2 + w^2 \circ^2}} \qquad (70)$$

An alternative form for (70) is obtained by recalling that $ct = \sqrt{\circ^2 + z^2}$,

$\sin ¶ = \frac{\circ}{\sqrt{\circ^2 + z^2}}$, $\cos ¶ = \frac{z}{\sqrt{\circ^2 + z^2}}$

$$J' = \left(\frac{z^2}{(\rho^2 + z^2)} + \frac{w^4 \rho^2 z^2 U(\theta, t)}{I(\theta, t)(\rho^2 + z^2)^2} + \frac{\rho^2 \cos\varphi}{c I(\theta, t)\sqrt{\rho^2 + z^2}} \right) \frac{c}{\sqrt{c^2 + w^2 \rho^2}} \qquad (71)$$

By similar manipulations we obtain another form of (70) in terms of θ and t

$$J' = \left(\cos^2\theta + \frac{w^4 \sin^2\theta \cos^2\theta\, U(\theta, t)}{I(\theta, t)} + \frac{t \sin^2\theta \cos\varphi}{I(\theta, t)} \right) \frac{1}{\sqrt{1 + w^2 t^2 \sin^2\theta}} \qquad (72)$$

Now we may use J' in (53) since $\frac{dV}{dV'} = \frac{1}{J'}$ and find the field,

$$\mathbf{G}' = -\frac{k_G m'}{4\pi(\rho^2 + z^2)} \frac{1}{J'} \hat{\mathbf{v}}' = -\frac{k_G m'}{4\pi \left(z^2 + \frac{\rho^2 z^2 w^4 U(\theta, t)}{I(\theta, t)(\rho^2 + z^2)} + \frac{\rho^2 \cos\varphi \sqrt{\rho^2 + z^2}}{c I(\theta, t)} \right) \frac{c}{\sqrt{c^2 + w^2 \rho^2}}} \hat{\mathbf{v}}'$$

(73)

The formula above expresses $\mathbf{G}'$ in terms of ρ, a quantity observed by observer O. If we wish to express it in terms of what observer O' sees, we must substitute ρ according to (54). Namely, $\rho' = c \sin\theta\, I(\theta, t) = \frac{ct \sin\theta\, I(\theta, t)}{t} = \frac{\rho}{t} I(\theta, t)$

The particular case when $z = 0$ implies that

$$I(\theta, t)\Big]_{z=0} = I(\frac{\pi}{2}, t) = \int_0^t \frac{dt}{\sqrt{1 + w^2 t^2}} = \frac{1}{w} \operatorname{arcsinh} wt = \frac{1}{w} \operatorname{arcsinh} \frac{w\rho}{c} \qquad (74)$$

Using (74) into (73) and setting $z = 0$, we find the field at the horizontal plane at the origin, which forms a disc as the signals at the horizontal plane rotate and expand outward logarithmically:

$$\mathbf{G}'\Big]_{z=0} = -\frac{k_G m'(c^2 + w^2 \rho^2) \operatorname{arcsinh} \frac{w\rho}{c}}{4\pi \rho^3 c w} \hat{\mathbf{v}}' = -\frac{k_G m'(c^2 + w^2 \rho^2) \frac{w\rho'}{c}}{4\pi c \rho^3 w} \hat{\mathbf{v}}' \qquad (75)$$

where we have used the fact that $\cos\varphi\Big]_{z=0} = \frac{c}{\sqrt{c^2 + w^2 \rho^2}}$

The direction of the field $\mathbf{G}'$ is given by its cylindrical components: $G'_z = -|\mathbf{G}'| \cos\theta$, $G'_\rho = -|\mathbf{G}'| \sin\theta \cos\varphi$, $G'_\varphi = -|\mathbf{G}'| \sin\theta \sin\varphi$ or the unit vector $\hat{\mathbf{v}}'$ is given by $\hat{\mathbf{v}}' = (v_\rho, v_\varphi, v_z) = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$. A plot of the field strength appears in Figure 2

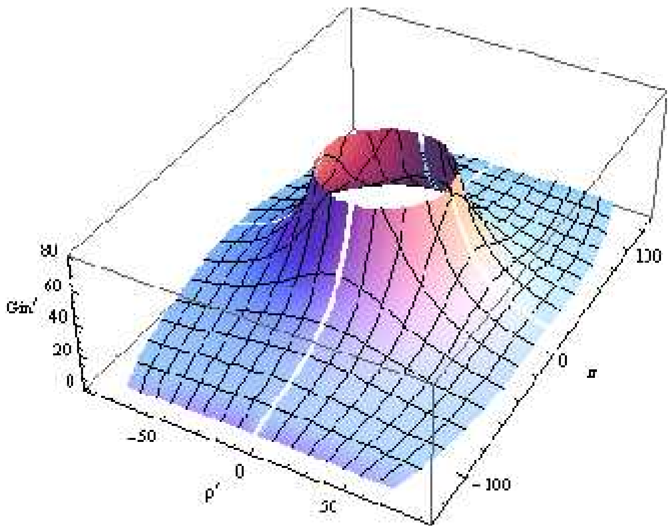


Figure 2 $|\mathbf{G}'|$ versus $°'$ and z . The effect of w is to decrease the scale as it increases keeping the same general shape .

