# On the Logical Inconsistency of Einstein's Length Contraction

Stephen J. Crothers

Tasmania, Australia
thenarmis@yahoo.com
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**ABSTRACT**

Length contraction is a principal feature of the Special Theory of Relativity. It is purported to be independent of position, being a function only of uniform relative velocity, via the Lorentz Transformation. However, it is not possible for a system of clock-synchronised stationary observers to assign by the Lorentz Transformation, a common definite length to any object in a 'moving system'. Consequently, the Theory of Relativity is false due to an insurmountable intrinsic logical contradiction.

# 1 Introduction

In previous papers [1, 2] I proved that Einstein's system of clock-synchronised stationary observers is logically inconsistent with the Lorentz Transformation. Herein I assure by mathematical construction a system of clock-synchronised observers and assume the Lorentz Transformation, then prove that Einstein's 'length contraction' is false because there is no common determinable length contraction for all observers in the 'stationary system' $K$ at any given time $t$ of $K$.

The Lorentz Transformation is,

$$\begin{aligned} &\tau = \beta\left(t - vx/c^2\right), \qquad \xi = \beta\left(x - vt\right), \\ &\eta = y, \qquad \zeta = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}. \end{aligned} \tag{1}$$

According to Special Relativity a moving 'rigid body'* undergoes a length contraction in the direction of its motion. If the length of a body in the $x$-direction in the 'stationary system' $K$ is $l_0$, then according to the 'stationary system' $K$ the length of the very same body in the $\xi$-direction of the moving system $k$ is $l'_0 = l_0/\beta = l_0\sqrt{1 - v^2/c^2}$. However, at any time $t > 0$ of the 'stationary system' $K$ there is always a place $x^*$ in $K$ from which the length of the moving body is not $l_0/\beta$.

# 2 Einstein's rigid sphere

Einstein [3, §4] considered a rigid sphere of radius $R$:

> "*We envisage a rigid sphere*[1] *of radius R, at rest relatively to the moving system k, and with its centre at the origin of co-ordinates of k. The equation of the surface of this sphere moving relatively to the system K with velocity v is*
>
> $$\xi^2 + \eta^2 + \zeta^2 = R^2.$$
>
> *The equation of this surface expressed in x, y, z at the time t = 0 is*
>
> $$\frac{x^2}{\left(\sqrt{1 - v^2/c^2}\right)^2} + y^2 + z^2 = R^2.$$
>
> *A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion - viewed from the stationary system - the form of an ellipsoid of revolution with the axes*
>
> $$R\sqrt{1 - v^2/c^2}, R, R.$$
>
> "*Thus, whereas the Y and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear modified by the motion, the X dimension appears shortened in the ratio* $1 : \sqrt{1 - v^2/c^2}$, *i.e. the greater the value of v, the greater the shortening.*
>
> "[1] *That is, a body possessing spherical form when examined at rest.*"

Einstein's rigid sphere "*at rest relatively to the moving system k*" is illustrated in figure 1. The radius of the sphere at rest is $R$ in all directions. Since Einstein's rigid sphere moves only in the $X$-direction, the radius $R$ in that direction is purported to shorten to $R\sqrt{1 - v^2/c^2}$, according to the 'stationary system' $K$. This is easily seen by setting $y = z = 0$ in Einstein's equation for the "*ellipsoid of revolution*", from which it immediately follows that $x = R\sqrt{1 - v^2/c^2}$. The 'stationary system' $K$ however contains observers at different locations. Einstein does not specify the location of any such observer of his distorted sphere. Evidently his length contraction is the same for all his stationary observers since his contracted rigid sphere is "*viewed from the stationary system*".

It is evident from Einstein's equation for "*an ellipsoid of revolution*" that his ellipsoid is centred at the origin of coordinates $x = y = z = 0$ for the 'stationary system' $K$. Hence

*Although Einstein utilised rigid bodies, these bodies *change their lengths* when they are in motion.

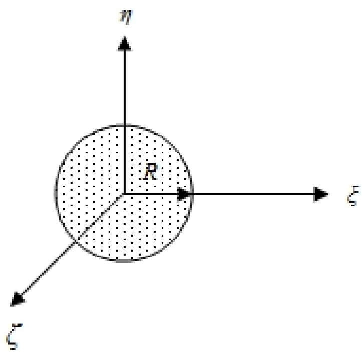


Fig. 1: Initial conditions: a rigid sphere of radius $R$ centred at the origin of coordinates for the 'moving system' $k$. The sphere is at rest with respect to $k$. In the $k$ system the sphere has the equation $\xi^2+\eta^2+\zeta^2=R^2$. When $t=0$ in the 'stationary system' $K$, the time $\tau=0$ at the origin $\xi=0$ but at $\xi=R$ the time is $\tau=-Rv/c^2$, by the Lorentz Transformation.

