FORCES OF NATURE:
Gravitational force in the context of electrical force

Four forces of nature:
We all are familiar with the four force of nature, we deal with them, work with them and come across all the way along with our day-to-day life with or without knowing them. And they are:

1. Gravitational force
2. Electromagnetic force
3. Weak nuclear force and
4. Strong nuclear force

We are almost familiar with the above mentioned forces, but most of the time may not be experienced some forces intentionally. For example, radium, which is resultant of weak nuclear force, even though we come across many time with this force, even if we do so, many among us doesn’t get the root cause for it, precisely the actual physics behind it.

Gravity is the oldest known force among all the forces listed herein. And its weakest force too. Electric force is much stronger than the Gravitational force, but lesser than the strong nuclear force. The gravitational force is dealt with massive object or in other words it may be studied by considering mass of the objects. The strange thing about the Gravitational force is, even after the two bodies being apart, it could totally be found even though they are not being in physical contact. Now let us discuss how actually the force is resolved and compared with Electric force.

We all are familiar with the Newton’s famous formula,

\[ F = ma \] (1)

that, perhaps revolutionized the industrial development in the world. There are many different ways to look after at equation (1). Probably one might straight forwardly tells that ‘a’ stands for acceleration. If we judge a current position and position later, taking the difference between them and dividing it by time gives ‘velocity’. Then find velocity now and velocity later, take the difference and divide it by time, then we get ‘acceleration’. So the acceleration itself really requires three measurements two for each velocity, but we talk about acceleration right now. Because we could make three measurements arbitrarily near each other and limit in which time difference between them goes to zero we can talk about velocity right now and acceleration right now. In simple example we can say, if we find the needle of our car odometer points at 60 kmph then it’s our velocity and if we step on the acceleration pedal we feel the push and that’s the acceleration right now that’s the property of that instance.

So we know acceleration, the question is can we go find the mass using the force and acceleration from equation (1)? Let us make clear about the main difference or perhaps a misconception about difference between the “mass” and “weight”. Well weight is resultant where the object being near the earth (or any other massive object in the space-time continuum- Albert Einstein’ General theory of Relativity) due to gravity and the mass is related with the counts of the atomic and subatomic particles and could be found anywhere the same.
The force can be measured in many forms for example, force experienced from the driving car, that is \( F = ma \). The force experienced by the string tied with its one end to the object of mass \( M \) and other to the stand gives a function for the force along with the \( X \) and would be given by:

\[
F = -kX \tag{2}
\]

where the \( k \) is force constant and negative sign is because the force will be in opposite direction caused by the spring. Now the difference between the equation (1) and (2) is that, the equation (1) says that, if we know the force we can tell the acceleration but it’s our job to go find out everything that what force is acting on a particle. If the object is connected by the spring and if we pull the spring it exist a force so much that in equation (2) to find out the force acting on the object.

Another kind of force that we can find on the surface of the earth. If we drop something it seems to accelerate towards the ground and everything accelerates with same amount \( g \). according to Newton’s law and by equation (1) itself anything get to accelerate because of force acting on the body. The force on any mass \( m \) must be given by

\[
F = mg \tag{3}
\]

Here something special about this force is that while the spring force where the spring is touching/being in contact with the mass, will be pulling it and if we walk on ground we will move forward while being in contact with ground and exerting some force on the ground. But the pull of the gravity is bit strange. Because there is not at all real contact between the object that is falling and the ground (earth). In the early days when the gravity was first taken into account, it was a great abstraction to believe that the things can reach out and pull things which are not touching them and gravity was formal and first discovered force among four forces of nature.

If we take equation (3) and extend it for a two massive objects (say earth and the moon, for example) which are being apart from one other with some long distance, say \( r \) then force between them is given by

\[
F = \frac{G M_1 M_2}{r^2} \tag{4}
\]

This says that the force between the earth and moon decreases with increase in the distance \( (1/r^2) \). So everything accelerates. The reason is the force. Many times many forces can be acting on a body and if we put all the forces acting on a body that explains the acceleration we’re done and sometimes it won’t.

The overall advantage of \( F = Ma \) is that, and is known good for the reasons:

1. It defines mass
2. It helps calculates force using on bodies by seeing how they accelerate
3. And finally to find the acceleration of bodies, given the forces acting on it.

This is cycle of Newtonian dynamics.

Of course the gravity is the very first force discovered by the man unlike it we’re going to list another kind of force which is listed above. To describe the force let us take a demonstration perhaps an experiment as bellow:
Take a comb and a piece of paper, comb your hairs bring the comb to next to paper, then we may able to lift that piece of paper. Well that’s not the force of gravity which causes the attraction between the comb and paper perhaps gravity doesn’t care whether we comb hair or not and when we shake the comb attached with piece of paper, the paper would fall down. So we would think there may be a new force that doesn’t overcome the gravitational force and felled down. But it’s actually a mistake, the fact is, the new force that we are talking about is stronger than the gravitation force.

In fact its enormous strong even though it’s so strong then we got to understand why the paper piece fell down when the comb would be shook, the reason is, the comb will try to pull the paper, but what is it that is trying to pull the paper piece down, guess what makes the entire earth will be trying to pull the paper down, the whole earth try to pull the paper down and when we shook the comb we would provide the necessary in need force along with the pull created by earth and hence the paper fell down. That is how we could compare the Gravitational field with the so called new force, the Electric force.

The electric force is much stronger than the gravity perhaps its takes the entire planet to compensate the tiny force created between the comb and a piece of paper. The whole idea about this new force is given by a coulomb’s law. It says that certain entities will have some properties called ‘charge’, if any of the object, say the comb and the paper, will have ‘q’, the charge that comb and the paper have, and will be measured in coulombs.