5 The $\mathbf{G}''$ field created by a body rotating without slippage (far away observer O'' case A.II)

From (13) to (19),which apply for case A.II above, we have,

$$°'' = ° \frac{c}{\sqrt{c^2 + w^2 °^2}} \tag{76}$$

$$z'' = z \tag{77}$$

$$¯'' = ¯ + wt \tag{78}$$

The angle of deflection is

$$\tan ¨'' = wt(1 + w^2 t^2 \sin^2 ¶) \tag{79}$$

And the angle of inclination is

$$\tan ¶'' = \tan ¶ \frac{\sqrt{1 + w^2 t^2 (1 + w^2 t^2 \sin^2 ¶)^2}}{(1 + w^2 t^2 \sin^2 ¶)^2} \tag{80}$$

Since $\frac{\partial ¯''}{\partial ¯} = 1$ and $\frac{\partial °''}{\partial °} = \frac{c^3}{(c^2 + w^2 °^2)^{\frac{3}{2}}}$, the Jacobian is given by

$$J'' = \frac{\partial °''}{\partial °} \frac{\partial ¯''}{\partial ¯} = \frac{c^3}{(c^2 + °^2 w^2)^{\frac{3}{2}}} \tag{81}$$

Using the same arguments as in section 4 we may express the $\mathbf{G}''$ field as in (53)

$$\mathbf{G}'' = -\frac{k_G m'}{4® \left(°^2 + z^2 \right)} \hat{\mathbf{v}}'' \frac{dV}{dV''} \tag{82}$$

Where $\frac{dV}{dV''} = \frac{1}{J''}$ and $m'' = m'$. The equality $m'' = m'$ is shown in section 6, where aq calculation of m' is also presented. Note also, that $\hat{\mathbf{v}}''$ is the unit vector in the direction of the field.

Then (82) becomes

$$\mathbf{G}'' = -\frac{k_G m'(c^2 + {\circ}^2 w^2)^{\frac{3}{2}}}{4® c^3\left({\circ}^2 + z^2\right)}\hat{\mathbf{v}}'' \tag{83}$$

Now using (76) we find that

$${\circ} = {\circ}''\frac{c}{\sqrt{c^2 - w^2 {\circ}''^2}} \tag{84}$$

Substituting this into (83) we express $\mathbf{G}''$ in terms of ${\circ}''$,

$$\mathbf{G}'' = -\frac{km'c^3}{4® (c^2 {\circ}''^2 + (c^2 - w^2 {\circ}''^2)z^2)\sqrt{c^2 - w^2 {\circ}''^2}}\hat{\mathbf{v}}'' \tag{85}$$

Setting ${\circ} = 0$ in (83) we obtain the field along the z axis which is the simple Newtonian gravitational field, except for the increased mass m' due to rotation.
The direction of the field in cylindrical coordinates is given by the unit vector $\hat{\mathbf{v}}'' = (\sin¶''\cos¨'', \sin¶''\sin¨'', \cos¶'')$.
At ${\circ}'' = c/w$ the field blows to infinity. This corresponds to $t \to \infty$ and to $\tan¨'' \to \infty$ or that the field has turned 90 degrees with respect to the radial direction thus a “barrier” is formed at ${\circ}'' = c/w$.

Figure 3 shows how $|\mathbf{G}''|$ looks

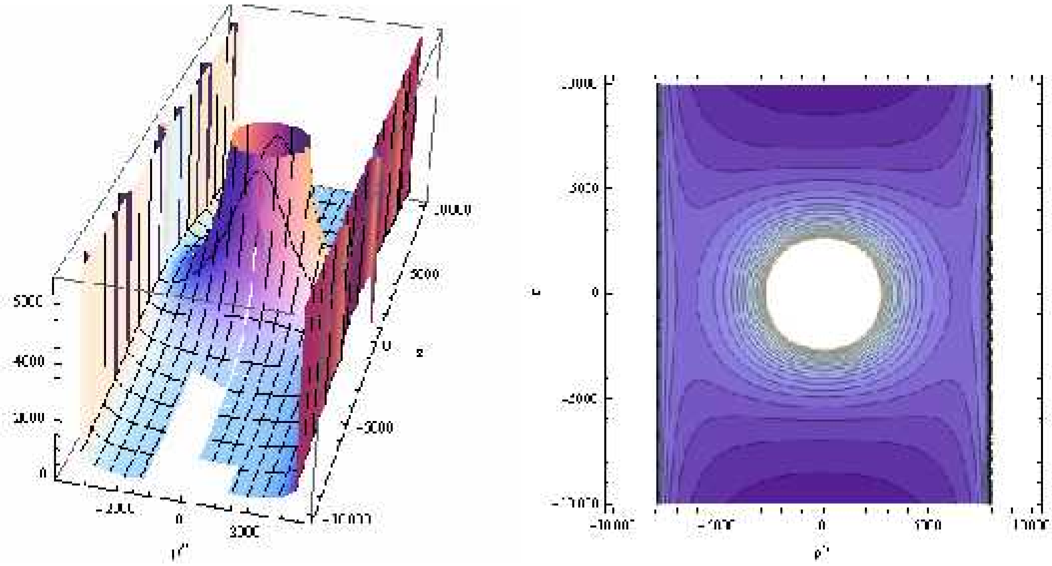


Figure 3 The graph of $|\mathbf{G}''|$ vs. ${\circ}''$ on the left appears as a contour plot on the right. We may imagine the minimum locus of Figure 4 placed on the contour plot as well as the lines at ${\circ}'' = \pm\frac{c}{w}\sqrt{\frac{2}{3}}$, where the minimum in the direction of the signal path occurs. The magnitude of $|\mathbf{G}''|$ is decreasing in the ${\circ}''$ direction until it reaches a minimum and then increases and blows to infinity. In the z direction it decreases as $1/z^2$

Now take the derivative

$$\frac{\partial}{\partial{\circ}}|\mathbf{G}''| = \frac{k_G m'}{4® c^3({\circ}^2 + z^2)}{\circ}\sqrt{c^2 + w^2{\circ}^2}(3w^2z^2 + w^2{\circ}^2 - 2c^2) \tag{86}$$