Einstein [3, §4] superposed the two coordinate systems for $K$ and $k$ respectively, so that their origins coincide at the 'stationary system' time $t=0$, illustrated in figure 2. In this case it is imagined that the sphere is moving at a constant speed $v$ in the common $X$-direction according to the 'stationary system' $K$.

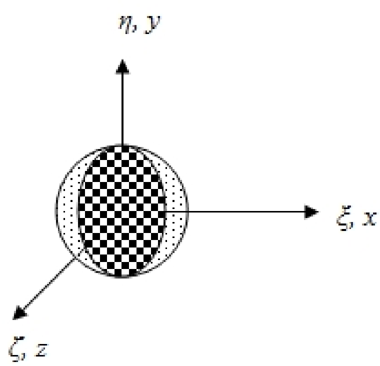


Fig. 2: Subsequent conditions: a rigid sphere of radius $R$ centred at the origin of both coordinate systems. The sphere is at rest with respect to $k$ but moving at a constant speed $v$ with respect to $K$, in the common $X$-direction. The ellipsoid is the 'shortened sphere' observed from the stationary system $K$. In the $k$ system the sphere has the equation $\xi^2+\eta^2+\zeta^2=R^2$. In the $K$ system it is not a sphere, but an ellipsoid, with equation $\frac{x^2}{(1-v^2/c^2)}+y^2+z^2=R^2$. Here the time $t=0$ at all time-synchronised points in the 'stationary system' $K$, but for the 'moving system' $k$ the time is, according to $K$, $\tau=0$ at $\xi=0$ but $\tau=-Rv/c^2$ at $\xi=R$.

Einstein set $t=0$ at the common origin of coordinates, so that, by the Lorentz Transformation (1), $\xi=\beta x$. Consequently, at the common origin, $x=0$ and $\xi=0$. Referring to figure 2, when $t=0$ at all time-synchronised points in the 'stationary system' $K$, at $\xi=0$ the $k$-time is, according to $K$, $\tau=0$, but at $\xi=R$ the $k$-time is $\tau=-Rv/c^2$, by the Lorentz Transformation. Einstein did not mention this. If $t>0$, then $\xi=\beta(x-vt)$ and the equation of the "*ellipsoid of revolution*" according to the 'stationary system' $K$ is,

$$\frac{(x-vt)^2}{\left(\sqrt{1-v^2/c^2}\right)^2}+y^2+z^2=R^2. \tag{2}$$

This ellipsoid is centred at $x=vt, y=0, z=0$ of the 'stationary system' $K$. The first term of equation (2) is not constant, but varies with the 'time' $t$. To avoid this awkward problem, Einstein set $t=0$. However, it follows from the Lorentz Transformation that for any time $t>0$ there is always a place $x^*$ in the 'stationary system' $K$, from which the moving sphere of radius $R$ in $k$ is, for instance, a sphere of radius $R$ in $K$.

Since length contraction supposedly occurs only in the direction of motion, consider a 'rigid rod' of length $l_0$ in the as yet 'stationary system' $k$ and the 'statonary system' $K$, as shown in figure 3.

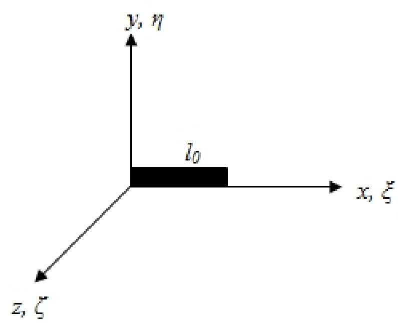


Fig. 3: A rigid rod of length $l_0$ in the stationary system $K$, and in the as yet stationary system $k$.

Attach the coordinate system for $k$ to the rod and imagine the system $k$ with rod to have a constant speed $v$ in the positive direction of the $x$-axis of $K$, as shown in figure 4. Let the time $t$ of the 'stationary system' $K$ be reckoned from $t=0$ when the $y$ and $\eta$ axes coincide. After a time $t>$ the $k$ system advances to a distance $vt$ from the origin of the $K$ system, for example, as shown in figure 5.

