Solving Polignac's and Twin Prime Conjectures Using Information-Complexity Conservation

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Abstract: Prime numbers and composite numbers are intimately related simply because the complementary set of composite numbers constitutes the set of natural numbers with the exact set of prime numbers excluded in its entirety. In this research paper, we use our 'Virtual container' (which predominantly incorporates the novel mathematical tool coined Information-Complexity conservation with its core foundation based on this [complete] prime-composite number relationship) to solve the intractable open problem of whether prime gaps are infinite (arbitrarily large) in magnitude with each individual prime gap generating prime numbers which are again infinite in magnitude. This equates to solving Polignac's conjecture which involves analysis of all possible prime gaps = 2, 4, 6,... and [the subset] Twin prime conjecture which involves analysis of prime gap = 2 (for twin primes). In conjunction with our cross-referenced 2017-dated research paper entitled "Solving Riemann Hypothesis Using Sigma-Power Laws" (http://viXra.org/abs/1703.0114), we advocate for our ambition that the Virtual container research method be considered a new method of mathematical proof especially for solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution'.

Keywords: Information-Complexity conservation; Polignac's conjecture; Riemann hypothesis; Twin prime conjecture

Mathematics Subject Classification: 11A41, 11M26

1. Introduction

The following are preambles of this paper leading up to the core research materials on prime numbers by which we provide the proofs for Polignac's and Twin prime conjectures. Previous unsuccessful attempts to solve celebrated open problems such as Riemann hypothesis, Polignac's and Twin prime conjectures have frustratingly been littered with false claims and counter claims. The historical Riemann hypothesis, proposed over 150 years ago by the famous German mathematician Bernhard Riemann (September 17, 1826 – July 20, 1866) in 1859, stood out well in this regard. Not surprisingly we shall observe that our alternative proofs essentially on Polignac's & Twin conjectures in this 2017-dated research paper will have to be very inspirational, innovative and different when compared to any previous attempts.

The set of prime numbers 2, 3, 5, 7, 11, 13,... consist of rational numbers. The set of nontrivial zeros of Riemann zeta function whose values are traditionally given by parameter t [rounded off to six decimal places] 14.134725, 21.022040, 25.010858, 30.424876, 32.935062, 37.586178,... consist of irrational [transcendental] numbers. The sets of prime numbers and nontrivial zeros of Riemann zeta function are countable infinite sets (CIS) and are Incompletely Predictable [Pseudorandom] numbers obeying Complex Elementary Fundamental Laws. To render all our claims and statements meaningful in this paper, we hereby declare that the terms "Completely Predictable" and "Incompletely Predictable" particularly in the context of numbers are to be defined next.

Completely Predictable numbers, and Incompletely Predictable numbers, are (respectively) defined as numbers whose associated details are able to be fully specified without, and with, needing to know the associated details of preceding numbers in the neighborhood. This property is demonstrated below using the example of odd number '99' (generated by a 'simple' formula and is thus a Completely Predictable number obeying Simple Elementary Fundamental Laws) and prime number '97' (generated by a 'complicated' algorithm and is a Incompletely Predictable number obeying Complex Elementary Fundamental Laws).

Randomly picked odd number '99': Can we completely predict its associated details (i) specifying whether it can be classified as an odd number and (ii) whether its precise position can be specified without needing to know all preceding odd numbers? '99' satisfy the Odd number Label "Always end with a digit of 1, 3, 5, 7, or 9" and hence is truly an odd number. Its precise position can be calculated as follows: i = (99 + 1) / 2 = 50. This implies that 99 is the 50^{th} odd number. Note that '99' is odd and composite but not prime as it consists of factors derived as 99 = 3 X 33 = 3 X 3 X 11. Therefore the answer is affirmative to both parts of our proposed question, and '99' is a Completely Predictable number.

Randomly picked prime number '97': Can we completely predict its associated details (i) specifying whether it can be classified as a prime number and (ii) whether its precise position can be specified without needing to know all preceding prime numbers? '97' satisfy the Prime number Label "Always evenly divisible only by 1 or itself & must be whole numbers greater than 1" as tested by the trial division method resulting in 97 = 1 X 97. Hence '97' is truly prime. However, its precise position can only be determined by knowing all preceding 24 prime numbers to eventually determine that 97 is the 25^{th} prime number. We would have already realize that '97' is both odd [(97 + 1) / 2 = 49^{th} odd number] and prime as it also satisfy the Odd number Label "Always end with a digit of 1, 3, 5, 7, or 9". Therefore the answer is affirmative to the first part of our proposed question but negative to the second part, and '97' is a Incompletely Predictable number.

We importantly discern that all nontrivial zeros of Riemann zeta function will manifest the Incompletely Predictable number property twice as the set of numerical digits after the decimal point in each nontrivial zero [as a transcendental number] are CIS and can again be classified as Incompletely Predictable numbers. Immediately one would intuitively sense that the mathematical equations required to deal with (for instance) "discrete" Incompletely Predictable prime numbers as opposed to (for instance) "continuous" Incompletely Predictable nontrivial zeros would have to be, respectively, of a "discrete" nature as opposed to a "continuous" nature. The 'overall' infinity components in prime numbers and nontrivial zeros of Riemann zeta function have previously been solved in a relatively straightforward manner as outlined next.

In relation to Riemann zeta function, Hardy in 1914 [1] and Hardy & Littlewood in 1921 [2] proved that there are infinitely many nontrivial zeros on the critical line by considering moments of certain functions related to this function. The ancient Euclid's proof on the infinitude of prime numbers predominantly by *reductio ad absurdum* (proof by contradiction), occurring well over 2000 years ago (c. 300 BC), is the earliest known but not the only possible proof for this simple problem in number theory. Since then dozens of proofs have been devised to show that prime numbers are indeed infinite in magnitude such as the three chronologically listed below with the strangest candidate likely to be Furstenberg's Topological Proof.

1. Goldbach's Proof using Fermat numbers (written in a letter to Euler, July 1730)

2. Furstenberg's Topological Proof in 1955 [3]

3. Filip Saidak's Proof in 2006 [4]

Then solving Riemann hypothesis [and Polignac's & Twin prime conjectures] (respectively) demands the rigorous proof that each and every single one of the infinitely many nontrivial zeros lies on the critical line [and that each and every single one of the infinitely many prime numbers is derived from the infinite (arbitrarily large) magnitude of prime gaps with each prime gap generating its own (unique) infinite magnitude of prime numbers]. In addition, there is a massive unification of prime numbers and Riemann hypothesis in the sense that a crucial primary or direct by-product arising out of the rigorous proof for Riemann conjecture / hypothesis is theorized to result in complete formalization of prime number theorem which relates to prime counting function for prime numbers.

