On the Logical Inconsistency of Einstein's Length Contraction

Stephen J. Crothers

Tasmania, Australia
thenarmis@yahoo.com
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ABSTRACT

Length contraction is a principal feature of the Special Theory of Relativity. It is purported to be independent of position, being a function only of uniform relative velocity, via systems of clock-synchronised stationary observers and the Lorentz Transformation. However, a system of stationary observers reports not length contraction but length expansion. Two observers in a system of clock-synchronised observers assign a common length contraction, but at the expense of time dilation and of being stationary. Systems of clock-synchronised stationary observers are logically inconsistent with the Lorentz Transformation. Consequently, the Theory of Relativity is false due to an insurmountable intrinsic logical contradiction.

1 Introduction

In previous papers [1, 2] I proved that Einstein's system of clock-synchronised stationary observers is logically inconsistent with the Lorentz Transformation. Herein I highlight that a system of stationary observers K report length expansion, not length contraction, and that two observers in a system of clock-synchronised observers report length contraction but not time dilation.

The Lorentz Transformation is,

$$\begin{aligned} &\tau = \beta\left(t - vx/c^2\right), \quad &\xi = \beta\left(x - vt\right), \\ &\eta = y, &\zeta = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}. \end{aligned} \tag{1}$$

According to Special Relativity a moving 'rigid body'* undergoes a length contraction in the direction of its motion. If the length of a body in the x-direction in Einstein's 'stationary system' K is l_0, then according to the 'stationary system' K the length of the very same body in the ξ-direction of the moving system k is $l_0' = l_0/\beta = l_0\sqrt{1 - v^2/c^2}$.

2 Einstein's rigid sphere

Einstein [3, §4] considered a rigid sphere of radius R:

> "*We envisage a rigid sphere*[1] *of radius R, at rest relatively to the moving system k, and with its centre at the origin of co-ordinates of k. The equation of the surface of this sphere moving relatively to the system K with velocity v is*
>
> $$\xi^2 + \eta^2 + \zeta^2 = R^2.$$
>
> *The equation of this surface expressed in x, y, z at the time $t = 0$ is*
>
> $$\frac{x^2}{\left(\sqrt{1 - v^2/c^2}\right)^2} + y^2 + z^2 = R^2.$$
>
> *A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion - viewed from the stationary system - the form of an ellipsoid of revolution with the axes*
>
> $$R\sqrt{1 - v^2/c^2}, R, R.$$
>
> "*Thus, whereas the Y and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear modified by the motion, the X dimension appears shortened in the ratio $1 : \sqrt{1 - v^2/c^2}$, i.e. the greater the value of v, the greater the shortening.*
>
> "[1] *That is, a body possessing spherical form when examined at rest.*"

Einstein's rigid sphere "*at rest relatively to the moving system k*" is illustrated in figure 1. The radius of the sphere at rest is R in all directions. Since Einstein's rigid sphere moves only in the X-direction, the radius R in that direction is purported to shorten to $R\sqrt{1 - v^2/c^2}$, according to his 'stationary system' K. This is easily seen by setting $y = z = 0$ in Einstein's equation for the "*ellipsoid of revolution*", from which it immediately follows that $x = R\sqrt{1 - v^2/c^2}$. Einstein's 'stationary system' K however contains observers at different locations. Einstein does not specify the location of any such observer of his distorted sphere. Evidently his length contraction is the same for all his clock-synchronised stationary observers since his contracted rigid sphere is "*viewed from the stationary system*".

*Although Einstein utilised rigid bodies, these bodies *change their lengths* when they are in motion.

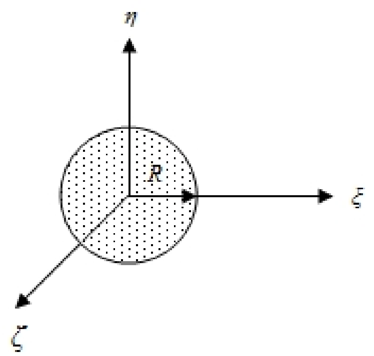

Fig. 1: Initial conditions: a rigid sphere of radius R centred at the origin of coordinates for the 'moving system' k. The sphere is at rest with respect to k. In the k system the sphere has the equation $\xi^2 + \eta^2 + \zeta^2 = R^2$. When $t = 0$ in Einstein's 'stationary system' K, the time $\tau = 0$ at the origin $\xi = 0$ but at $\xi = R$ the time is $\tau = -Rv/c^2$, by the Lorentz Transformation.