This quantity will be non positive for $3w^2z^2 + w^2 \circ^2 - 2c^2 \le 0$ or for

$$\circ^2 \le \frac{2c^2}{w^2} - 3z^2 \tag{87}$$

and hence $|\mathbf{G}''|$ will be minimum for

$$\circ^2_{\min} = \frac{2c^2}{w^2} - 3z^2 \tag{88}$$

The locus of pairs ($\circ, z$), where the minimum in the $\circ$ direction occurs represents an ellipse and the value of the field at a minimum is found by substituting (88) into (83)

$$|\mathbf{G}''|_{\min} = \frac{\sqrt{27} k_G m' w^2 \sqrt{c^2 - w^2 z^2}}{8 \text{®} c^3} \tag{89}$$

To see how the minimum looks to observer O'' we must find the minimum for $|\mathbf{G}''|$ with respect to $\circ''$. We take $\frac{\partial}{\partial \circ''}|\mathbf{G}''| = \frac{\partial}{\partial \circ}|\mathbf{G}''| \frac{\partial \circ}{\partial \circ''}$ and since $\frac{\partial \circ}{\partial \circ''} \ge 0$ the minimums with respect to $\circ$ found above, correspond to the minimums with respect to $\circ''$ and hence substituting (84) into (88) we find that

$$c^2 \circ''^2 + c^2 z^2 - w^2 z^2 \circ''^2 = \frac{2c^4}{3w^2} \tag{90}$$

This equation which is symmetric in $\circ'', z$ represents the locus of pairs of ($\circ'', z$) where $|\mathbf{G}''|$ is minimum in the $\circ''$ direction for each particular z. The locus appears in Figure 4

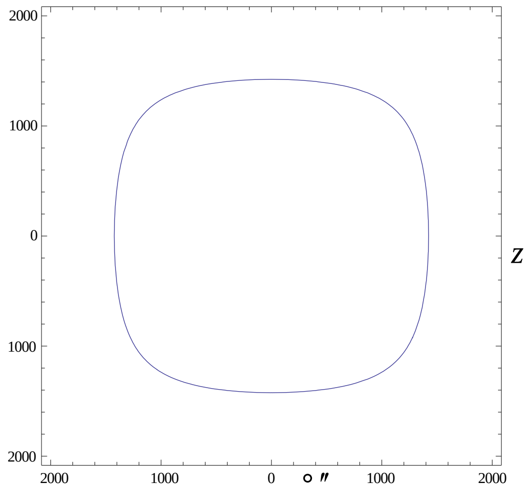


Figure 4 The shape depicts the pair of points ($\circ'', z$) where $|\mathbf{G}''|$ is minimum in the $\circ''$ direction. The graph is symmetric in $\circ'', z$

If we are interested in the derivative of $|\mathbf{G}''|$ along the path of a signal with inclination ¶ , we will take the derivative with respect to t. In this case

$$\frac{\partial}{\partial t}|\mathbf{G}''| = \frac{\partial}{\partial t}\frac{k_G m'}{4\text{®} c^3 c^2 t^2}(c^2 + w^2 c^2 t^2 \sin^2 \text{¶})^{\frac{3}{2}} = \frac{k_G m'}{4\text{®} c^2 t^3}\sqrt{1 + w^2 t^2 \sin^2 \text{¶}}\,(w^2 t^2 \sin^2 \text{¶} - 2) \tag{91}$$

and the minimum is reached at

$$t_{\min} = \frac{\sqrt{2}}{w \sin ¶} \tag{92}$$

This locus (as $¶$ varies) is a cylinder of radius $° = \frac{c\sqrt{2}}{w}$, which corresponds using (76) to $°'' = \frac{c}{w}\sqrt{\frac{2}{3}}$.

6 The rotating mass

Until now we have considered bodies as point masses. If we allow them to have dimensions the rotation makes them suffer a relativistic increase in mass. We will here examine the change in the mass from the rest mass m_{stat} to the mass that O' (or O'') observes.

Let $´ = \frac{dm_{stat}}{dV}$ and $´' = \frac{dm'}{dV'}$ be the density of the mass for observer O and O' respectively. Where V and V' is the volume for the two observers respectively. Also we will accept that a point mass that is at the distance $°'$ and has angular velocity w' according to O', will appear to him as having mass according to the transformation of special relativity, $m' = \frac{m_{stat}}{\sqrt{1 - \frac{w'^2 °'^2}{c^2}}} = \frac{m_{stat}\sqrt{c^2 + w^2 °^2}}{c}$ and therefore, for a small mass $dm = ´\, dV$ that revolves with angular velocity w at distance $°$ from the axis, $dm = ´\, dV = \frac{´' c dV'}{\sqrt{c^2 + w^2 °^2}} = dm' \frac{c}{\sqrt{c^2 + w^2 °^2}}$. But

$$m' = \int_{V'} ´' dV' \tag{93}$$

and substituting for $´'$ we obtain,

$$m' = \int_{V'} \frac{´ \sqrt{c^2 + w^2 °^2}}{c} \frac{dV}{dV'} dV' = \int_{V} \frac{´ \sqrt{c^2 + w^2 °^2}}{c} dV \tag{94}$$

Assume now that the stationary mass has uniform density,

$$m' = \frac{´}{c} \int_{V} \sqrt{c^2 + w^2 °^2}\, dV \tag{95}$$

As an example assume that the mass is spherical. Changing to spherical coordinates and integrating (see Appendix A) we find,

$$m' = \frac{®´ r^3}{2} - \frac{®´ c^2 r}{2w^2} + \frac{®´}{2w^3 c}(c^2 + w^2 r^2)^2 \arcsin\left(\frac{wr}{\sqrt{c^2 + w^2 r^2}}\right) \tag{96}$$

which is the same as

$$m' = \frac{4®r^{3}´}{3} + \frac{®´\ w^2r^5}{2c^2} + \frac{®´}{2c}(c^2+w^2r^2)^2\sum_{i=1}^{\infty}\frac{(-1)^i w^{2i-2}}{2i+1}\left(\frac{r}{c}\right)^{2i+1} \qquad (97)$$

where r is the stationary radius of m_{stat}. We note from (96) that as $w \to \infty$, $m' \to \infty$.

while, from (97) we see that for $w = 0$, $m' = \frac{4®r^{3}´}{3} = m_{stat}$ as expected.