Roughly speaking akin to the 'proof by contradiction' technique, our lemmas, propositions & theorems in relation to Polignac's and Twin prime conjectures in this paper are comparatively simple and prominently based on compulsorily using 'Virtual container' **to "contain" the infinite magnitude of all known prime numbers [consisting of prime numbers with 'small gaps' and prime numbers with 'large gaps' alike] but without needing to know their true identities** simply because otherwise we have to infinitely often prove (without complete certainty) that every single prime number will comply with various mathematical "properties" (satisfying Condition X or Y or Z). This inevitable mathematical snag could be perceived as the classical equivalent of persistently encountering the (fatal) mathematical error "undefined" when dividing a non-zero number N by zero; viz. N÷0 = "infinitely large arbitrary number" is undefined, whereas the reciprocal 0÷N = 0 is clearly defined. Phrased in another way: Once successfully obtained, our Virtual container can then be used to rigorously prove Polignac's and Twin prime conjectures (with complete certainty) via the required finite steps of subsequent / concurrent correct analysis on various mathematical "pseudo-properties" (once again satisfying Condition X or Y or Z) as derived from this Virtual container. Furthermore, if this Virtual container research method is apparently the only (solitary) way to ultimately solve the two conjectures, then these conjectures are to be regarded as belonging to the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution'.

The Virtual container for prime numbers must be endowed with the following key properties and behaviors. This container (i) must mathematically incorporate the ability to accurately and completely "contain" the relevant prime numbers without being "contaminated" by non-prime numbers entities, and (ii) must mathematically not incorporate the ability to either fully or partially "calculate" the identities of relevant prime numbers in an intrinsic manner.

We will first provide a brief synopsis on this research paper pertaining to important treatises on prime numbers. The main aim of this paper is to solve Polignac's conjecture which can be succinctly stated as whether prime gaps are infinite (arbitrarily large) in magnitude with each individual prime gap generating prime numbers which are again infinite in magnitude. Polignac's conjecture involves analysis of all possible prime gaps = 2, 4, 6,... which slowly become infinitely large at prime number examination on larger ranges (in the opposite direction to that of the smallest possible prime gap = 2). Bearing in mind that Twin prime conjecture involves analysis of prime gap = 2 (for twin primes), we can regard this conjecture as a mathematical subset of Polignac's conjecture. We use our Virtual container research method, which neatly incorporates the novel mathematical tool coined Information-Complexity conservation, to solve those conjectures. Having obtained the relevant Virtual container [which is essentially embodied in the relevant Theorem I to IV below], the subsequent / concurrent correct analysis of this container will result in the rigorous proofs for these two conjectures to mature. The proofs will only succeed with deployment of sound mathematical principles derived from Set theory, Dimensional analysis homogeneity, and our Information-Complexity conservation. Prime numbers and composite numbers are intimately related simply because the complementary set of composite numbers constitutes the set of natural numbers with the exact set of prime numbers excluded in its entirety. The Information-Complexity conservation has its core foundation based on this [complete] prime-composite number relationship. In addition, a key mathematical law dubbed "Plus-Minus Composite Gap 2 Number Alternating Law" arising naturally from Information-Complexity conservation forms a vital mathematical bridge in achieving those rigorous proofs.

The phrase "If I have seen a little further it is by standing on the shoulders of giants" used by Isaac Newton in a 1676-dated letter to his rival Robert Hooke is eminently applicable to all modern researchers, scientists, and authors alike from the current 21st Century. Generally speaking, a relevant valid finding arising from the finalized recent / past research would usually represent a mathematical step closer to achieving that elusive proof for a well-defined conjecture.

In 2013, Yitang Zhang proved a spectacular landmark mathematical result showing that there is some unknown number 'N' smaller than 70 million such that there are infinitely many pairs of primes that differ by 'N' [5]. Without going into specific details concerning optimizing Zhang's bound, subsequent Polymath Project collaborative efforts employing a new refinement of the GPY sieve in 2013 lowered 'N' to 246; and assuming the Elliott-Halberstam conjecture and its generalized form have managed to further lower 'N' to 12 and 6, respectively. Thus 'N' has (intuitively) more than one valid values such that there are infinitely many pairs of primes that differ by each of those 'N' values. No matter what, we can only theoretically lower 'N' to 2 (in regards to prime numbers with 'small gaps'), and unfortunately there are still an infinite number of prime gaps (in regards to prime numbers with 'large gaps') that will require "the proof that each will generate a set of infinite prime numbers". Colloquially speaking, we are just observing the 'tip of the iceberg' in regards to the infinitely large (or arbitrarily large) number of prime gaps. In other words, the belief here is that we can justifiably use our Virtual container (predominantly achieved through Information-Complexity conservation) to mathematically contain the complete set of infinite prime numbers generated by all infinite prime gaps. Then subsequent / concurrent correct analysis of this Virtual container will ultimately yield the reward for successfully proving those two conjectures on prime numbers.

Remark 1.1. Relevant Virtual container research method could be applied to both Completely Predictable and Incompletely Predictable entities. We provide further clarification on the Virtual container concept below.

Relevant Virtual containers must be correctly utilized and understood as constituting the basic foundation underlying any research methodology to ensure that mathematical proofs using this Virtual container technique will be valid. Carefully explained with various examples below, we must justify the responsible use of this core 'Virtual container research method' which is essential for mathematically enabling theoretical derivation of various convincing proofs in this and our other research papers to mature. This is now demonstrated when applied to (i) Completely Predictable entities [which we will classify as belonging to the 'General-Class-of-Mathematical-Problems with Multiple-Proof-Solutions'] using "discrete" even & odd numbers as the two nominated examples and "continuous" y = 2x & y = 2x - 1 equations as the two nominated examples; and (ii) Incompletely Predictable entities [which we will classify as belonging to the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution'] using "discrete" prime & composite numbers as the two nominated examples and "continuous" Riemann zeta function as the solo nominated example.

We can arbitrarily and usefully create three groups of entities: (i) Completely Predictable entities, (ii) Incompletely Predictable entities, and (iii) Completely Unpredictable [Completely Chaotic] entities. Only certain correctly selected and naturally occurring physical processes can ever give rise to true [measured] random numbers since these physical processes are totally indeterministic / chaotic and are thus Completely Unpredictable entities. In this sense, the [computational] pseudorandom number generators based solely on deterministic logic can never be regarded as true random number sources (or true Completely Unpredictable entities).

Consider x for all real number values ≥ 1. Let y be the set of real numbers such that y = 2x. Then this y = 2x "continuous" linear equation is literally the Virtual container mathematically "containing" the [complete set] straight line of infinite length commencing from the Cartesian point (x=1,y=2). [We note that by applying integral calculus to the continuous y = 2x function for the interval [1, +∞], viz. $\int_1^\infty (2x)dx$, will result in the "area of infinite size enclosed by the curve (straight line) and the x-axis".] This straight line will fully represent the y = 2x output real number values for all the specified x ≥ 1 input real number values. Computating y = 2x values an infinite number of times will not, through this procedure *per se*, result in us ever obtaining the gradient or slope of 2 for this equation. This gradient / slope can be obtained by more than one ways – either via trigonometrically calculating the tangent of this y = 2x straight line or via rigorously analyzing the intrinsic property of this y = 2x equation using differential calculus (viz. dy/dx = d(2x)/dx = 2).

We can carry out an identical treatment to the y = 2x - 1 "continuous" equation for the same x ≥ 1 real number values to obtain the infinite length straight line commencing this time from the Cartesian point (x=1,y=1). Its gradient of 2 can similarly be obtained either using the tangent method or differential calculus.