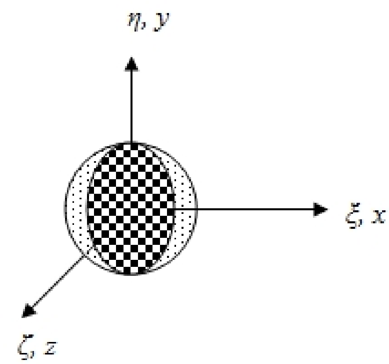

Fig. 2: Subsequent conditions: a rigid sphere of radius R centred at the origin of both coordinate systems. The sphere is at rest with respect to k but moving at a constant speed v with respect to K, in the common X-direction. The ellipsoid is the 'shortened sphere' observed from the Einstein's clock-synchronised stationary system K. In the k system the sphere has the equation $\xi^2+\eta^2+\zeta^2 = R^2$. In the K system it is not a sphere, but an ellipsoid, with equation $\frac{x^2}{(1-v^2/c^2)} + y^2 + z^2 = R^2$. Here the time $t = 0$ at all points in Einstein's 'clock-synchronised stationary system' K, but for the 'moving system' k the time is, according to K, $\tau = 0$ at $\xi = 0$ but $\tau = -Rv/c^2$ at $\xi = R$.

It is evident from Einstein's equation for "*an ellipsoid of revolution*" that his ellipsoid is centred at the origin of coordinates $x = y = z = 0$ for his 'clock-synchronised stationary system' K. Hence Einstein [3, §4] superposed the two coordinate systems for K and k respectively, so that their origins coincide at his 'clock-synchronised stationary system' time $t = 0$, illustrated in figure 2. In this case it is imagined that the sphere is moving at a constant speed v in the common X-direction according to Einstein's 'clock-synchronised stationary system' K.

Einstein set $t = 0$ at the common origin of coordinates, so that, by the Lorentz Transformation (1), $\xi = \beta x$. Consequently, at the common origin, $x = 0$ and $\xi = 0$. Referring to figure 2, when $t = 0$ at all time-synchronised points in the 'stationary system' K, at $\xi = 0$ the k-time is, according to K, $\tau = 0$, but at $\xi = R$ the k-time is $\tau = -Rv/c^2$, by the Lorentz Transformation. Einstein did not mention this*. If $t > 0$, then $\xi = \beta(x - vt)$ and the equation of the "*ellipsoid of revolution*" according to the 'stationary system' K is,

$$\frac{(x - vt)^2}{\left(\sqrt{1 - v^2/c^2}\right)^2} + y^2 + z^2 = R^2. \tag{2}$$

This ellipsoid is centred at $x = vt, y = 0, z = 0$ of the 'stationary system' K. The first term of equation (2) is a function of the 'time' t. To avoid this awkward problem, Einstein set $t = 0$. However, it follows from the Lorentz Transformation for a system of stationary observers that there is no place in the 'stationary system' K from which the moving sphere of radius R in k undergoes length contraction. All stationary observers report length expansion.

*Engelhardt [4] recently proved that Einstein's clock-synchronisation is inconsistent with the Lorentz Transformation.

Since length contraction supposedly occurs only in the direction of motion, consider a 'rigid rod' of length l_0 in the as yet 'stationary system' k and the 'stationary system' K, as shown in figure 3.

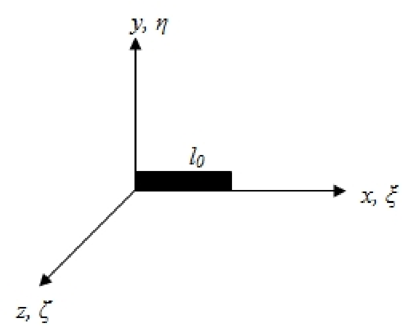

Fig. 3: A rigid rod of length l_0 in the stationary system K, and in the as yet stationary system k.

Now imagine the system k with rod to have a constant speed v in the positive direction of the x-axis of K, as shown in figure 4. Let the time t of the 'stationary system' K be reckoned from $t = 0$ when the y and η axes coincide. After a time $t > 0$ the k system advances to a distance vt from the origin of the K system, as shown in figure 4.

