Electron Stability Approach to Finite Quantum Electrodynamics

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This paper analyses electron stability and applies the resulting stability principle to resolve divergence issues in quantum electrodynamics without mass and charge renormalization. Stability is enforced by requiring that the positive electromagnetic field energy be balanced by a negative interaction energy between the observed electron charge and a local vacuum potential. Then in addition to the observed core mechanical mass m, an electron system consists of two electromagnetic mass components $m^{\pm}_{em}$ of equal magnitude M but opposite sign; consequently, the net electromagnetic mass is zero. Two virtual, electromagnetically dressed mass levels $m \pm M$, constructed to form a complete set of mass levels and isolate the electron-vacuum interaction, provide essential S-matrix corrections for radiative processes involving infinite field actions. Total scattering amplitudes for radiative corrections are shown to be convergent in the limit $M \to \infty$ and equal to renormalized amplitudes when Feynman diagrams for all mass levels are included. In each case the infinity in the core mass amplitude is canceled by the average amplitude for dressed mass levels, which become separated in intermediate states and account for the stabilizing interaction energy between an electron and its surrounding polarized vacuum.

I. INTRODUCTION

A long-standing enigma in particle physics is how an elementary charged particle such as an electron can be stable in the presence of its own electromagnetic field (see [1, 2] and cited references). Critical accounting for electron stability is essential since radiative corrections in quantum field theory involve self-interactions that can change the mass and charge of an electron. This analysis seeks to identify, understand, and account for the hidden interaction that energetically stabilizes an electron such that its mass and charge assume their physically observed values.

The agreement between renormalized quantum electrodynamics (QED) theory and experiment confirms the effect of vacuum fluctuations on the dynamics of elementary particles to astounding accuracy. For example, electron anomalous magnetic moment calculations currently agree with experiment to about 1 part in a trillion [3, 4]. This achievement is the result of more than six decades of effort since the relativistically invariant form of the theory took shape in the works of Feynman, Schwinger, and Tomonaga (see Dyson's unified account [5]). The agreement leaves little doubt that QED predictions are correct; however, the renormalization technique [6, 7] used to overcome divergence issues in radiative corrections offers little insight into the underlying physics behind electron stability in the high-energy regime. Recall that divergent integrals occur in scattering amplitudes for self-energy processes and arise in sums over intermediate states of arbitrarily high-energy virtual particles. This stymied progress until theoretical improvements were melded with renormalization to isolate the physically significant parts of radiative corrections by absorbing the infinities into the electron mass and charge. Although the renormalization method used to eliminate ultraviolet divergences (UVD) results in numerical predictions in remarkable agreement with experiments, redefinition of fundamental physical constants remains an undesirable feature of the current theory.

Our main purpose is to develop an alternative to mass and charge renormalization in QED. We begin by revisiting the classical self-energy problem where we define an energetically stable charge. From the resulting stability principle, we construct two virtual, electromagnetically dressed mass levels to isolate the interaction between the observed charge and the polarized vacuum. Since all UVD may be resolved into primitive divergences [8], we limit our scope to these. The total scattering amplitude for a radiative correction with a primitive divergence includes contributions from core and dressed core mass levels; therefore, three Feynman diagrams are defined, one for each mass level. In particular, S-matrix corrections for the electromagnetically dressed core are simply constructed using core amplitudes from the literature and account for the action of the vacuum back on the electron via an opposing vacuum current. After defining divergent integrals for dressed core amplitudes, we apply the theory to radiative processes to verify that renormalized results are obtained for vacuum polarization, electron self-energy, and vertex corrections.

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II. FORMULATION

Regarding an electron as a point particle [9], the classical electrostatic self-energy $e^2/2a \equiv \alpha\Lambda_\circ$ diverges linearly as the shell radius $a \to 0$, or energy cutoff $\Lambda_\circ \to \infty$, where $-e$ is the charge and $\alpha = e^2/4\pi\hbar c$ is the fine structure constant. However, Weisskopf [10, 11] showed using Dirac's theory [12] that the charge is effectively dispersed over a region the size of the Compton wavelength due to pair creation in the vacuum near an electron, and the self-energy only diverges logarithmically. Feynman's calculation [13] in covariant QED yields an electromagnetic mass-energy

$$m_{em} = \frac{3\alpha m}{2\pi}\left(\ln\frac{\Lambda_\circ}{mc^2} + \frac{1}{4}\right) , \tag{1}$$

where m is the electron mass. In the absence of a compensating negative energy, (1) signals an energetically unstable electron. It is the key ultraviolet divergence problem in QED, whose general resolution will result in convergent amplitudes for all radiative corrections containing primitive divergences. In this section we derive a stability condition and apply it to develop corrections to scattering amplitudes for divergent processes.