By carrying out this same identical treatment using the same y = 2x and y = 2x - 1 as "discrete" equations by considering x for all integer number values ≥ 1 [instead of x for all real number values ≥ 1], we easily obtain (respectively) the complete set of even and odd numbers [with both sets of infinite magnitude in size]. Again these "discrete" equations are the Virtual containers "containing" all relevant even and odd numbers. Computing even and odd numbers infinitely often will not *per se* enable us to ever conclude that the gap between any two consecutive even numbers (even gap) and any two consecutive odd numbers (odd gap) will both always equal to 2. Again this "gradient-equivalent" even gaps and odd gaps can simply be obtained by transforming those equations from their "discrete" formats into the equivalent "continuous" formats [viz. "discrete" $\Delta x = 1 \longrightarrow$ "continuous" $\Delta x = 0$] to obtain their gradients either using the tangent method or differential calculus. Then even and odd gaps, both equal to 2, is numerically identical and mathematically equivalent to the relevant obtained gradients, both also equal to 2.

Two crucial points to note here are (i) the two equations y = 2x and y = 2x - 1 in both their "discrete" and "continuous" formats are totally independent of each other as we can successfully obtain their respective gradient or gap values by just analyzing the relevant equation by itself, and (ii) there are more than one ways to obtain those gradient and gap values as clearly illustrated above using the tangent method or differential calculus. Specifically by this two points, we imply that Completely Predictable entities will always belong to the 'General-Class-of-Mathematical-Problems with Multiple-Proof-Solutions'.

We now logically apply in an analogous manner similar treatment to (a) "discrete" prime & composite numbers and (b) "continuous" Riemann zeta function. They are all of infinite magnitude in size and are typical representations of Incompletely Predictable entities. In relation to the "continuous" Riemann zeta function, the axes intercepts at the Origin (viz. nontrivial zeros), the x-axis (viz. 'usual' Gram points) and the y-axis (viz. Gram[x=0] points) all consisting of "continuous" transcendental numbers are the Incompletely Predictable entities of interest that we wish to study. As we shall subsequently observe in all the proofs depicted in this and our other research papers on prime numbers and Riemann zeta function, both (a) the sets of infinite prime & composite numbers (b) the axes intercepts sets of infinite nontrivial zeros, infinite Gram points and infinite Gram [x=0] points are totally dependent on each other in the following sense. There is one solitary way to solve their associated open problems as we can only suceed in rigorously obtaining the relevant proofs when (a) prime & composite numbers are simultaneously analyzed together using relevant Virtual container [largely represented by Information-Complexity conservation derived via certain non-negotiable mathematical steps being correctly taken] to "contain" them, and (b) nontrivial zeros, Gram points and Gram [x=0] points are all simultaneously analyzed and "contained" using relevant Virtual container [largely represented by Sigma-Power Laws derived via certain non-negotiable mathematical steps being correctly taken]. In other words, we are only able to solve those open problems when (i) prime & composite numbers are dependently analyzed together to derive this 'solitary-style' relevant Virtual container to "contain" them, and the axes intercepts of Riemann zeta function are dependently analyzed together with the derived 'solitary-style' relevant Virtual container that "contains" them, and (ii) those representative Virtual containers can only be derived via certain non-negotiable mathematical steps being correctly undertaken. Satisfying criteria (i) and (ii) is the *sine qua non* of the requirements to fulfill the condition that Incompletely Predictable entities will belong to the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution'.

We intuitively sense that in order to ultimately prove any open problems related to prime numbers and Riemann zeta function would subsequently / concurrently require correctly analyzing certain [finite] intrinsic properties and behaviors arising from those representative Virtual containers. In particular, we are now dealing with entities such as Incompletely Predictable varying gaps [which is equivalent to Incompletely Predictable varying 'gradients'] between consecutive prime numbers (prime gaps) & between consecutive composite numbers (composite gaps); and our hereby conjured-for-illustration-purpose Incompletely Predictable varying gaps [which is equivalent to Incompletely Predictable varying 'gradients'] between consecutive nontrivial zeros (dubbed nontrivial zero gaps), between consecutive Gram points (dubbed Gram points gaps) & between consecutive Gram[x=0] points (dubbed Gram[x=0] points gaps). Finally, we epitomize the tell-tale sign indicating Virtual container use in our paper by including relevant statements / sentences incorporating expressions with wordings such as "...containing each and every conceivable prime number [but not its actual identity]..." or "...containing each and every conceivable nontrivial zeros [but not its actual identity]...".

2. Dual source of prime number infiniteness from prime gaps

In terms of prime gaps, one can argue that the potential mathematical source(s) of prime number infiniteness could feasibly arise in two ways via (i) one or more than one or all of the prime gap(s) with those nominated prime gap(s) each generating an infinite magnitude of distinct prime numbers & / or (ii) the infinite (arbitrarily large) magnitude of prime gaps collectively able to generate an infinite magnitude of prime numbers with the proviso that the criterion in (ii) will hold true even if none of the individual prime gaps were to ever generate an infinite magnitude of prime numbers. Stated differently for the "none of the prime gaps that will each generate an infinite magnitude of prime numbers" scenario, we are alleging that groups of these [imaginary] finite prime numbers derived from each prime gap are [wrongly] classified as countable finite sets (CFS) but this arrangement will still culminate in producing a [incomplete] countable infinite set (CIS) of prime numbers as long as there are infinitely many prime gaps. We now arrive at the realization that rigorously proving Polignac's and Twin prime conjectures in essence would mathematically translate to needing to show that (i) the [solitary] set of all prime gaps and (ii) the [infinite] sets of all prime numbers arising in totality from each of those prime gaps, must all be CIS. We next outline below some interesting properties of prime numbers.

English mathematician John Horton Conway coined the term 'jumping champion' in 1993. An integer n is a jumping champion if n is the most frequently occurring difference (prime gap) between consecutive prime numbers < x for some x. Example: for any x with 7 < x < 131, n = 2 (indicating twin prime numbers) is the jumping champion. It has been conjectured that (i) the only jumping champions are 1, 4 and the primorials 2, 6, 30, 210, 2310,... and (ii) jumping champions tend to infinity. Their required proofs will likely need the proof of the k-tuple conjecture. [For i = 1, 2, 3,...; primordial $P_i\#$ is the analog of the usual factorial for prime numbers (2, 3, 5, 7,...). For instance $P_1\# = 2$, $P_2\# = 2 X 3 = 6$, $P_3\# = 2 X 3 X 5 = 30$, $P_4\# = 2 X 3 X 5 X 7 = 210$, $P_5\# = 2 X 3 X 5 X 7 X 11 = 2310$, etc.]

We now look at the data of all prime numbers obtained when extrapolated out over a wide range of x values. Generally speaking, as the sequence of prime numbers carries on, prime numbers with ever larger prime gaps will tend to appear. For the given range of x values, we say that prime gap = n_2 is a 'maximal prime gap' if prime gap = n_1 < prime gap = n_2 for all $n_1 < n_2$. In other words, the largest such prime gaps in the sequence are called maximal prime gaps. The ratio [n_2] / [ln(prime number with prime gap = n_2)] is called the 'merit' of prime gap = n_2 (maximal prime gap). Thus the merit of a prime gap is a normalized number representing how "soon" in the sequence a prime gap appears relative to the logarithm of the larger prime. To the best of our knowledge, there is no clear-cut correlation between the largest known merit value obtained and either the relative size of the relevant prime number with prime gap = n_2 or the relative size of that prime gap = n_2.