Now, according to Special Relativity, the length of the 'moving' rod l'_0 is the same at any time t and place x of observer in the 'stationary system' K, because length con-

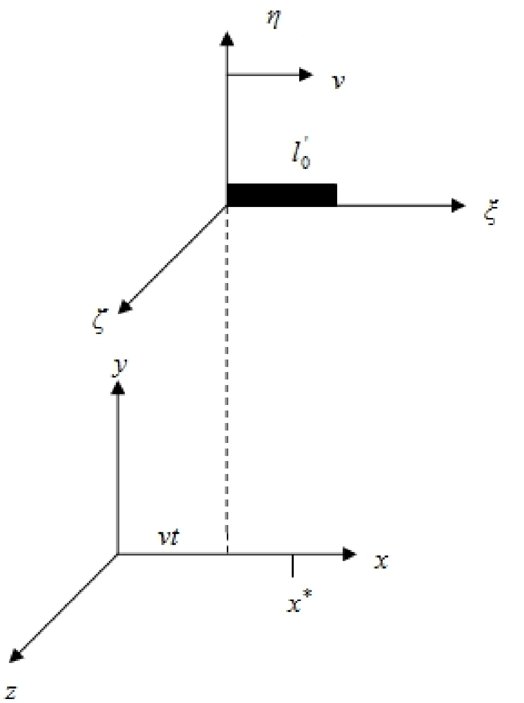

Fig. 4: After time $t > 0$ the k system advances a distance vt and the observers in system K determine the length l_0' of the moving rod from their vantage points x^*.

traction is independent of the value of t and position of the rod in either system, depending only on the constant relative speed v. According to the Lorentz Transformation (1), $\xi = \beta(x - vt)$. Thus, when $t = 0$, $x = \xi/\beta$, and so if $\xi = l_0$ at rest relative to the 'moving system' k, then $x = l_0' = l_0/\beta = l_0\sqrt{1 - v^2/c^2}$.

I have shown elsewhere that the Lorentz Transformation between systems of observers stationary with respect to their own systems is [1, §2],

$$\begin{aligned}
&\tau = \beta\left(t_\kappa - vx_\kappa/c^2\right),\ x_\kappa = \kappa x_1,\ \eta = y,\ \zeta = z,\\
&\xi_\kappa = \beta\left(x_\kappa - vt_\kappa\right) = \beta\left[\left(\kappa/\beta^2 + v^2/c^2\right)x_1 - vt_1\right],\\
&t_\kappa = t_1 + (\kappa - 1)\,vx_1/c^2,\\
&\beta = 1/\sqrt{1 - v^2/c^2},\ \kappa \in \Re.
\end{aligned} \tag{2}$$

Similarly I have previously shown [1, §5] that the Lorentz Transformation between systems of observers that are clock-synchronised with respect to their own systems is,

$$\begin{aligned}
&\tau_\kappa = \beta\left(t - vx_\kappa/c^2\right) = \kappa\tau_1, && \xi_\kappa = \beta\left(x_\kappa - vt\right),\\
&x_\kappa = (1 - \kappa)\,c^2t/v + \kappa x_1, && \eta = y,\ \zeta = z,\\
&\beta = 1/\sqrt{1 - v^2/c^2}, && 1 - v/c < \kappa < 1 + v/c.
\end{aligned} \tag{3}$$

At any instant of time in the stationary system K, let a rigid rod in the moving system k have a length $\Delta\xi = l_0$ when at rest relative to the moving system k. Then by (2), at any instant of time,

$$\Delta\xi = l_0 = \xi_\sigma - \xi_\rho = \frac{(\sigma - \rho)\,x_1}{\beta} = \frac{\Delta x}{\beta}.$$

Therefore,

$$\Delta x = \beta l_0 = \frac{l_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{4}$$

Hence, all the stationary observers x_σ of the stationary system K observe not length contraction, but length expansion.

By (3), at any instant of time,

$$\Delta\xi = \xi_\sigma - \xi_\rho = \beta\left(x_\sigma - x_\rho\right) = \beta\,(\sigma - \rho)\,x_1 = \beta\Delta x. \tag{5}$$

Therefore,

$$\Delta x = \frac{\Delta\xi}{\beta} = \Delta\xi\sqrt{1 - \frac{v^2}{c^2}}. \tag{6}$$

This is Einstein's 'length contraction' equation. Clock-synchronised observers x_σ and x_ρ of the system K observe the same length contraction, at the expense of being stationary observers and at the expense of time dilation [1, §7].

3 Conclusions

For $t \geq 0$ none of the observers in a system of stationary observers K report length contraction for a rigid body in the 'moving system' k. They all report a common length expansion. No system of stationary observers can be clock-synchronised and obey the Lorentz Transformation. For any $t \geq 0$ two observers in a 'clock-synchronised system' K observe the same 'length contraction', but at the expense of being stationary observers and at the expense of time dilation. Consequently Einstein's length contraction is inconsistent with the Lorentz Transformation. Einstein's assumption that a system of clock-synchronised stationary observers is consistent with the Lorentz Transformation is false. Hence, the Theory of Relativity is false.

References

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[3] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905

[4] Engelhardt, W., Einstein's third postulate, *Physics Essays*, **29**, 4 (2016)