To ensure that the total electron mass is its observed value, renormalization theory posits that a negatively infinite 'bare' mass must exist to counterbalance m_{em}. For lack of physical evidence, negative matter is naturally met with some skepticism (see Dirac's discussion [14] of the classical problem, for example). Nevertheless, energies that hold an electron together are expected to be negative, and we can understand their origin by first considering the source for the electrical energy required to assemble a classical charge in the rest frame. Recall that the work done in assembling a charge from infinitesimal parts is equal to the electromagnetic field energy. Since the agents that do the work must draw an equivalent amount of energy from an external energy source (well), the well's energy is depleted and the total energy

$$\mathcal{E} = mc^2 + \mathcal{E}^+_{em} + \mathcal{E}_w \tag{2}$$

of the system including matter, electromagnetic field $\mathcal{E}^+_{em}$, and energy well $\mathcal{E}_w$ is constant. Now consider an electron and suppose that the surrounding vacuum is polarized to form a potential energy well. Then, the resulting vacuum potential Φ_{vac} confines the observed core charge akin to a spherical capacitor as shown in Fig. 1, and the interaction energy

$$\mathcal{E}_w \to \mathcal{E}^-_{em} \equiv -e\Phi_{vac} \tag{3}$$

is assumed to just balance $\mathcal{E}^+_{em}$ resulting in a stability condition

$$\begin{aligned} \mathcal{E}^+_{em} + \mathcal{E}^-_{em} &= 0 \\ m^+_{em} + m^-_{em} &= 0 \, , \end{aligned} \tag{4}$$

where the mass-energy equivalence $\mathcal{E}^\pm_{em} = m^\pm_{em}c^2$ has been used to obtain an equivalent expression in terms of electromagnetic masses. Therefore, the net mass-energy of a free electron is attributed entirely to the observed core mechanical mass m. In contrast to Poincaré's theory [15] wherein internal non-electromagnetic stresses hold an electron together, external vacuum electrical forces are assumed to provide charge stabilization and energy balance via a steady state polarization field surrounding the electron. Corresponding to a divergent self-action process, we require a mechanism whereby the core charge interacts locally with the polarized vacuum according to (3).

The energy of the core charge in the potential well of Fig. 1 is

$$\mathcal{E}^-_{core} = mc^2 + \mathcal{E}^-_{em} \equiv m_b c^2 \, , \tag{5}$$

where m_b may be identified with the bare mass, and

$$m_b + m^+_{em} = m \tag{6}$$

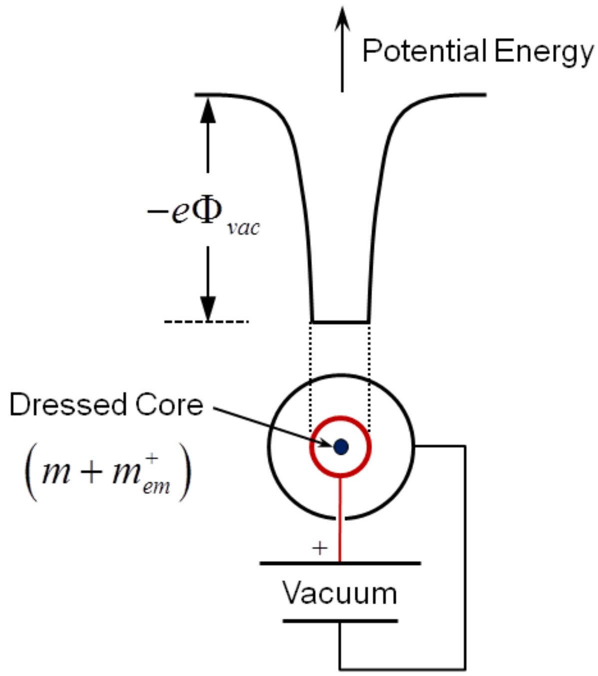

FIG. 1: Effective vacuum potential confines core electron charge similarly to spherical capacitor. Since the stability principle requires $\mathcal{E}_{em}^{+}-e\Phi_{vac}=0$, the total energy of core electron in the well and dressed in its electromagnetic field is just its observed mass-energy.

captures the mass renormalization condition which is equivalent to (2) with (3) and (4). However, notice that (4) is a more restrictive condition than (6) if (5) is not given. The bare mass corresponds to a core electron dressed in negative electromagnetic energy; hence, its characterization as a 'bare mechanical mass' is a misnomer (see [16] for example). Only the core mass is observable, and only it is expected to appear in the Lagrangian if one takes (4) seriously. In renormalization theory, however, one starts with a bare electron, self-interaction dresses it with positive electromagnetic energy, and (6) is subsequently applied to redefine the mass. On the other hand, suppose we start with the observed electron charge; then taking into account (2), (3), and (4), m_{em}^{+} and m_{em}^{-} are always present, and the total mass reduces to the observed core mechanical mass. Starting with this premise, we can formulate a finite theory of radiative corrections that accounts for all possible electromagnetically dressed intermediate states, and no asymmetry necessitating a redefinition of mass and charge is introduced. For the ensuing development, relativistic notation defined in [17] is employed, and natural units are assumed; that is, $\hbar=c=1$.