The term 'first occurrence prime gaps' commonly refers to first occurrences of maximal prime gaps whereby maximal prime gaps can also be perceived here as prime gaps of "at least of this length". The CIS of 'maximal prime gaps' and the (complementary) CIS of 'non-maximal prime gaps' can fully be derived and depicted as below. We endorse non-maximal prime gaps with the interesting nickname 'slow jumpers' in this paper. We coin the term 'slow jumpers' here because non-maximal prime gaps always lag behind their maximal prime gaps counterparts for their onset appearances in the prime number sequence. This is tabulated for the first 17 prime gaps in Table 1 consisting of maximal prime gaps and non-maximal [slow jumper] prime gaps.

Prime gap	Following the prime number
1*	2
2*	3
4*	7
6*	23
8*	89
10	139
12	199
14*	113
16	1831
18*	523
20*	887
22*	1129
24	1669
26	2477
28	2971
30	4297
32	5591

Table 1. First 17 prime gaps depicted in the format utilizing maximal prime gaps [depicted with the asterisk symbol (*)] and non-maximal prime gaps [depicted without this symbol].

Note that the progressive resultant prime numbers generated here in Table 1 solidly represent only a single prime number for each prime gap and this will always be less than the complete set of all prime numbers generated from, for instance, the Sieve of Eratosthenes. The initial seven of the [majority] "missing" prime numbers are 5, 11, 13, 17, 19, 29, 31,...; and they all belong to the subset of prime numbers with 'residual' prime gaps which must be the potential source of prime numbers in relation to the proposal that each of the prime gap of 2, 4, 6, 8,... will generate its specific CIS of prime numbers.

Remark 2.1. Maximal and non-maximal prime gaps supply crucial indirect evidence to intuitively and philosophically support, but does not prove, the mathematical statement "Each prime gap will generate an infinite magnitude of prime numbers on its own accord".

From the above brief analysis on prime number distribution, we easily deduce that [predominantly the groups of] prime numbers with jumping champion prime gaps, [the individual / groups of] prime numbers with maximal prime gaps, and [the individual / groups of] prime numbers with non-maximal prime gaps would seem to make perpetual repeating appearances amongst the complete CIS of prime numbers. A vitally crucial observation is that all prime numbers generated by (i) non-maximal (slow jumper) prime gaps and (ii) maximal prime gaps, will still not generate the complete CIS of prime numbers. This is simply because, apart from the one-off prime gap = 1 associated with the very first prime number 2, all other [infinite magnitude] prime gaps 2, 4, 6,... must each generate more than one, if not a CIS of, prime numbers in order to account for all prime numbers. This clear-cut observation constitutes crucial indirect evidence to intuitively and philosophically support [but does not prove] the proposition that each prime gap will likely generate an infinite magnitude of prime numbers on its own accord.

Although not crucial for the purpose of this paper, we could potentially study exciting behaviors from the subset of prime numbers with 'residual' prime gaps as obtained when the subset of prime numbers with maximal prime gaps and the subset of prime numbers with non-maximal prime gaps are progressively removed from the complete set of prime numbers with all prime gaps. This can mathematically be visualized as: Complete set of prime numbers with all prime gaps = Subset of prime numbers with maximal prime gaps + Subset of prime numbers with non-maximal prime gaps + Subset of prime numbers with 'residual' prime gaps. In addition, prime numbers with 'residual' prime gaps must include all the correctly selected prime numbers which are representative of all prime gaps 2, 4, 6,.... (except the one-off prime gap = 1 for the very first prime number).

3. Information-Complexity conservation

An Equation or Algorithm is simply a Black Box generating the necessary Output (with qualitative structural 'Complexity') when supplied with the given Input (with quantitative data 'Information'). In Set theory, the infinite sets are sets that are not finite sets, and they are further subdivided into two groups: "discrete" countable infinite sets and "continuous" uncountable infinite sets; with the later being conceptually larger than the former in magnitude [despite both being treated as objects endowed with the infinity property]. A set is countable if we can count its elements. If the set is finite, we can easily count its elements. If the set is infinite, being countable means that we are able to put the elements of the set in order (just like natural numbers are in order). The infinite sets of rational number and irrational number are, respectively, countable and uncountable. These two sets together give rise to the infinite set of real number which are uncountable.

The two-dimensional complex plane is typically specified by a one-dimensional real number line for horizontal or x-axis, and a one-dimensional imaginary number (i = $\sqrt{-1}$) line for vertical or y-axis. Complex numbers, each defined with a (pure) real number component and a (pure) imaginary number component, lie on this plane. Real numbers could alternatively be perceived as complex numbers with their imaginary number component being zero. In regards to the uncountable infinite set of real number line (with the understanding that every positive real number has its mirror image negative real number counterpart), this line is further seen to consist of both countable and uncountable infinite sets. The set and subsets of real numbers with some of their properties are comparatively illustrated below using the following legends & abbreviations: =, <, >, >>, ⊂, ∈, CFS, CIS & UIS denoting (respectively) 'equal to', 'less than', 'greater than', 'much greater than', 'subset of', 'belongs to', 'countable finite set', 'countable infinite set' & 'uncountable infinite set'.

We can use the cardinality relation to describe the size of a set by comparing it with standard sets. Any set X with cardinality less than that for the set of natural numbers (set N), or |X| < |N|, is said to be a CFS. Any set X that has the same cardinality as set N, or |X| = |N|, is said to be a CIS endowed with "cardinality of the natural numbers". Any set X with cardinality greater than that for set N, or |X| > |N| (for example, when set X = real numbers), is said to be a UIS endowed with "cardinality of the continuum". From smallest to biggest sets, natural numbers (CIS) ⊂ integer numbers (CIS) ⊂ rational numbers (CIS) ⊂ algebraic numbers (CIS) ⊂ real numbers (UIS). By definition, for relatively "smaller" set X (= even numbers or odd numbers or prime numbers or composite numbers) ∈ set of natural numbers, we [counterintuitively] note that |X| still = "cardinality of the natural numbers". The set of natural numbers has cardinality (of the natural numbers) that is strictly less than the set of real numbers having cardinality (of the continuum) as it can be shown that there does not exist a bijective function from natural numbers to real numbers using Cantor's diagonal argument or Cantor's first uncountability proof.

Irrational numbers (UIS) = (I) Transcendental numbers (UIS) + (II) Algebraic numbers (CIS) with (I) >> (II). Almost all real and complex numbers are transcendental. All irrational numbers can imperatively be depicted as numbers with non-repeating decimal point digits of infinite length, with those decimal point digits being Incompletely Predictable. An algebraic number is any real or complex number that is a root of a non-zero polynomial in one variable with rational coefficients (or equivalently - by clearing denominators - with integer coefficients). All integers and rational numbers are algebraic, as are all roots of integers. Thus a transcendental number is a real or complex number that is not algebraic [with criteria as just stated]; and it "transcends" the power of algebra to display it in its totality.

Real numbers (UIS) = (I) Irrational numbers (UIS) + (II) Rational numbers (CIS) with (I) > (II). If real numbers are to be the union of two countable sets, they would have to be [incorrectly] countable; so the irrational numbers must be [correctly] uncountable by following this 'proof by contradiction' argument.