The arguments are similar for observer O'':

$$m'' = \frac{m_{stat}}{\sqrt{1-\frac{w^2∘''^2}{c^2}}} \qquad (98)$$

And using (13) we find that $m'' = m'$.

7 The $\mathbf{G}''$ field for rotation with slippage (observer O'' case B.II)

When the rotation is with slippage the angular velocity of space around the rotating body decreases exponentially with distance from the body. In fact we assume that the angular velocity w is given by $w = w_0 e^{-(ª∘+«z)}$. In this case which is studied in [1] the signals start radially from the origin , then turn sideways until they reach a maximum of sideways turn and then gradually return back to the radial direction. Hence, the **G** field that is created is different from the no slippage case although it keeps the basic characteristic of the sideways turn and having a barrier when w_0 is big as in the case of no slippage as we will see. For the far away observer O'' the field is denoted as $\mathbf{G}''$
In this case the transformation is given by case B.II above (recall (31) and following),

$$∘'' = ∘\frac{c}{\sqrt{c^2+w_0^2∘^2e^{-2(ª∘+«z)}}} \qquad (99)$$

$$¯'' = ¯ + \int_0^t w_0e^{-ct}\ dt = ¯ + \frac{w_0(1-e^{-ct}\ )}{c} \qquad (100)$$

$$z'' = z \qquad (101)$$

$$\tan ¨'' = wt\frac{1+w^2t^2\sin^2¶}{1+c\ \ t^3w^2\sin^2¶} = \tan ¨\ \frac{1+\frac{w^2∘^2}{c^2}}{1+(ª∘+«z)\frac{w^2∘^2}{c^2}} \qquad (102)$$

Where $\ = ª\sin¶ + «\cos¶$, $\tan ¨ = \frac{w\sqrt{∘^2+z^2}}{c}$

$$\tan¶'' = \tan¶\ \frac{wt}{\sqrt{1+w^2t^2\sin^2¶}}\sqrt{1+\frac{(1+c\ \ t^3w^2\sin^2¶)^2}{w^2t^2(1+w^2t^2\sin^2¶)^2}} \qquad (103)$$

The Jacobian of the transformation is,

$$J'' = \frac{\partial(∘'',¯'',z'')}{\partial(∘,¯,z)} = \frac{\partial∘''}{\partial∘}\frac{\partial¯''}{\partial¯} \qquad (104)$$

But $\dfrac{\partial ¯''}{\partial ¯} = 1$, and $\dfrac{\partial °''}{\partial °} = \dfrac{c}{\sqrt{c^2 + w_0^2 °^2 e^{-2(ª° + «z)}}} - \dfrac{c w_0^2 °^2 e^{-2(ª° + «z)}(1 - ª°)}{(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{3}{2}}}$, which after some manipulation gives

$$\frac{\partial °''}{\partial °} = \frac{c(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})}{(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{3}{2}}} \tag{105}$$

Hence,

$$J'' = \frac{c(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})}{(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{3}{2}}} \tag{106}$$

and

$$\mathbf{G}'' = -\frac{k_G m'}{4®(°^2 + z^2)} \frac{\hat{\mathbf{v}}''}{J''} = -\frac{k_G m'}{4®(°^2 + z^2)} \frac{(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{3}{2}}}{c(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})} \hat{\mathbf{v}}'' \tag{107}$$

For the direction of the field we need the unit vector in cylindrical coordinates, which as usual is given by

$$\hat{\mathbf{v}}'' = (v_{°}, v_{¯}, v_z) = (\sin ¶'' \cos ¨'', \sin ¶'' \sin ¨'', \cos ¶'') \tag{108}$$

The minimums and maximums of $\mathbf{G}''$ are difficult to calculate. However, it is possible to make the following observations,

7.1 Observations on the extremes of $\mathbf{G}''$

(a) Recall that whatever we find for $°$ has a corresponding value for $°''$ since by (105), which is positive for all $°$, we know that $°''$ is monotonically increasing in $°$. The derivative of $|\mathbf{G}''|$ with respect to $°$ is

$$\frac{\partial |\mathbf{G}''|}{\partial °} = \frac{k_G m'}{4®} \left\{ -\frac{2°(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{3}{2}}}{c(°^2 + z^2)^2 (c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})} + \frac{3(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{1}{2}} w_0^2 °\, e^{-2(ª° + «z)}(1 - ª°)}{c(°^2 + z^2)(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})} \right.$$
$$\left. - \frac{(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})^{\frac{3}{2}} w_0^2 ª °^2 e^{-2(ª° + «z)}(3 - 2ª°)}{c(°^2 + z^2)(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})^2} \right\} \tag{109}$$

Setting it equal to zero and simplifying we obtain,

$$-2(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)}) + 3e^{-2(ª° + «z)} w_0^2 (1 - ª°)(°^2 + z^2)(c^2 + ª\, w_0^2 °^3 e^{-2(ª° + «z)})$$
$$-e^{-2(ª° + «z)} w_0^2 ª°(c^2 + w_0^2 °^2 e^{-2(ª° + «z)})(3 - 2ª°)(°^2 + z^2) = 0 \tag{110}$$

The left hand side of the equation tells us that the first term is always negative the second term starts positive and at $° = \frac{1}{ª}$ turns negative, the third term starts negative and at $° = \frac{3}{2a}$ turns positive. Therefore, for the interval for $\frac{1}{a} < ° < \frac{3}{2a}$, the derivative is always negative.