Equations (2) and (4) suggest that a stable electron consists of three rest mass components: a core mass m and two electromagnetic masses $m_{em}^{\pm}$ that are assumed large in magnitude but finite until the final step of the development. We can think of $m_{em}^{\pm}$ as components of an electromagnetic vacuum (zero net energy) which are tightly bound to the core mass and inseparable from the core and each other, at least for finite field actions. Considering all non-vanishing masses constructed from the set $\{m,\, m_{em}^{+},\, m_{em}^{-}\}$, we are led to define a complete set of mass levels $m+\lambda M$, where $\lambda=\{0,\pm 1\}$ and $M\equiv|m_{em}^{\pm}|$. Associated 4-momenta are $p+\lambda P_M$, where $\{p,\, P_M\}$ correspond to $\{m,\, M\}$, respectively.

Consider a free particle state $|p,\, m\rangle$ with momentum satisfying $p^2\equiv p_\mu p^\mu=m^2$, where $p^\mu=(p^\circ,\,\vec{p})$ and $p_\mu=g_{\mu\nu}p^\nu$ are contravariant and covariant momentum 4-vectors, respectively. We employ the relativistic normalization

$$\langle p',\, m\,|p,\, m\rangle=2E\left(\vec{p},\, m\right)(2\pi)^3\,\delta\left(\vec{p}-\vec{p}\,'\right)\ ,$$

where $E\left(\vec{p},\, m\right)=\sqrt{\vec{p}^{\,2}+m^2}$. Spin is omitted in $|p,\, m\rangle$ since it is inessential to the subsequent development, and the rest mass is included because it is the fundamental particle characteristic which varies in DCM corrections to the S-matrix [see Eq. (16)]. Metric tensor $g_{\mu\nu}$ has non-zero components

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1\,.$$

Now suppose we regard an electron as a superposition

$$|\psi_{dcm}\rangle = \frac{1}{\sqrt{2}} \sum_{\lambda=\pm1} |\Upsilon_\lambda(p)\rangle \tag{7}$$

of electromagnetically dressed core mass (DCM) states

$$|\Upsilon_\lambda(p)\rangle = |p + \lambda P_M\,,\, m + \lambda M\rangle\ , \tag{8}$$

wherein the 4-momentum is dispersed per an uncertainty $\Delta p \equiv \lambda P_M$. Dressed core mass states are normalized according to

$$\begin{aligned}\langle \Upsilon_{\lambda'}(p')|\, \Upsilon_\lambda(p)\rangle &= 2E\left(\vec{p} + \lambda \vec{P}_M\,,\, m + \lambda M\right)(2\pi)^3\,\delta\left(\vec{p} - \vec{p}\,' + (\lambda - \lambda')\,\vec{P}_M\right)\,, \\ &\simeq 2E\left(\vec{P}_M\,,\, M\right)(2\pi)^3\,\delta\left(\vec{p} - \vec{p}\,'\right)\delta_{\lambda\lambda'}\end{aligned}$$

where the latter form follows upon assuming $M \gg m$ and requiring the vector components to satisfy

$$\left|P_M^i\right| \gg \left|p^i - p'^i\right|\ ,\ i = 1,2,3$$

thereby excluding a zero in the delta-function argument at infinity for $\lambda' \neq \lambda$. The expected momentum and mass are given by

$$\frac{\langle \psi_{dcm}\,|\{p_{op}, m_{op}\}|\,\psi_{dcm}\rangle}{\langle \psi_{dcm}\,|\psi_{dcm}\,\rangle} = \{p, m\}\ ,$$

where $\{\,p_{op}, m_{op}\}$ are corresponding operators. Therefore, the composite state (7) is energetically equivalent to the core mass state $|p,m\rangle$ as required by (2) and (4). A core electron dressed with positive or negative energy as in (8) is a transient state that is sharply localized within a spacial interaction region $r \simeq \hbar/Mc$ in accordance with Heisenberg's uncertainty principle [18] $\Delta p^\mu \Delta x^\mu \geq \hbar/2$ (no implied sum over μ). Scattering amplitudes for low-energy processes are assumed unaffected because the energies are insufficient to induce a separation of tightly bundled states (8) in (7). For infinite field actions, however, DCM states may become separated in intermediate states with infinitesimally small lifetimes; in this case, we shall need to account for both core and DCM scattering amplitudes. To account for all possible intermediate states in QED and satisfy (4), both mass levels $m \pm M$ are required; this generalizes the classical model depicted in Fig. 1 which assumed that only an electron dressed with positive energy interacts with the vacuum potential well.

Since the interaction region reduces to a point as $M \to \infty$ for DCM states, self-interaction effects vanish, and a dressed electron interacts only with the polarized vacuum. The vacuum potential is generated by a net positive current in close proximity to the core electron charge since $\Phi_{vac} > 0$. Therefore, suppose a dressed electron is located at space-time position x_1 such that it is constrained to interact only with an opposing vacuum current as indicated in Fig. 2. The current density at a neighboring point $x_2 \neq x_1$ is distinct from that of the dressed core and reversed in sign; that is,

$$\mathrm{sgn}\left[j_\mu(x_2)\right] = -\mathrm{sgn}\left[j_\mu(x_1)\right]\ . \tag{9}$$