Rational numbers (CIS) = (I) Fractions (CIS) + (II) Integer numbers (CIS) with (I) > (II). A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q. Since q may be equal to 1, every integer is a rational number. Fractions can imperatively be depicted as numbers with non-repeating decimal point digits of finite length type or repeating decimal point digits of infinite length type, with both sets of decimal point digits being Completely Predictable. Thus integers can specially be depicted either as the integer number itself followed by a (redundant) non-repeating decimal point digit '0' or as fractions with numerator given by the integer number itself and denominator given by the (redundant) number '1'.

Integers (CIS): -∞ ,..., -3, -2, -1, 0, 1, 2, 3,..., +∞. Whole numbers (CIS): 0, 1, 2, 3,..., ∞. Natural numbers (CIS): 1, 2, 3, 4,..., ∞. Let x be the set consisting of either one or two number(s) such that x ∈ natural numbers, whole numbers or integers. Whenever relevant in this paper, we consider x for the relevant upper & / or lower boundary(ies) of interest in the study on a chosen set of numbers (such as even, odd, prime, and composite numbers).

Lemma 3.1. Natural numbers (CIS): 1, 2, 3, 4,..., ∞. The natural counting function Natural-$\pi(x)$, defined as the number of natural numbers ≤ x, is Completely Predictable to be simply = x.

Proof The formula for generating natural numbers with 100% certainty is N_i = i whereby N_i is the i^{th} natural number and i = 1, 2, 3,..., ∞. For a given N_i number, its i^{th} position is simply i. Natural gap (G_{Ni}) = N_{i+1} - N_i, with G_{Ni} always = 1. Thus there are x natural numbers ≤ x. The (coined) natural counting function, denoted here by Natural-$\pi(x)$, is defined as the number of natural numbers ≤ x - this is Completely Predictable to be simply = x. The proof is now complete for Lemma 3.1.

Lemma 3.2. The even counting function Even-$\pi(x)$, defined as the number of even numbers ≤ x, is Completely Predictable to be simply = floor(x/2).

Proof. Even numbers (CIS): 2, 4, 6, 8,..., ∞. The formula for generating even numbers with 100% certainty is E_i = iX2 whereby E_i is the i^{th} even number and i = 1, 2, 3,..., ∞ abiding to the mathematical label "All natural numbers always ending with a digit 0, 2, 4, 6 or 8". For a given E_i number, its i^{th} position is calculated as i = E_i / 2. Even gap (G_{Ei}) = E_{i+1} - E_i, with G_{Ei} always = 2. Thus there are $\lfloor \frac{x}{2} \rfloor$ even numbers ≤ x. The (coined) even counting function, denoted here by Even-$\pi(x)$, is defined as the number of even numbers ≤ x - this is Completely Predictable to be simply = floor(x/2). The proof is now complete for Lemma 3.2.

Lemma 3.3. The odd counting function Odd-$\pi(x)$, defined as the number of odd numbers ≤ x, is Completely Predictable to be simply = ceiling(x/2).

Proof. Odd numbers (CIS): 1, 3, 5, 7,..., ∞. The formula for generating odd numbers with 100% certainty is O_i = (iX2)-1 whereby O_i is the i^{th} odd number and i = 1, 2, 3,..., ∞ abiding to the mathematical label "All natural numbers always ending with a digit 1, 3, 5, 7, or 9". For a given O_i number, its i^{th} position is calculated as i = (O_i + 1) / 2. Odd gap (G_{Oi}) = O_{i+1} - O_i, with G_{Oi} always = 2. Thus there are $\lceil \frac{x}{2} \rceil$ odd numbers ≤ x. The (coined) odd counting function, denoted here by Odd-$\pi(x)$, is defined as the number of odd numbers ≤ x - this is Completely Predictable to be simply = ceiling(x/2). The proof is now complete for Lemma 3.3.

Lemma 3.4. The prime counting function Prime-$\pi(x)$, defined as the number of prime numbers ≤ x, is Incompletely Predictable and always need to be calculated using the Sieve of Eratosthenes algorithm.

Proof. Prime numbers (CIS): 2, 3, 5, 7, 11, 13, 17,..., ∞. The algorithm for generating all prime numbers P_i whereby P_1 (= 2), P_2 (= 3), P_3 (= 5), P_4 (= 7),..., ∞ with 100% certainty is based on the Sieve of Eratosthenes abiding to the mathematical label "All natural numbers apart from 1 that are evenly divisible by itself and by 1". Suffice to state here that although we can check the primality of a given odd number [check whether a given odd number is a prime number or not] by trial division, we can never determine its position without knowing the positions of preceding prime numbers. All prime numbers must be odd numbers and the only even prime number is 2. Prime gap (G_{Pi}) = P_{i+1} - P_i, with G_{Pi} constituted by all even numbers except the 1^{st} G_{P1} = 3 - 2 = 1. The prime counting function, denoted here by Prime-$\pi(x)$ [which is traditionally denoted simply by $\pi(x)$], is defined as the number of prime numbers ≤ x - this is Incompletely Predictable and always need to be calculated via the mentioned algorithm. We notice that by the very definition of prime gap above, every prime number [represented here with the aid of 'n' notation instead the usual 'i' notation] can be written as $P_{n+1} = 2 + \sum_{i=1}^{n} G_{Pi}$ with '2' denoting P_1. Here i & n = 1, 2, 3, 4, 5, ..., ∞. The proof is now complete for Lemma 3.4.

Lemma 3.5. The composite counting function Composite-$\pi(x)$, defined as the number of composite numbers ≤ x, is Incompletely Predictable and always need to be calculated indirectly as the set of natural numbers minus the set of prime numbers [obtained using the Sieve of Eratosthenes algorithm].

Proof. Composite numbers (CIS): 1, 4, 6, 8, 9, 10, 12,..., ∞. Composite numbers have the mathematical label "All natural numbers other than that are evenly divisible by itself and by 1". The algorithm for generating all composite numbers C_i whereby C_1 (= 1), C_2 (= 4), C_3 (= 6), C_4 (= 8),..., ∞ with 100% certainty is also based on the Sieve of Eratosthenes albeit in an indirect manner by simply selecting [the excluded] non-prime natural numbers to be composite numbers. We define the (coined) term Composite gap G_{Ci} as C_{i+1} - C_i with G_{Ci} constituted by 1 & 2 except the 1^{st} G_{C1} = 4 - 1 = 3. The (coined) composite counting function, denoted by Composite-$\pi(x)$, is defined as the number of composite numbers ≤ x - this is Incompletely Predictable and always need to be [indirectly] calculated via the mentioned algorithm. Applying similar ideas from prime numbers, we notice that by the very definition of composite gap above, every composite number [represented here with the aid of 'n' notation instead the usual 'i' notation] can be written as $C_{n+1} = 1 + \sum_{i=1}^{n} G_{Ci}$ with '1' denoting C_1. Here i & n = 1, 2, 3, 4, 5, ..., ∞. We crucially mention at this point that, in stark contrast to the equation "containing" but not identifying all prime numbers [outlined in the proof for Lemma 3.4 above] with prime gaps constituted by all even numbers [dealing with 'unfriendly' CIS property] except the 1^{st} G_{P1} = 3 - 2 = 1; the equivalent equation "containing" but not identifying all composite numbers deals with 'friendly' CFS property for composite gaps which are constituted by 1 & 2 except the 1^{st} G_{C1} = 4 - 1 = 3. Also we reinforce from the contents associated with Remark 1.1 above that we could conceptually and usefully visualize both prime gaps and composite gaps respectively as "prime gradients" and "composite gradients". The proof is now complete for Lemma 3.5.