(b) The derivative of $|\mathbf{G}''|$ with respect to w_0^2 will give us more information,

$$\frac{\partial|\mathbf{G}''|}{\partial(w_0^2)} = \frac{k_G m'}{4®c(°^2 + z^2)} °^2 e^{-2(a° + «z)} \frac{\frac{3}{2}\sqrt{c^2 + w_0^2 °^2 e^{-2(a° + «z)}}(c^2 + a w_0^2 °^3 e^{-2(a° + «z)}) - a°(c^2 + w_0^2 °^2 e^{-2(a° + «z)})^{\frac{3}{2}}}{(c^2 + a w_0^2 °^3 e^{-2(a° + «z)})^2}$$

(111)

Setting it equal to zero and simplifying we obtain,

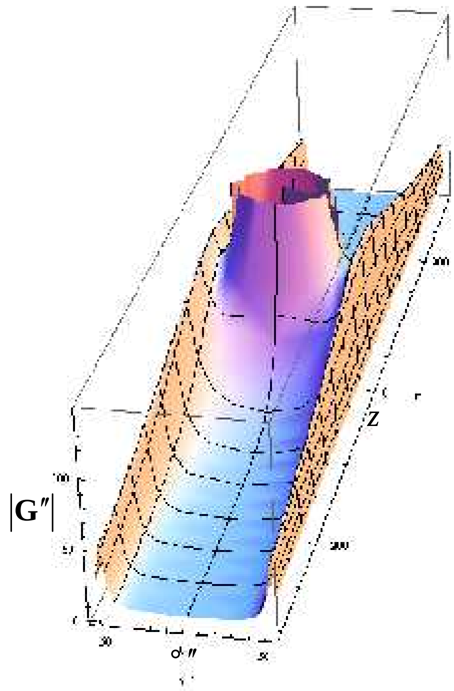
$$\frac{3}{2}(c^2 + a w_0^2 °^3 e^{-2(a° + «z)}) - a°(c^2 + w_0^2 °^2 e^{-2(a° + «z)}) = 0 \quad (112)$$

And solving

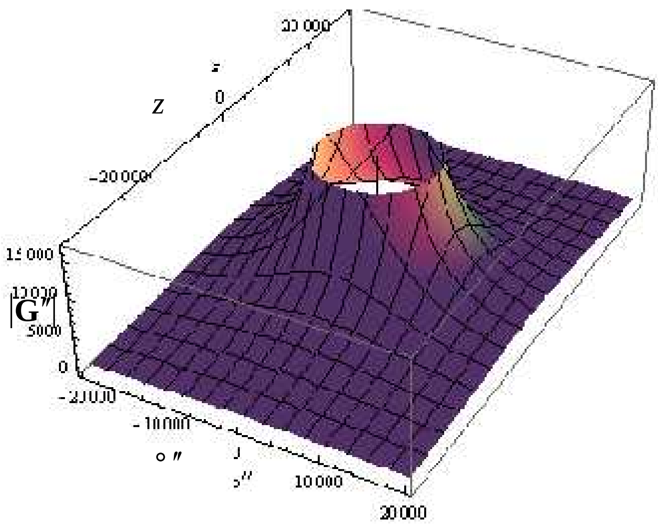
$$w_0^2 = \frac{2c^2(a° - \frac{3}{2})}{a°^3 e^{-2(a° + «z)}} \quad (113)$$

For $a° - \frac{3}{2} < 0$ the derivative (111) starts at $w_0^2 = 0$ with positive value. Therefore, as we start increasing w_0^2, $|\mathbf{G}''|$ is increasing but no maximum is reached as long as $a° - \frac{3}{2} < 0$ because (113) is impossible. (A positive left hand side cannot equal to a negative right hand side) and hence (112) cannot hold and no maximum is reached. In other words, in the interval $0 < ° < \frac{3}{2a}$, $|\mathbf{G}''|$ increases indefinitely as w_0 increases while in the interval $\frac{3}{2a} < °$ a maximum is reached for some value of w_0^2. This means that by increasing w_0 enough I can make $\max\limits_{0<°<\frac{3}{2}a} [|\mathbf{G}''|]$ exceed $|\mathbf{G}''|$ for all values of $°$, such that $\frac{3}{2a} < °$. In short for w_0 big enough the maximum of $|\mathbf{G}''|$ over $°$ is always in the interval $0 < ° < \frac{3}{2a}$. Also this maximum is not in the neighbourhood of $° = 0$ when $z \neq 0$, because at these points $|\mathbf{G}''|$ has a finite value independent of w_0.
Combining now this result with our observation (a), (that in the interval for $\frac{1}{a} < ° < \frac{3}{2a}$, the derivative is always negative), we may restrict this interval even further so that the maximum of $|\mathbf{G}''|$ over $°$, to be denoted as $°_{G\max}$, is always in the interval $0 < °_{G\max} < \frac{1}{a}$. Still we have not shown whether there is only one or more local max in $0 < ° < \frac{1}{a}$. However, the graph of $|\mathbf{G}''|$ (see Figure 5a) indicates that there is