With core current defined by the normal product [19, 20]

$$j_\mu(x_1) = -\frac{e}{2}\left[\bar{\psi}\gamma_\mu\psi - \bar{\psi}_c\gamma_\mu\psi_c\right]_{x_1} = -eN\left[\bar{\psi}\gamma_\mu\psi\right]_{x_1}\ ,$$

where γ^μ are Dirac matrices, the vacuum current operator at x_2 may be generated by interchanging the field operator ψ with its charge conjugate ψ_c to satisfy (9) and model an exchange of core and vacuum electrons via the e^+e^- annihilation process suggested in Fig. 2 , then

$$j_\mu(x_2) = eN\left[\bar{\psi}\gamma_\mu\psi\right]_{x_2}\ .$$

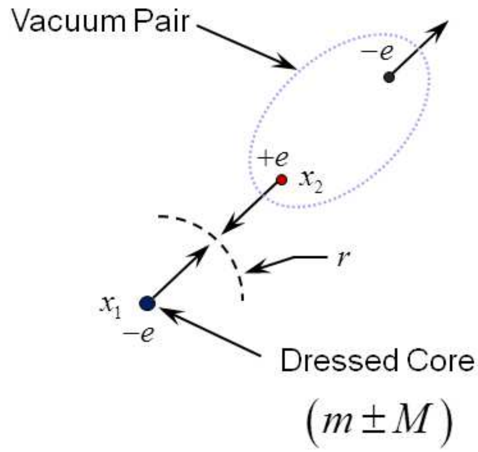

FIG. 2: Dressed core electron interacts with opposing vacuum current resulting in an exchange of the core and vacuum electrons and a sign reversal of the DCM scattering amplitude relative to the core.

Similarly to (9), the Hamiltonian density at nearby points must satisfy

$$\mathrm{sgn}\left[\mathcal{H}_{int}(x_2)\right] = -\mathrm{sgn}\left[\mathcal{H}_{\mathrm{int}}(x_1)\right] \ , \tag{10}$$

where $\mathcal{H}_{\mathrm{int}}(x) = j_\mu(x)\, A^\mu(x)$ in the interaction representation [21], and $A^\mu(x)$ is the radiation field. From (10) we anticipate a sign reversal in the DCM scattering amplitude relative to that for the core mass since 2^{nd} order S-matrix [8] corrections involve a product $\mathcal{H}_{int}(x_1)\, \mathcal{H}_{int}(x_2)$.

For radiative corrections containing primitive divergences, evaluation of S-matrix corrections for DCM states entails a core mass replacement

$$m \to m + \lambda M \tag{11}$$

in fermion lines internal to loops as indicated in Fig. 3; that is, in each fermion propagator [22]

$$iS_F(p, m) = \frac{i}{\not p - m + i\varepsilon} \ ,$$

where $\not p = \gamma_\mu p^\mu$. Resulting loop-operator amplitudes are averaged over mass levels; that is, $\lambda = \pm 1$. For an external line entering a loop, the momentum is similarly modified

$$p \to p + \lambda P_M \ , \tag{12}$$

since the propagator is required to have a pole at $m + \lambda M$. For processes containing infrared divergences, in which a fictitious photon mass $m_\gamma \neq 0$ is introduced [13] to regulate singularities for soft photon emissions, inspection of fermion self-energy and vertex functions in Sec. IV reveals that a replacement

$$m_\gamma \to m_\gamma + \lambda M_\gamma \tag{13}$$

is also needed in the modified photon propagator

$$iD_F^{\alpha\beta}(k) = \frac{-ig^{\alpha\beta}}{k^2 - m_\gamma^2 + i\varepsilon} \ .$$

Since resulting amplitudes involve a ratio m/m_γ, we require $M_\gamma = \eta m_\gamma$ with $\eta = M/m$ to ensure reduction to known results.

The total 2^{nd} order loop-operator associated with a self-energy or vertex part is defined by

$$\Omega = \Omega_{core} + \Omega_{dcm} \ , \tag{14}$$

where Ω_{core} accounts for self-interaction effects involving the core mass and Ω_{dcm} enforces stability via interaction of DCM states with the polarized vacuum. Ω_{dcm} is evaluated by substituting (11), (12), and (13) into known Ω_{core}. In addition to mass m, Ω_{core} depends on external momenta $\{p, q, k\}$ for Feynman diagrams in Fig. 3. For notational simplicity, any dependence on an external momentum parameter is suppressed during construction of Ω_{dcm} because $\{p, q\}$ are implicitly dependent on the core mass. Since Ω_{core} and Ω_{dcm} are both divergent for loop corrections, their improper integrals must be regulated using an energy cutoff $\Lambda_\circ$ or by dimensional regularization. Assuming an energy cutoff, the net amplitude (14) is convergent and reduces to expected results if we define

$$\Omega = \lim_{\Lambda_\circ \to \infty} \left[\Omega_{core}\left(m, \Lambda_\circ\right) + \Omega_{dcm}(m, \Lambda_\circ) \right] \ , \tag{15}$$

where

$$\Omega_{dcm}\left(m, \Lambda_\circ\right) = -\frac{1}{2} \lim_{\eta \to \infty} \sum_{\lambda = \pm 1} \Omega_{core}\left(m + \lambda M, \, \Lambda\right)\Big|_{M = \eta m, \, \Lambda = \eta \Lambda_\circ} \ . \tag{16}$$