The following are consistent mathematical relationships amongst the various groups of rational numbers: [Positive] Integers (CIS) = Whole numbers (CIS) = Number '0' (CFS) + Natural numbers (CIS). Natural numbers (CIS) = (I) Even numbers (CIS) + (II) Odd numbers (CIS) with (I) = (II). Natural numbers (CIS) = (I) Prime numbers (CIS) + (II) Composite numbers (CIS) with (I) < (II). Composite numbers (CIS) = (I) Even numbers (CIS) + (II) [Odd numbers (CIS) - Prime numbers (CIS)] with (I) > (II) and Odd numbers > Prime numbers. Prime numbers (CIS) = (I) Natural numbers

(CIS) - (II) Composite numbers (CIS) with (I) > (II).

Prime numbers < composite numbers and they are (A) mutually exclusive to each other. In fact, (B) composite numbers are the exact complementary counterparts of prime numbers simply because the Incompletely Predictable composite numbers = Completely Predictable natural numbers - Incompletely Predictable prime numbers. Relationships (A) and (B) allow us to forge a useful mental picture of "monotonously, slowly and eternally increasing prime and composite numbers which are inseparable" with composite numbers doing so at a relatively faster rate than prime numbers. Better visualization of this picture could desirably be achieved by graphing various derived formulations of relevant counting functions for both elements.

The set of natural number can be visualized to consist of two 'mutually exclusive, complementary and inseparable' subset groupings of either (i) Completely Predictable even and odd numbers, or (ii) Incompletely Predictable prime and composite numbers. Denote 'A' to represent natural, even, odd, prime, and composite numbers. The counting function A-$\pi(x)$ is the number of A ≤ x with x belonging to the set of natural number. As a prelude to outlining the all-important Information-Complexity conservation concept below, we can easily define and compute in a progressive manner the entity 'Grand-Total Gaps for A at x' (Grand-Total ΣA_x-Gaps) and their associated properties.

Proposition 3.6. For any given x value, the designated Information or Input is always validly represented by $\Sigma \text{Natural}_x$-Gaps = x - 1 for [one set of] the Completely Predictable natural numbers.

Proof. INPUT: Set of natural numbers (for x = 10): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Natural-$\pi(x)$ = 10. There are x - 1 = 9 Natural-Gaps each of '1' magnitude: 1, 1, 1, 1, 1, 1, 1, 1, 1. $\Sigma \text{Natural}_x$-Gaps = 9 X 1 = 9. This equates to "x - 1" which we can regard as INFORMATION for Completely Predictable numbers. The proof is now complete for Proposition 3.6.

Proposition 3.7. For any given x value, the designated Complexity or Output is always validly represented by $\Sigma \text{EvenOdd}_x$-Gaps = 2x - 4 for [two sets of] the Completely Predictable even and odd numbers.

Proof. OUTPUT: Set of even and odd numbers (for x = 10): 2, 4, 6, 8, 10 and 1, 3, 5, 7, 9. Even-$\pi(x)$ = 5 and Odd-$\pi(x)$ = 5. There are predictably $\lfloor \frac{x}{2} \rfloor$ - 1 = 4 Even-Gaps each of '2' magnitude: 2, 2, 2, 2. ΣEven_x-Gaps = 4 X 2 = 8, and $\lceil \frac{x}{2} \rceil$ - 1 = 4 Odd-Gaps each of '2' magnitude: 2, 2, 2, 2. ΣOdd_x-Gaps = 4 X 2 = 8. Grand-Total $\Sigma \text{EvenOdd}_x$-Gaps = 8 + 8 = 16. This equates to "2x - 4" which we can regard as COMPLEXITY for Completely Predictable numbers. The proof is now complete for Proposition 3.7.

Proposition 3.8. For any given x value, the designated Complexity or Output is always validly represented by $\Sigma \text{PrimeComposite}_x$-Gaps = 2x - 4 for [two sets of] the Incompletely Predictable prime and composite numbers.

Proof. OUTPUT: Set of prime and composite numbers (for x = 12): 2, 3, 5, 7, 11 and 1, 4, 6, 8, 9, 10, 12. Prime-$\pi(x)$ = 5 and Composite-$\pi(x)$ = 7. There are four Prime-Gaps of 1, 2, 2, 4 magnitude and six Composite-Gaps of 3, 2, 2, 1, 1, 2 magnitude. ΣPrime_x-Gaps = 1 + 2 + 2 + 4 = 9. $\Sigma \text{Composite}_x$-Gaps = 3 + 2 + 2 + 1 + 1 + 2 = 11. Grand-Total $\Sigma \text{PrimeComposite}_x$-Gaps = 9 + 11 = 20. This equates to "2x - 4" which we can regard as COMPLEXITY for Incompletely Predictable numbers. The proof is now complete for Proposition 3.8.

Incredibly, this (defacto) baseline "2x - 4" Grand-Total Gaps for the Incompletely Predictable numbers output is identical to that for the Completely Predictable numbers output. This common Grand-Total Gaps ingredient present in two outputs representing two vastly different groups of number is part of what we regard as fulfilling Information-Complexity conservation.

Let both x & N ∈ 1, 2, 3,..., ∞. We utilize the word 'Dimension' here to contextually denote the relevant Dimension 2x - N whereby (i) the allocated [infinite] N integer values will result in Dimensions of the types 2x - 4, 2x - 5, 2x - 6, ..., 2x - ∞ for the Prime-Composite mathematical landscape below and (ii) the allocated [finite] N integer values for the Even-Odd mathematical landscape in Appendix 1 below will result in Dimensions of the type 2x - 4. For both Prime-Composite and Even-Odd groupings, we have not included the very first (one-off) Dimension 2x - 2. [The term "mathematical landscape" is self-explanatorily employed in this paper to denote tabulated and graphed data showing specific mathematical patterns and features.]

Using the relevant data, we have now painstakingly tabulate (in Table 2) and graphically map (in Figure 1) the all-important [Incompletely Predictable] Prime-Composite mathematical landscape for a relatively larger x = 64 as demonstrated below (and ditto for the [Completely Predictable] Even-Odd mathematical landscape as demonstrated in Appendix 1 at the end of this paper). Legend: C = composite, P = prime, Y = Dimension 2x - 4 (for visual clarity). Of utmost importance, we note that this Prime-Composite mathematical landscape made up of the relevant Dimensions will intrinsically incorporate prime and composite numbers in an integrated manner; and that there will be infinite times whereby relevant Dimensions will deviate away from the 'baseline' Dimension 2x - 4 simply because prime and composite numbers are infinite in magnitude. For comparison, we have repeated this whole procedure for the [Completely Predictable] Even-Odd mathematical landscape in Appendix 1 and note the complete lack of deviation away from the 'baseline' Dimension 2x - 4 apart from the one-off point of deviation as manifested by the initial Dimension 2x - 2.