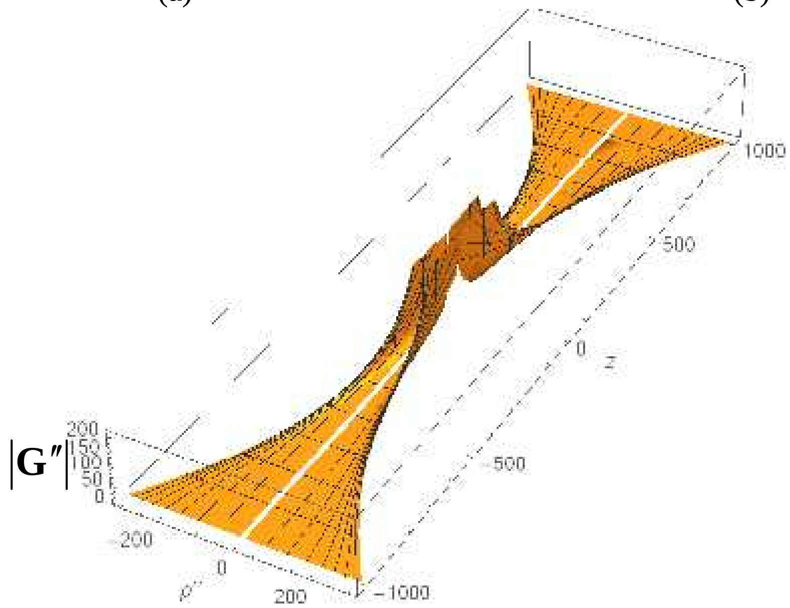
only one internal maximum in the ° direction and that it appears for big enough w_0, otherwise, when w_0 is not big enough, the graph of $|\mathbf{G}''|$ is monotonically decreasing in both ° and z (see Figure 5b). In Figure 5 the plot is against °″ not ° but recall that there is a monotonic relation between °″ and ° and hence the existence and number of extremes are the same. Only due to the contraction of °″ the maximum of the field at $°''_{G\max}$ becomes very acute and looks like a wall to observer O''.



(a)



(b)



(c)

Figure 5 . $|\mathbf{G}''|$ versus °″ and z (a) for big w_0 ($w_0 \approx 10^7$ rad/s) and (b) for small w_0

The graph shows the strength of the field, not its direction, which changes according to the angle of deflection ¨″.When w_0 is big (a), the field rises sharply in the radial direction forming a barrier that diminishes in the z direction. (b) For small enough w_0 , the field has no such interior maximum and drops gradually both in °″ and z , while cos ¨″ does not diminish as much. In (c) a variation of case (a) appears where by choice of ª and « the barrier appears to widen in the z axis direction.

7.2 Observations on °″

From (99) we have,

$$°″ = \frac{°}{\sqrt{1 + \frac{w_0^2 °^2 e^{-2(ª° + «z)}}{c^2}}} \tag{114}$$

It is obvious that °″ ≤ ° . For large w_0 , °″ becomes as small as we like and we are talking about the *microcosmos* (sub atomic level). On the other hand, small w_0 keeps °″ at a magnitude comparable to ° , which we call the *macrocosmos*. More formally,

--If $\frac{w_0 °}{c} << e^{ª° + «z}$ then °″ ≈ ° and $|\mathbf{G}''| \simeq \frac{k_G m'}{4®(°^2 + z^2)}$, which is the usual Newtonian field. This for example holds for cosmic level (macrocosmos), where w_0 is small, while ° is very big.

-- If $\frac{w_0 °}{c} >> e^{ª° + «z}$ (This is achieved for the microcosmos level where w_0 is big, while ° remains small), then

$$°″ \simeq \frac{°}{\sqrt{\frac{w_0^2 °^2 e^{-2(ª° + «z)}}{c^2}}} = \frac{c}{w_0} e^{ª° + «z} \tag{115}$$

In particular, for ª° + «z << 1 we see that

$$°″ \simeq \frac{c}{w_0} \tag{116}$$

at the microcosmos level. It is remarkable that this is also the limit for °″ for the no slippage case, where the value of $|\mathbf{G}''|$ blows to infinity. It is interesting to note that the radius in this case is inversely proportional to the angular velocity w_0 .The fact that in (116) °″ is independent of ° means that the signals of the field for a range of values of ° are mapped and concentrated on $\frac{c}{w_0}$ and is, therefore, a maximum for $|\mathbf{G}''|$. This is in very good harmony with plots of $|\mathbf{G}''|$, where indeed the maximum appears at $°″_{G\max} \simeq \frac{c}{w_0}$. This is also in agreement with the requirement of section 7.1 that $°_{G\max} \leq \frac{1}{ª}$ since it is implied by the requirement ª° + «z << 1.

7.3 The maximum deflection of the direction of **G″**

The maximum of the deflection angle ¨″, is not easy to calculate however, we expect it to happen very close to the maximum of ¨ or the minimum of cos ¨ , in the direction of the signal path is found in [1] when we take $\frac{\partial}{\partial t}\cos$ ¨ . This occurs at

$$ct_{¨\,\max} = \frac{1}{\text{a}\sin¶ + «\cos¶} \tag{117}$$

Where ¶ is the angle of inclination of the signal from the z axis. Corresponding to this maximum are

$$°_{¨\,\max} = \frac{\sin¶}{\text{a}\sin¶ + «\cos¶} \tag{118}$$

And

$$z_{¨\,\max} = \frac{\cos¶}{\text{a}\sin¶ + «\cos¶} \tag{119}$$

And the locus of points where the max occurs is a rhombus by revolution which in the first quadrant follows the equation

$$\text{a}°_{¨\,\max} + «z_{¨\,\max} = 1 \tag{120}$$

Using (114) we find

$$°''_{¨\,\max} = \frac{°_{¨\,\max}}{\sqrt{1+\frac{w_0^2 °_{¨\,\max}^2 e^{-2(\text{a}°_{¨\,\max}+«z_{¨\,\max})}}{c^2}}} = \frac{°_{¨\,\max}}{\sqrt{1+\frac{w_0^2 °_{¨\,\max}^2 e^{-2}}{c^2}}} = \frac{\sin¶}{\sqrt{(\text{a}^2+\frac{w_0^2}{c^2e^2})\sin^2¶ + «^2\cos^2¶}} \tag{121}$$