The overall minus sign in (16) ensures that the core charge associated with a DCM state interacts with an opposing vacuum current as required by (9). Scaling rules

$$M = \eta m \tag{17}$$

$$\Lambda = \eta \Lambda_\circ \tag{18}$$

are required for consistent definition of the integrals – they ensure that $\Lambda \gg M$ for arbitrarily large M, synchronize cutoff to $\Lambda_\circ$, and yield a well defined limit as $\eta \to \infty$ in (16). As verified in Sec. IV, the operator Ω_{dcm} is independent of $\{P_M, M\}$ for $M \gg m$. In contrast to the regulator technique of Pauli and Villars [23], the above method employs physically meaningful DCM levels (albeit virtual only), and the same principle applies to all self-energy processes in QED without introduction of auxiliary constraints.

III. DIVERGENT INTEGRALS

Here we develop integration formulae required for evaluation of DCM corrections using cutoff and dimensional regularization. In the p-representation, loop diagrams involve four-dimensional integrals over momentum space, and the real parts of scattering amplitudes contain integrals of the form [24]

$$D\left(\Delta\right) = \frac{1}{i\pi^2} \int \frac{d^4 p}{\left(p^2 - \Delta\right)^n} = \frac{\left(-1\right)^n}{\pi^2} \int \frac{d^4 p_\varepsilon}{\left(p_\varepsilon^2 + \Delta\right)^n} \ , \tag{19}$$

where Δ depends on the core mass, momentum parameters external to the loop, and integration variables. On the right side of (19), a Wick rotation has been performed via a change of variables $p = (i p_\varepsilon^\circ, \vec{p}_\varepsilon)$, so that the integration can be performed in Euclidean space where $p_\varepsilon^2 = p_\varepsilon^\circ p_\varepsilon^\circ + \vec{p}_\varepsilon \cdot \vec{p}_\varepsilon$. Integrals for the divergent case ($n = 2$) must be regulated such that they are consistently defined for core and dressed core masses. For the core mass, D is regularized using a cutoff $\Lambda_\circ$ on $s = |p_\varepsilon|$. In four-dimensional polar coordinates, we have

$$D\left(\Delta, \Lambda_\circ\right) = \frac{1}{\pi^2} \int d\Omega \int_0^{\Lambda_\circ} ds \, \frac{s^3}{\left[s^2 + \Delta\right]^2} \ . \tag{20}$$

For DCM states, Δ depends on $|m \pm M| \simeq \eta m$ with $\eta \gg 1$, and the domain of integration in (20) must be scaled according to (18); consequently, we need to evaluate

$$D_{dcm} = D\left[\Delta\left(\eta m\right), \, \eta \Lambda_\circ\right] \ .$$

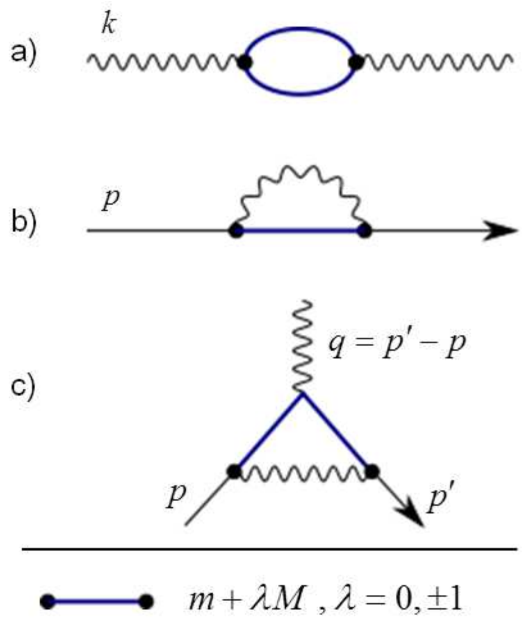

FIG. 3: Baseline radiative corrections a) Vacuum polarization, b) Fermion self-energy, and c) Vertex involve the core mass only in internal fermion lines. Two additional diagrams, obtained by replacing the core mass with electromagnetically dressed mass levels $m \pm M$, are required for each radiative process to account for interaction with an opposing vacuum current and ensure stability.