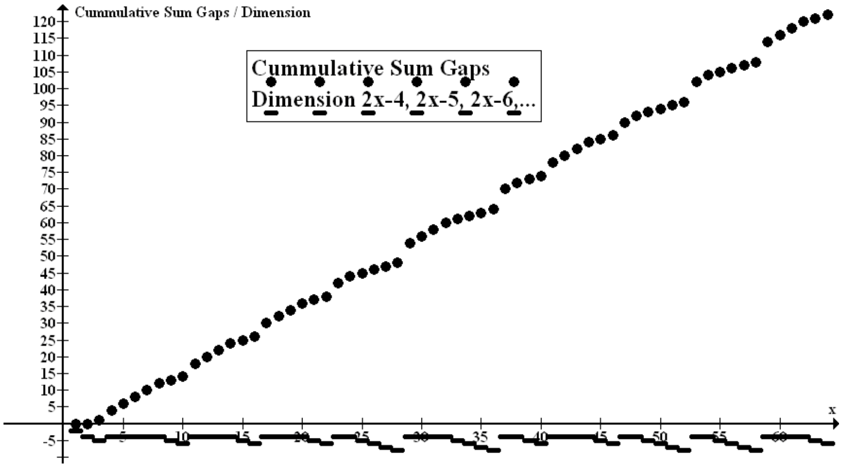

Figure 1: Prime-Composite mathematical (graphed) landscape using data obtained for x = 64.

x	P_i or C_i, Gaps	ΣPC_x-Gaps	Dimension	x	P_i or C_i, Gaps	ΣPC_x-Gaps	Dimension
1	C1, 3	0	2x-2	33	C22, 1	61	2x-5
2	P1, 1	0	Y	34	C23, 1	62	2x-6
3	P2, 2	1	2x-5	35	C24, 1	63	2x-7
4	C2, 2	4	Y	36	C25, 2	64	2x-8
5	P3, 2	6	Y	37	P12, 4	70	Y
6	C3, 2	8	Y	38	C26, 1	72	Y
7	P4, 4	10	Y	39	C27, 1	73	2x-5
8	C4, 1	12	Y	40	C28, 1	74	2x-6
9	C5, 1	13	2x-5	41	P13, 2	78	Y
10	C6, 2	14	2x-6	42	C29, 2	80	Y
11	P5, 2	18	Y	43	P14, 4	82	Y
12	C7, 2	20	Y	44	C30, 1	84	Y
13	P6, 4	22	Y	45	C31, 1	85	2x-5
14	C8, 1	24	Y	46	C32, 2	86	2x-6
15	C9, 1	25	2x-5	47	P15, 6	90	Y
16	C10, 1	26	2x-6	48	C33, 1	92	Y
17	P7, 2	30	Y	49	C34, 1	93	2x-5
18	C11, 2	32	Y	50	C35, 1	94	2x-6
19	P8, 4	34	Y	51	C36, 1	95	2x-7
20	C12, 1	36	Y	52	C37, 1	96	2x-8
21	C13, 1	37	2x-5	53	P16, 6	102	Y
22	C14, 2	38	2x-6	54	C38, 1	104	Y
23	P9, 6	42	Y	55	C39, 1	105	2x-5
24	C15, 1	44	Y	56	C40, 1	106	2x-6
25	C16, 1	45	2x-5	57	C41, 1	107	2x-7
26	C17, 1	46	2x-6	58	C42, 1	108	2x-8
27	C18, 1	47	2x-7	59	P17, 2	114	Y
28	C19, 2	48	2x-8	60	C43, 2	116	Y
29	P10, 2	54	Y	61	P18, 6	118	Y
30	C20, 2	56	Y	62	C44, 1	120	Y
31	P11, 6	58	Y	63	C45, 1	121	2x-5
32	C21, 1	60	Y	64	C46, 1	122	2x-6

Table 2: Prime-Composite mathematical (tabulated) landscape using data obtained for x = 64.

In Figure 1, Dimensions 2x - 4, 2x - 5, 2x - 6, ..., 2x - ∞ are symbolically represented by -4, -5, -6, ..., ∞ with 2x - 4 displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this (Grand-Total Gaps) 'baseline' Dimension on a perpetual basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by ever larger negative integers will correspond to prime numbers associated with ever larger prime gaps and this phenomenon will generally happen at ever larger x values. In other words, at ever larger x values, Prime-$\pi(x)$ will overall become larger but with a DECELERATING trend whereas Composite-$\pi(x)$ will overall become larger but with an ACCELERATING trend. This highlights the inevitable mathematical event of ever larger prime gaps occurring at ever larger x values. We note that there is a complete presence of Chaos & Fractals phenomena being manifested in our graph.

The definitive derivation of the data in Table 2 is given next and this is clearly illustrated by two examples given for position x = 31 & 32. For i & x ∈ 1, 2, 3,..., ∞; ΣPC_x-Gap = ΣPC_{x-1}-Gap + Gap value at P_{i-1} or Gap value at C_{i-1} whereby (i) P_i or C_i at position x is determined by whether the relevant x value belongs to a prime (P) or composite (C) number, and (ii) both ΣPC_1-Gap and ΣPC_2-Gap = 0. Example for position x = 31: 31 is a prime number (P11). Our desired Gap value at P10 = 2. Thus ΣPC_{31}-Gap (58) = ΣPC_{30}-Gap (56) + Gap value at P10 (2). Example for position x = 32: 32 is a composite number (C21). Our desired Gap value at C20 = 2. Thus ΣPC_{32}-Gap (60) = ΣPC_{31}-Gap (58) + Gap value at C20 (2).

Finally, we easily observe the 'overall magnitude of composite numbers to be always greater than that of prime numbers' criterion to hold true from x = 8 onwards. For instance, position x = 61 corresponds to prime number 61 which is the 18^{th} prime number, whereas [the one lower] position x = 60 corresponds to composite number 60 which is the [much higher] 43^{rd} composite number.

4. Polignac's and Twin prime conjectures

We have already established that prime and composite numbers are mutually exclusive, complementary, inseparable, and infinite in magnitude. With the letter 'Y' symbolizing (baseline) Dimension 2x - 4 and prime gap at $P_i = P_{i+1} - P_i$ with P_i & P_{i+1} respectively symbolizing consecutive "first" & "second" prime number in any P_i-P_{i+1} pairings, we can conveniently denote (i) Dimensions YY grouping [depicted by 2x - 4 initially appearing twice in (iii) below] as representing the signal for appearances of prime number pairings other than twin primes (with prime gap = 2) such as cousin primes (with prime gap = 4), sexy primes (with prime gap = 6), etc; (ii) Dimension YYYY grouping as representing the signal for appearances of prime number pairings as twin primes (with prime gap = 2); and (iii) Dimension (2x - ≥4)-Progressive-Grouping allocated to the 2x - 4, 2x - 4, 2x - 5, 2x - 6, 2x - 7, 2x - 8,..., 2x - ∞ as elements of the PRECISE and PROPORTIONATE countable finite set (CFS) Dimensions representation of an individual prime number P_i with its associated prime gap namely, Dimensions 2x - 4 & 2x - 4 pairing = twin prime (with both of its prime gap & CFS cardinality = 2); 2x - 4, 2x - 4, 2x - 5 & 2x - 6 pairing = cousin prime (with both of its prime gap & CFS cardinality = 4); 2x - 4, 2x - 4, 2x - 5, 2x - 6, 2x - 7 & 2x - 8 pairing = sexy prime (with both of its prime gap & CFS cardinality = 6); and so on. Then the higher order [which is traditionally defined as closest possible] prime groupings of three prime numbers as prime triplets, of four prime numbers as prime quadruplets, of five prime numbers as prime quintuplets, etc can each be mathematically deemed to consist of relevant serendipitous groupings required-by-law to always respect the following unwritten mathematical rule: With the exception of the three 'outlier' prime numbers 3, 5, & 7; groupings of any three prime numbers as the P, P+2, P+4 combination (viz. manifesting two consecutive twin primes with prime gap = 2) is a mathematical impossibility. The 'anomaly' that one of every three sequential odd numbers is a multiple of three, and hence this particular number cannot be prime, would clearly explain this mathematical impossibility. Then the closest possible prime grouping must be of either P, P+2, P+6 format or P, P+4, P+2 format.