Now, according to Special Relativity, the length of the 'moving' rod $l_0'$ is the same at any time $t$ and place $x$ of observer in the 'stationary system' $K$, because length contraction is independent of the value of $t$ and position of the rod in either system, depending only on the constant relative speed $v$. According to the Lorentz Transformation (1), $\xi=\beta(x-vt)$. Thus, when $t=0$, $x=\xi/\beta$, and so if $\xi=l_0$ at rest relative to the 'moving system' $k$, then $x=l_0'=l_0/\beta$ $=l_0\sqrt{1-v^2/c^2}$. But when $t>0$,

$$l_0=\beta(x-vt)\,. \tag{3}$$

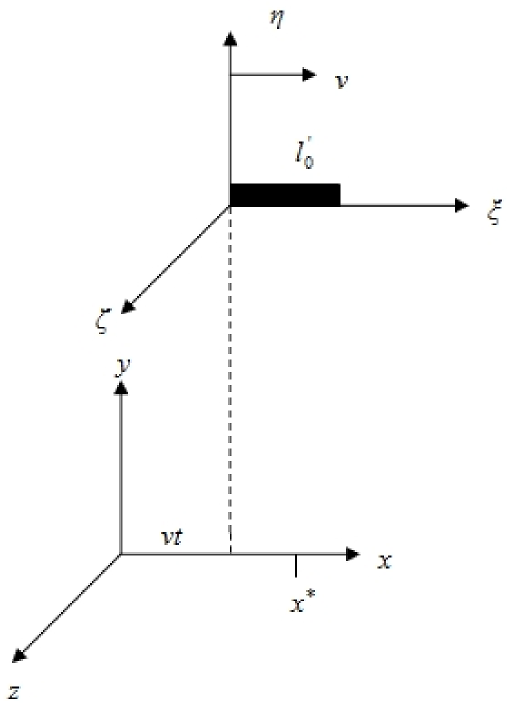


Fig. 4: After time $t > 0$ the $k$ system advances a distance $vt$ and the observers in system $K$ determine the length $l'_0$ of the moving rod from their vantage points $x^*$.

To assure a system of clock-synchronised observers set,

$$\sigma l_0 = \beta \left( x^* - vt \right), \tag{4}$$

where $0 \leq \sigma$. Solving (4) for $x^*$ gives,

$$x^* = \frac{\sigma l_0 + \beta vt}{\beta}. \tag{5}$$

When $t = 0$, $x^* = \sigma l_0/\beta$. If the length of the rigid rod at rest relative to the 'moving system' $k$ is $l_0$ there is always an observer located at $x^*$ in the clock-synchronised system $K$ from which the rod has the length $l'_0 = \sigma l_0$, $0 \leq \sigma$, for any synchronised time $t$ in $K$; sample values tabulated.

| $\sigma$ | $x^*$ | $l'_0$ |
|---|---|---|
| 0 | $vt$ | 0 |
| 1/2 | $(l_0 + 2\beta vt)/2\beta$ | $l_0/2$ |
| 1 | $(l_0 + \beta vt)/\beta$ | $l_0$ |
| 2 | $(2l_0 + \beta vt)/\beta$ | $2l_0$ |
| $1/\beta$ | $\left(l_0 + \beta^2 vt\right)/\beta^2$ | $l_0/\beta$ |
| $\beta$ | $(l_0 + vt)$ | $\beta l_0$ |

Only for the observer at $x^* = \left(l_0 + \beta^2 vt\right)/\beta^2$ does Einstein's 'length contraction' equation hold. Therefore, only for the observer at $x = R/\beta^2$ does Einstein's 'length contraction' hold for his moving rigid sphere, not for his observer at $x = 0$, not for his observer at $x = R/\beta$, not for his observer at $x = R$, or anywhere else, as seen by setting $t = 0$ in the table of sample values. Einstein's 'length contraction' depends upon the position of the observer in $K$. Furthermore, although clock-synchronised, observers $x^*$ are not stationary, contrary to Einstein's assumption of a system of clock-sychronised stationary observers $K$.

## 3 Conclusions

For $t \geq 0$ none of the observers in the 'stationary system' $K$ can assign any common definite length $l'_0$ to a body in the 'moving system' $k$. Consequently there is no common determinable length contraction from the 'stationary system' $K$. Consequently Einstein's length contraction is inconsistent with the Lorentz Transformation. Einstein's assumption that a system of clock-synchronised stationary observers is consistent with the Lorentz Transformation is false. Hence, the Theory of Relativity is false.

## References

[1] Crothers, S.J., On the Logical Inconsistency of the Special Theory of Relativity, 6th March 2017, http://vixra.org/abs/1703.0047

[2] Crothers, S.J., On the Logical Inconsistency of Einstein's Time Dilation, 9th March 2017, http://vixra.org/abs/1703.0093

[3] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905