With a change of variables $s = \eta t$ and taking the limit $\eta \to \infty$, we obtain

$$D_{dcm} = D\left(\Delta_\circ, \Lambda_\circ\right) \ , \tag{21}$$

where

$$\Delta_\circ = \lim_{\eta \to \infty} \eta^{-2} \Delta(\eta m) \ . \tag{22}$$

For example, considering the standard divergent integral [24]

$$D_\circ \equiv D\left(\Delta = m^2, \Lambda_\circ\right) = \ln \frac{\Lambda_\circ^2}{m^2} - 1 + O\left(\frac{m^2}{\Lambda_\circ^2}\right) \ , \tag{23}$$

we see that $D_\circ$ is invariant under scaling rules (17) and (18); that is,

$$D_\circ = D\left(M^2, \Lambda\right) \ . \tag{24}$$

This result finds immediate application in the reduction of the fermion self-energy correction using Dyson's expansion method [8]

$$\Sigma_{core} = mA + (\not{p} - m)\, B + \Sigma \, ,$$

where $\{A, B\}$ depend on $D_\circ$, and Σ is convergent; refer to Jauch & Rohrlich [24]. Applying (16), (24), and noting for convenience from (39) that

$$\lim_{M \to \infty} \sum_{\lambda = \pm 1} \Sigma\left(m + \lambda M\right) = 0 \, ,$$

it follows that

$$\Sigma_{dcm} = -\left[mA + \left(\not{p} - m\right)B\right] \ ,$$

which shows that $\Sigma = \Sigma_{core} + \Sigma_{dcm}$ is correctly isolated.

In contrast to the cutoff method, dimensional regularization evaluates a Feynman diagram as an analytic function of space-time dimension d. For $n = 2$ and $d^4p \rightarrow d^dp$ in (19), D may be evaluated using [17, 25]

$$\begin{aligned} D\left(\Delta, \sigma\right) &= \pi^{-\sigma}\Gamma\left(\sigma\right)\Delta^{-\sigma} \\ &= \frac{1}{\sigma} - \gamma - \ln\Delta + O\left(\sigma\right) \ , \end{aligned} \tag{25}$$

where $\sigma = 2 - d/2$ and $\gamma = 0.577...$ is Euler's constant. For $\sigma \neq 0$, the limit $\Lambda_\circ \rightarrow \infty$ may be taken since σ regulates the integral. For DCM states, D_{dcm} must yield consistent results for both cutoff and dimensional regularization methods. Considering the requirements used to derive (21) and employing appendix formulae in [25], we conclude

$$D_{dcm} = D\left(\Delta_\circ, \sigma\right) \ . \tag{26}$$

Therefore, the net S-matrix correction computed from (15) is convergent and involves a factor

$$\delta D = D - D_{dcm} = -\ln\frac{\Delta}{\Delta_\circ} \ , \tag{27}$$

where we have manually negated D_{dcm} as required by (16). For examples, compare (27) with the photon and fermion self-energy expressions in (35) and (39).

IV. APPLICATION TO LOOP PROCESSES

Let us apply the foregoing theory with integration formulae given above to verify that the net amplitudes for radiative corrections are convergent and agree with results obtained via renormalization theory. To this end, the approximations

$$\left(m + \lambda M\right)^2 \simeq \eta^2 m^2 \tag{28}$$

$$\left(p + \lambda P_M\right)^2 \simeq P_M^2 = M^2 + \delta\left(P_M^2\right) \simeq \eta^2 m^2 \tag{29}$$

$$\left(m_\gamma + \lambda M_\gamma\right)^2 \simeq \eta^2 m_\gamma^2 \ , \tag{30}$$

utilizing (17) with $\eta \gg 1$, will be useful for reduction of DCM corrections. They ensure that the regulated integral D_{dcm} in (21) or (26) and Ω_{dcm} in (16) are independent of individual mass levels ($\lambda = \pm 1$) for $M \gg m$. In the expansion of P_M^2 about M^2 on the right side of (29), the off-shell term $\delta\left(P_M^2\right)$ is assumed bounded and therefore negligible compared to M^2. Dimensional and cutoff regularization approaches will be used to illustrate the method.

A. Vacuum polarization

Fig. 3 (a) results in a photon propagator modification [8]

$$iD_F^{\alpha\beta} \rightarrow iD_F^{\alpha\beta} + iD_F^{\alpha\mu}\left(-i\Pi_{\mu\nu}\right)iD_F^{\nu\beta} \ ,$$

where $\Pi_{\mu\nu} \equiv \Pi_{\mu\nu}^{core} + \Pi_{\mu\nu}^{dcm}$ is a polarization tensor generalized to include the DCM correction, and whose core mass term

$$\Pi_{\mu\nu}^{core}\left(k,\,m\right)=\frac{ie^{2}}{\left(2\pi\right)^{4}}\int d^{4}p\,\mathrm{Tr}\left[\gamma_{\mu}S_{F}(p,m)\gamma_{\nu}S_{F}(p-k,m)\right] \tag{31}$$

follows from the Feynman-Dyson rules [5, 13]. In consequence of Lorentz and gauge invariance [7] or by direct calculation, it factors into

$$\Pi_{\mu\nu}^{core}\left(k,\,m\right)=\Pi_{core}\left(k^{2},\,m\right)\left(k_{\mu}k_{\nu}-g_{\mu\nu}k^{2}\right)\ , \tag{32}$$

where $\Pi_{core}\left(k^{2},m\right)$ is a scalar function. After dimensional regularization, reduction using Dirac matrix algebra, and Feynman parameterization, (25) is employed to cast Π_{core} into a form equivalent to that given in Mandl & Shaw [17]

$$\Pi_{core}\left(k^{2},\,m\right)=\frac{2\alpha}{\pi}\int_{0}^{1}dz\;z\left(1-z\right)D\left(\Delta,\sigma\right)\ , \tag{33}$$