--For the *macrocosmos* case where $\frac{w_0 °}{c} << e^{\text{a}°+«z}$ or because of (120) $\frac{w_0 °_{¨\,\max}}{c} << e$, we may approximate (121) by

$$°''_{¨\,\max} \simeq °_{¨\,\max} = \frac{\sin¶}{\text{a}\sin¶ + «\cos¶} = \frac{1}{\text{a} + «\cot¶} \le \frac{1}{\text{a}} \tag{122}$$

(Recall that also $°''_{G\max} \le °_{G\max} < \frac{1}{\text{a}}$ from section 7.1 but in the case of macrocosmos it is meaningless since there is no interior maximum for the **G** field and hence $°''_{G\max}$ and $°_{G\max}$ do not exist.) .

--For the *microcosmos* level where $\frac{w_0 °_{¨\,\max}}{c} >> e$ we may approximate (121) by

$$°''_{¨\,\max} = \frac{°_{¨\,\max}}{\sqrt{1+\frac{w_0^2 °_{¨\,\max}^2}{c^2e^2}}} \simeq \frac{ce}{w_0} \tag{123}$$

Comparing this with $°''_{G\max} \simeq \frac{c}{w_0}$, as we discussed in section 7.2, we see that the max deflection ¨ (which we used to approximate the maximum deflection of ¨″) of the field

occurs at a distance that is further away from the maximum magnitude of the field by a factor of e.
From (102) we find that

$$\left|\tan ¨\, ″\right|_{t=t_{¨\,\max}} = \frac{w_0}{ec} \qquad (124)$$

Since $t_{¨\,\max} = \dfrac{1}{c}$. Thus for big enough w_0 , $¨\, ″ \to 90^{\circ}$ degrees

8 The $\mathbf{G}'$ field for rotation with slippage (observer O', case B.I)

This case, where the observer O' is affected by the field, is more complicated in the formulas and we will just present it briefly,
The transformation is

$$°\,' = c \sin ¶\, I(¶, t, ª, «) \qquad (125)$$

$$¯\,' = \frac{®\,'}{®}\left(¯ + \int_0^t w_0 e^{-ct(ª \sin ¶ + « \cos ¶)} dt\right) \qquad (126)$$

$$z' = z \qquad (127)$$

Now the Jacobian of the transformation is

$$J' = \frac{®\,'}{®}\frac{\partial °\,'}{\partial °} = \frac{\partial °\,'}{\partial °}\frac{°}{°\,'}\frac{c^2}{\sqrt{c^2 + w_0^2 °^2 e^{-2(ª° + «z)}}} = \frac{\partial °\,'}{\partial °}\frac{°}{c \sin ¶\, I(¶, t, ª, «)}\frac{c^2}{\sqrt{c^2 + w_0^2 °^2 e^{-2(ª° + «z)}}} \qquad (128)$$

Then we may find $\dfrac{\partial °\,'}{\partial °}$ proceeding in a manner similar to that for the no slippage case.
We will then find that

$$J' = \left(\cos^2 ¶ + \frac{° \cos ¨ \cos ¶}{cI(¶, t, ª, «)} - \frac{\cos ¶\, U_2(¶, t)}{I(¶, t, ª, «)}\right)\frac{c}{\sqrt{c^2 + w_0^2 °^2 e^{-2(ª° + «z)}}} \qquad (129)$$

where

$$U_2(¶, t) = \int_0^t \frac{\partial}{\partial \cos ¶}\left(\sqrt{\frac{1 - w_0^2 t^2 e^{-2ct(ª \sin ¶ + « \cos ¶)} \cos^2 ¶}{1 + w_0^2 t^2 e^{-2ct(ª \sin ¶ + « \cos ¶)} \sin^2 ¶}}\right) dt \qquad (130)$$

Finally, as usual

$$\left|\mathbf{G}'\right| = \frac{k_G m'}{4®(°^2 + z^2)}\frac{1}{J'} \qquad (131)$$

It is difficult to plot $\left|\mathbf{G}'\right|$ but from the physics of the problem we expect a maximum to occur near the locus where the deflection of the signals is maximized. This locus which has the shape of a rhombus by revolution in the $(°, z)$ space, is studied in [1] and presented briefly and used above in section 7.

9 Summary of the results

To find the $\mathbf{G}'$ field (for the nearby observer O') and $\mathbf{G}''$ field (for the far away observer O'' and determine their properties, we started by assuming that a field consists

of signals and that the strength of a field is given by the number of signals per unit volume. We further assumed that these signals travel with the speed of light or they behave like light. This allowed us to apply the results for light signals for rotating frames to the case of gravitational signals emitted by a rotating body of mass m' to find the form and strength of the $\mathbf{G}'$ and $\mathbf{G}''$ fields it creates. We also calculated the relativistic mass m' of a rotating body from its stationary non rotating mass m_{stat}.
$\mathbf{G}'$ and $\mathbf{G}''$ are fields that include a bending and even winding of signals and can become very strong. Also depending on the value of the angular velocity of rotation w the field applies to both the microcosmos (subatomic level) and cosmological scale or macrocosmos.
We examined two cases:

(a) When rotation is without slippage (the angular velocity of rotation is constant independent of distance from the origin), the $\mathbf{G}'$ field (of observer O') exists for $|z| < c/w$. The field looks to that observer unbounded in the radial direction and restricted in the z direction to $|z| < c/w$. For observer O'', however, who is outside $|z| < c/w$, the field he perceives is $\mathbf{G}''$ and is restricted in the radial by $°'' \leq \frac{c}{w}$.