**Coulombs law:**

We now know that the coulomb is used to measure the charge, perhaps the Electrical charge. One coulomb can be given as “if we take a body of one coulomb of charge and another body of charge of one coulomb of charge and they are separated by one meter apart then the force is given as

$$F = \frac{q_1q_2}{4\pi\epsilon_0r^2} \rightarrow (5)$$

Where $q_1$ and $q_2$ are two point charges

$\epsilon_0$\rightarrow$\text{Permittivity of free space (8.854 } \times 10^{-12} \text{ } c^2 N m^{-2}}$

$r$\rightarrow$\text{the distance between them. The distance here it means the distance between the two point charges (entities) mathematically called as point charges.}$

The coulombs say that if one entity with some charge called $q_1$ and some other entity $q_2$ they will exert a force on each other as in equation (5)

The force of attraction and repulsion between them is simply depends on the charge of $q_1$ and $q_2$. Then we got to have to study the couple things related with charge

1. **q is conserved**: that says the charge doesn’t change with time, electric charge may migrate from object A to object B but sum total of them remains same. For example, chemical reaction of any reaction including big particle accelerator where the atoms collide and all residing stuff fly our when bombarded each other the charge of the output products will always be equal to the incoming charges.
2. And it is local, that means locally conserved, means to say it can be thought as above in only one frame of reference according to Special Theory of Relativity (STR).

3. **q is Quantized:** The electrical charge is not continuous. All the charges that we’ve ever seen are multiple of certain basic unit turns out to be $\pm 1.6 \times 10^{-19}$ coulombs. Therefore, the charge is granular but not continuous.

So considering all above discussions Coulomb law alone we even cannot answer the attraction between the comb and the paper due to the multiple charges. But we can explain the effect by combining the coulombs law and superposition. That means we can add the sum of all the charges on comb and paper and due to polarization the neutral paper can shift its negative charges towards the edge pointing to the positively charged comb.

From equation (4) and (5) we can say that

$$\frac{GM_1M_2}{r^2} = \frac{q_1q_2}{r^2} \frac{1}{4\pi \varepsilon_0} \rightarrow (6)$$

$$GM_1M_2 = q_1q_2 (9 \times 10^9) \rightarrow (7)$$

The force between an apple falling down to earth and the force between two charges could be related as equation (6). How can we verify the laws from equation (4) and (5) we can say the force would be measured by keeping q1 and q2 charges and force exerted on the charge and measuring the distance by keeping one charge fixed and other varied provided we must know the mass of the charges as discussed from equation (1) and (2)?

That’s how newton actually did found $\frac{1}{r^2}$ formula [equation (5)]. He found acceleration of the apple is $\frac{1}{3600}$ the acceleration of the moon towards the earth and the moon. The moon was 60 times farther than the apple considering the distance of falling apple from the earth’s surface when newton was formulating) and the $60^2$ is 3600 that’s how he found $\frac{1}{r^2}$.

Even we can know the force from the equation (5) also depends on the first power of q1 and q2.

We said gravity is mare weaker than the electric current. Now let us see at what number it is said so, we it knows how precisely the strength is being taken into context for comparison.

So take any two bodies, take two elementary particles for example, an electron and proton. These two attract each other gravitationally and electrically. So we can wright the equation as to as:

$$\frac{F_g}{F_e} = \frac{GM_ep}{r^2} \frac{q_ep}{4\pi \varepsilon_0} \rightarrow (8)$$
Where
$M_e = $ mass of electron
$M_p = $ mass of proton
$q_1 = $ charge of electron
$q_2 = $ charge of proton

Equation (8) implies
\[
\frac{F_g}{F_e} = \frac{G M_e M_p}{q_1 q_2 (9 \times 10^9)}
\]

By neglecting the pre factors equation (9) implies
\[
\frac{F_g}{F_e} = \frac{10^{-11} \times 10^{-27} \times 10^{-30}}{10^{-38} \times 10^{10}}
\]
\[
\frac{F_g}{F_e} = 10^{-40}
\]

(Reference source- Yale university)

So the gravity is $10^{-40}$ times weaker than the electric force.

The whole of the universe is made up of elementary particles, which carry the charges, precisely the electric force (yes, of course we are discussing about the force) and on another hand we have the gravity. So it is obvious to think that, why the gravity over comes against the electric force when compared with huge masses, when we consider the sun and the earth even though both massive objects have huge amount of elementary particles, they are seemingly considered to fall under the bound of gravitational force. The reason is, the elementary particles will do have their respective opposite charges, electrons will be neutralized by protons and hence cancels the electric force, for example, the presence of exact amount of electrical charges will make some metals and other objects like plastic a bad conductor, perhaps the element is electrically neutralized. Electric force of an object can be hidden by neutralizing the element. But the gravity doesn’t. If we take, for instance earth and an exo-planet (a planet having the physiological properties exactly like earth, has the mass, plant similar to earth) for our comparison, even though if both objects have same mass, they won’t be able to overcome the gravitational pull between them and cease to attract each other gravitationally. That is to say, the charges can be neutralized, but not the mass. Hypothetically we say that graviton is the elementary particle which cease to exist in the frame of quantum field, and causes the gravitational pull between the masses at the quantum level. The electron and proton in the wood may not attract, the mass of the proton and the mass of the electron is not going to cancel out, but their charges do. And hence we cannot hide mass, and everything on the universe has mass, hence have the gravitational force and attract each other gravitationally.