where

$$\Delta=m^{2}-k^{2}z\left(1-z\right)\ .$$

Applying (16) and (26) we obtain

$$\begin{aligned}\Pi_{dcm}&=-\frac{1}{2}\lim_{\eta\to\infty}\left[\Pi_{core}\left(k^{2},\,m+\eta m\right)+\Pi_{core}\left(k^{2},\,m-\eta m\right)\right]\\&=-\frac{2\alpha}{\pi}\int_{0}^{1}dz\;z\left(1-z\right)D\left(\Delta_{\circ},\sigma\right)\ ,\end{aligned} \tag{34}$$

where

$$\Delta_{\circ}=m^{2}$$

follows from (22) using (28). We see that (34) is equivalent to the subtracted core amplitude evaluated on the light cone

$$\Pi_{dcm}=-\Pi_{core}(k^{2}=0,\,m)\,,$$

which is associated with a correction to the bare charge in renormalization theory, but here the correction represents an interaction between the core electron charge associated with a transient DCM state and a polarization current. Combining (33), (34), and using (25), we obtain

$$\begin{aligned}\Pi&=\Pi_{core}+\Pi_{dcm}\\&=-\frac{2\alpha}{\pi}\int_{0}^{1}dz\;z\left(1-z\right)\ln\left[1-\frac{k^{2}z\left(1-z\right)}{m^{2}}\right]\end{aligned} \tag{35}$$

in agreement with the renormalization result [17].

B. Fermion self-energy

The fermion self-energy operator for the core mass corresponding to the Feynman diagram in Fig. 3 (b) with $\lambda=0$ is given by

$$\Sigma_{core}\left(p,m\right)=\frac{-ie^{2}}{\left(2\pi\right)^{4}}\int d^{4}k\;\gamma_{\mu}\,S_{F}\left(p-k,m\right)\,\gamma^{\mu}\frac{1}{k^{2}-m_{\gamma}^{2}}\ . \tag{36}$$

Employing dimensional regularization, Σ_{core} simplifies to

$$\Sigma_{core}\left(p,m\right)=\frac{\alpha}{2\pi}\left\{S_1+\int_0^1 dx\ \left[2m-\not{p}x+\sigma\left(\not{p}x-m\right)\right]\ D\left(\Delta,\sigma\right)\right\}\ , \tag{37}$$

where $D\left(\Delta,\sigma\right)$ is given by (25) with

$$\Delta=\left(1-x\right)\left(m^2-xp^2\right)+xm_\gamma^2\ .$$

The integral expression in (37) is equivalent to a form given in Peskin & Schroeder [26], while the term

$$S_1=-\frac{1-\sigma}{4}\not{p}$$

follows from appendix formulae in [24] and represents a surface contribution arising from a term linear in k during reduction of (36).

Evaluation of Σ_{dcm} using (16) reduces to negating (37) and replacing $\Delta\rightarrow\Delta_\circ$ according to (26); we obtain

$$\Sigma_{dcm}\left(p,m\right)=-\frac{\alpha}{2\pi}\left\{S_1+\int_0^1 dx\ \left[2m-\not{p}x+\sigma\left(\not{p}x-m\right)\right]\ D\left(\Delta_\circ,\sigma\right)\right\}\ , \tag{38}$$

where

$$\Delta_\circ=m^2\left(1-x\right)^2+xm_\gamma^2$$

follows from (22) using (28), (29), and (30). Terms involving $(\lambda P_M,\lambda M)$ have canceled in the average over DCM levels yielding a function of the core mass and momentum only. The net correction $\Sigma=\Sigma_{core}+\Sigma_{dcm}$, including all three mass levels in Fig. 3 (b), is given by

$$\Sigma=\frac{\alpha}{2\pi}\int_0^1 dx\ \left(2m-\not{p}x\right)\ln\frac{m^2\left(1-x\right)^2+xm_\gamma^2}{\left(m^2-xp^2\right)\left(1-x\right)+xm_\gamma^2}\ , \tag{39}$$

where the limit $\sigma\rightarrow 0$ has been taken to recover four-dimensional space-time. With a change of variables $x=1-z$, (39) is seen to be identical to the renormalized result given in Bjorken & Drell [27].

The processes in Fig. 3 (b) result in a correction [5, 8] to the free particle propagator

$$iS_F\rightarrow iS_F^{'}=iS_F+iS_F\ \left(-i\Sigma\right)\ iS_F\ , \tag{40}$$

where the approximation for a modified propagator

$$iS_F^{'}\left(p,m\right)\simeq\frac{i}{\not{p}-m-\Sigma+i\varepsilon} \tag{41}$$

has the desired pole at $\not{p}=m$ since (39) vanishes on the mass shell

$$\Sigma\left(p^2=m^2\right)=0\ . \tag{42}$$

Upon identifying

$$m_{em}^{+}=\Sigma_{core}\left(\not{p}=m,\ m_\gamma=0\right) \tag{43a}$$

$$m_{em}^{-}=\Sigma_{dcm}\left(\not{p}=m,\ m_\gamma=0\right)\ , \tag{43b}$$

we see that (42) is equivalent to the stability principle (4). Setting $\sigma=0$ and using (19) with cutoff $\Lambda_\circ$, it follows that (43a) reduces to Feynman's result (1); for derivation, see [24]. In the language of renormalization theory, the bare mass in the propagator [17]

$$iS_F^{'}\left(p,m\right)\simeq\frac{i}{\not{p}-m_b-\Sigma_{core}+i\varepsilon} \tag{44}$$

must be renormalized using (6) with (43a).