(b) When rotation is with slippage ($w = w_0 e^{-a°-«z}$) the $\mathbf{G}''$ field is not restricted to within and $°'' \leq \frac{c}{w}$. We may distinguish two cases (i) The *macrocosmos* level for which $\frac{w_0 °}{c} << e^{a°+«z}$. This case holds, for example, for small w_0 and big $°$, and we have small contraction of space and the field behaves much like the usual $\frac{1}{r^2}$ field, except that there is a sideway component due to the rotation that makes $\cos ¨ < 1$, or the deflection angle become positive, with its maximum approximated at $°''_{¨ \max} \leq \frac{1}{a}$ and then gradually return to zero. (ii) The *microcosmos* level for which $\frac{w_0 °}{c} >> e^{a°+«z}$. Here, there is both a maximum of the magnitude of the field that rises sharply and can be very strong and occurs at $°''_{G\max} \simeq c/w_0$, and a maximum in the angle of deflection that occurs at approximately $°''_{¨ \max} \simeq \frac{ce}{w_0}$. This latter phenomenon we call a "barrier".

(c) Not much can be said about $\mathbf{G}'$, due to the complexity of the formulas. However, from the study on the signals emitted by the rotating body we expect a similar behavior to that of $\mathbf{G}''$.

10 References

1. Pechlivanides P. *On Rotating Frames and the Relativistic Contraction of the Radius (The Rotating Disc)*, http://vixra.org/abs/1411.0229, 2017

Appendix A
The spinning mass

$$m' = \frac{\text{´}}{c}\int_V \sqrt{c^2 + w^2 \circ^2}\, dV \qquad \text{(A.1)}$$

Changing to spherical coordinates

$$m' = \frac{4\text{®´}}{c}\int_0^r r^2 \left[\int_0^{\frac{\text{®}}{2}} \sqrt{c^2 + w^2 r^2 \cos^2 \text{¯}}\, \cos\text{¯}\, d\text{¯}\right] dr \qquad \text{(A.2)}$$

or

$$m' = \frac{4\text{®´}}{cw}\int_0^r r \left[\int_0^{\frac{\text{®}}{2}} \sqrt{c^2 + w^2 r^2 - w^2 r^2 \sin^2 \text{¯}}\, wr\cos\text{¯}\, d\text{¯}\right] dr \qquad \text{(A.3)}$$

$$m' = \frac{4\text{®´}}{wc}\int_0^r \frac{cwr^2}{2} dr + \underbrace{\frac{4\text{®´}}{wc}\int_0^r \frac{(c^2 + w^2 r^2)}{2} \arcsin\left(\frac{wr}{\sqrt{c^2 + w^2 r^2}}\right) r dr}_{I_1} \qquad \text{(A.4)}$$

The first term is $\frac{2\text{®´}\, r^3}{3}$ and the second term is

$$I_1 = \frac{\text{®´}}{2w^3 c}\int_0^r \arcsin\left(\frac{wr}{\sqrt{c^2 + w^2 r^2}}\right) d(c^2 + w^2 r^2)^2 \qquad \text{(A.5)}$$

Integrating by parts

$$I_1 = \frac{\text{®´}}{2w^3 c}(c^2 + w^2 r^2)^2 \arcsin\frac{wr}{\sqrt{c^2 + w^2 r^2}} - \frac{\text{®´}}{2w^3 c}\int_0^r wc(c^2 + w^2 r^2)^2 dr \qquad \text{(A.6)}$$

And finally,

$$I_1 = \frac{\text{®´}}{2w^3 c}(c^2 + w^2 r^2)^2 \arcsin\frac{wr}{\sqrt{c^2 + w^2 r^2}} - \frac{\text{®´}\, c^2 r}{2w^2} - \frac{\text{®´}\, r^3}{6} \qquad \text{(A.7)}$$

Therefore,

$$m' = \frac{\text{®´}\, r^3}{2} - \frac{\text{®´}\, c^2 r}{2w^2} + \frac{\text{®´}}{2w^3 c}(c^2 + w^2 r^2)^2 \arctan\frac{wr}{c} \qquad \text{(A.8)}$$

Using the expansion of

$$\arcsin\frac{wr}{\sqrt{c^2 + w^2 r^2}} = \arctan\frac{wr}{c} = \frac{wr}{c} - \frac{1}{3}\left(\frac{wr}{c}\right)^3 + \frac{1}{5}\left(\frac{wr}{c}\right)^5 - \frac{1}{7}\left(\frac{wr}{c}\right)^7 + \ldots \qquad \text{(A.9)}$$

which converges for $-1 \le \frac{wr}{c} \le 1$, we may write m' as

$$m' = \frac{4\text{®}\, r^3\text{´}}{3} + \frac{\text{®´}\, w^2 r^5}{2c^2} + \frac{\text{®´}}{2c}(c^2 + w^2 r^2)^2 \sum_{i=1}^{\infty} \frac{(-1)^i w^{2i-2}}{2i+1}\left(\frac{r}{c}\right)^{2i+1} \qquad \text{(A.10)}$$