C. Vertex

Since the scattering amplitude is a complex analytic function, it follows from Cauchy's formula that the real and imaginary parts are related by a dispersion relation. The imaginary part is divergence free; it may be obtained by replacing Feynman propagators with cut propagators on the mass shell according to Cutkosky's cutting rule [28] or, alternatively, via calculation in the Heisenberg picture as shown in Källén [29]. Since the imaginary part is easily evaluated, dispersion relations provide a convenient approach for constructing the real part. In particular, the vertex function for the core mass corresponding to Fig. 3 (c) with $\lambda = 0$ takes the form [17]

$$\Lambda^{\mu}_{core}(q, m) = \gamma^{\mu} F_1\left(q^2, m\right) + \frac{i}{2m}\sigma^{\mu\nu} q_{\nu} F_2\left(q^2, m\right) \ , \tag{45}$$

where

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right]$$

are spin matrices, and the form factors $\{F_1, F_2\}$ are defined by Hilbert transforms [29]

$$F_i = \int_{4m^2}^{4\Lambda_{\circ}^2} ds \, \frac{\Lambda_i\left(\frac{4m^2}{s}\right)}{s - q^2} \tag{46}$$

with imaginary parts given by

$$\Lambda_1(w) = \frac{\alpha}{4\pi} \frac{1}{\sqrt{1-w}} \left\{ (w-2)\ln\left[1 + 4\left(\frac{m}{m_{\gamma}}\right)^2 \left(\frac{1-w}{w}\right)\right] + 3 - 4w \right\} \tag{47}$$

$$\Lambda_2(w) = \frac{\alpha}{4\pi} \frac{w}{\sqrt{1-w}} \ . \tag{48}$$

For evaluation of the DCM correction using (16), only the divergent term F_1 needs to be considered since F_2 is convergent and its coefficient vanishes as $M \to \infty$. Upon substituting {(11),(12),(13)}, using {(28), (30)}, and performing a change of variables $s = \eta^2 t$ in F_1, we have

$$\Lambda^{\mu}_{dcm} = -\gamma^{\mu} \frac{1}{2} \sum_{\lambda=\pm 1} \lim_{\eta\to\infty} \int_{4m^2}^{4\Lambda_{\circ}^2} dt \frac{\Lambda_1\left(\frac{4m^2}{t}\right)}{t - \eta^{-2} Q_{\lambda}^2} \ , \tag{49}$$

where $Q_{\lambda} = q + \lambda\left(P_M^{'} - P_M\right)$. Assuming Q_{λ}^2 is bounded, we find

$$\Lambda^{\mu}_{dcm} = -\gamma^{\mu} F_1\left(q^2 = 0, m\right) \ ; \tag{50}$$

therefore, the total $\Lambda^{\mu} = \Lambda^{\mu}_{core} + \Lambda^{\mu}_{dcm}$, including the stability correction (50), is convergent and given by

$$\Lambda^{\mu}(q) = \gamma^{\mu}\left[F_1\left(q^2\right) - F_1(0)\right] + \frac{i}{2m}\sigma^{\mu\nu} q_{\nu} F_2\left(q^2\right) \ , \tag{51}$$

where the term in brackets reduces to a once-subtracted dispersion relation in agreement with renormalized QED, and the functional dependence on m is now omitted for simplicity.

Having verified that S-matrix corrections (16) based on the stability principle (4) yield a finite QED without renormalization, we pause to reflect on the last term in (51) whose identification marked a milestone in QED's development – substituting (48) into (46) yields

$$F_2\left(q^2 = 0\right) = \frac{\alpha}{2\pi} \equiv a^{(2)}$$

as the leading correction to the anomalous magnetic moment a of the electron first derived by Schwinger [30] and verified experimentally by Foley & Kusch [31].

V. CONCLUDING REMARKS

In this paper, we defined a stable electron model wherein a hidden interaction between an electromagnetically dressed electron and an opposing polarization current offsets the positive electromagnetic field energy. Concise rules for constructing S-matrix corrections for an electromagnetically dressed electron were developed and applied to resolve primitive divergence issues, and we maintained electron mass and charge as fundamental constants throughout. Given QED's agreement with experiment, our findings support existence of negative and positive electromagnetic mass components in virtual intermediate states of infinitesimally short duration. Since there is no renormalization in this approach, the electromagnetic coupling is a constant independent of the energy scale; therefore, QED is scale-invariant.

Since the core electron effectively resides in an external vacuum potential well in the proposed model, one is led to wonder whether the muon and tau could be excited states in such a potential well.

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