Note that prime groupings not respecting the traditional closest-possible-prime groupings above are also the norm [occurring infinitely often], and they simply indicate the continual presence of prime gaps ≥ 6 [by which we tentatively propose here to the wider scientific community to arbitrarily represent 'large gaps']. As prime numbers become sparser at larger range; the perpetual presence of prime gaps ≥ 6 of progressive greater magnitude will, in a general and gentle manner, occur ever more frequently.

Based on not dissimilar rationale to above, we can deduce that as prime numbers become sparser at larger range; the permanent presence of prime gaps 2 & 4 [by which we tentatively propose here to the wider scientific community to arbitrarily represent 'small gaps'] will, in a general and gentle manner, occur ever less frequently. Thus nature seems to dictate that in order to comply with Information-Complexity conservation, the permanent requirement, at larger range, of intermittently resetting to baseline Dimension 2x - 4 occurring four times in a row as denoted by Dimension YYYY grouping [indicating the occurrence of twin primes] is inevitable.

Crucially we can now insightfully understand the Dimension YYYY unique signal of twin prime appearances in full details. The initial two CFS Dimensions YY components of YYYY fully represent the "first" prime number component of the twin prime number pairing. The last two Dimensions YY components of YYYY signifying the appearance of the "second" prime number component of the twin prime number pairing is also the initial first-two-element component of the full CFS Dimensions representation for the "first" prime number component of the following non-twin prime number pairing. The seemingly "bizarre" uniqueness of twin primes (with prime gap = 2) is that they are represented by repeating the SINGLE Dimension 2x - 4 twice whereas in all other 'higher order' prime number pairings (with prime gaps ≥ 4), they will always require MULTIPLE Dimensions representations.

We now conveniently carry out the valid exercise of endowing all Dimensions with exponent / power / index of 1 for subsequent perusal in our on-going mathematical analysis. P_1=2 is represented by CFS as Dimension $(2x - 4)^1$ (with both of its prime gap & CFS cardinality = 1); P_2=3 is represented by CFS as Dimensions $(2x - 5)^1$ & $(2x - 4)^1$ (with both of its prime gap & CFS cardinality = 2); P_3=5 is represented by CFS Dimension $(2x - 4)^1$ & $(2x - 4)^1$ (with both of its prime gap & CFS cardinality = 2), etc.

Proposition 4.1. For any given x value in independent Case 1 and Case 2, the grand total number of Dimensions [Complexity] must exactly equal to (in Case 1) the two combined subtotal number of Dimensions [Complexity] to precisely represent each of the Completely Predictable even and odd numbers, and (in Case 2) the two combined subtotal number of Dimensions [Complexity] to precisely represent each of the Incompletely Predictable prime and composite numbers.

Proof. Natural numbers can directly be constituted from either (in Case 1) the combined even & odd numbers or (in Case 2) the combined prime & composite numbers. The correctly designated infinitely many CFS of Dimensions that can be used to precisely represent both (in Case 1) the combined even & odd numbers and (in Case 2) the combined prime & composite numbers are directly and proportionately constituted in both cases from the same countable infinite set (CIS) of Dimensions used to precisely represent natural numbers. Note that all the CFS of Dimensions that can be used to precisely represent (in Case 1) the combined even & odd numbers will persistently consist of the same [solitary] Dimension $(2x - 4)^1$ after the very first Dimension $(2x - 2)^1$. The proof is now complete for Proposition 4.1.

Proposition 4.2. For any given x value (except for the x = 1 value) in independent Case 1 and Case 2, in order to comply with Information-Complexity conservation; the Dimension $(2x - N)^1$ [Complexity] representations of (in Case 1) all Completely Predictable even and odd numbers and (in Case 2) all Incompletely Predictable prime and composite numbers, must respectively be given (in Case 1) by N = 4 and (in Case 2) by N ≥ 4.

Proof. Apart from the very first Dimension $(2x - 2)^1$ representation in groupings of (in Case 1) even & odd numbers and (in Case 2) prime & composite numbers; the smallest possible N value in Dimension $(2x - N)^1$ representation for both groupings must be 4. This Dimension $(2x - 4)^1$ simply represent the maximum possible (defacto) baseline "2x - 4" Grand-Total Gaps as per Proposition 3.7 for Case 1 & Proposition 3.8 for Case 2, thus intrinsically complying in full with Information-Complexity [Input-Output] conservation. The perpetual repeated deviation of N values away from the N = 4 (minimum) in Case 2 is simply representative of the infinite magnitude of both prime & composite numbers. The proof is now complete for Proposition 4.2.

The Information-Complexity of each prime number is conserved in that it must always remain constant as explained next using the prime number 61. At position x = 61 equating to P_{18}=61, it is exactly represented by CFS Dimensions $(2x - 4)^1$, $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$ & $(2x - 8)^1$ (with both its prime gap & CFS cardinality = 6). This Virtual container CFS Dimensions style of representation at that particular x = 61 position seems to only indicate an "unknown but absolutely correct" prime number with prime gap = 6 [without revealing its actual full identity which is the number 61 = P_{18} at position x = 61 with prime gap = 6] if the x = 61's associated full information [which can only be completely expressed by calculating all preceding CFS Dimensions / prime gaps prior to this particular CFS Dimensions / prime gap] is with-held from us. Put in a different manner, we can always confirm that '61' is prime by primality tests such as trial division but we will not glean the prime gap of 6 information associated with '61' unless it is displayed in the unique CFS Dimensions representation at [hidden] position x = 61 whereby we have now seemingly gained the extra "prime gap of 6 information". However on closer inspection, in order to ultimately arrive at this unique CFS Dimensions representation containing the extra "prime gap of 6 information" in prime number 61 at position x = 61, will still require prior expressing and calculating of all preceding CFS Dimensions / prime gaps.

By invoking certain broad principles such as expressed through the Universality of Physical & Mathematical Laws, Pigeonhole principle and Proof by contradiction technique, we can categorically make the following valid statements using sound mathematical judgment. The total number of individual CFS Dimensions required to represent each and every known prime numbers will have to be infinite in magnitude simply because prime numbers are [overall] infinite in magnitude. This is equivalent to the exact mathematical statement that the standalone Dimensions YY groupings [representing the signals for "higher order" non-twin primes appearances] &/or as the front Dimensions YY (sub)groupings [which by itself is fully representative of twin primes] from the Dimensions YYYY appearances, must always recur on an indefinite basis. Common sense alone would suggest that twin primes and the "higher order" cousin primes, sexy primes, etc should aesthetically all be infinite in magnitude simply because they should regularly and universally arise as part of the components in Dimensions YY and Dimensions YYYY appearances. We shall provide the rigorous proof for this statement in the following paragraphs.

An isolated prime is defined as a prime number P such that neither P-2 nor P+2 is prime. In other words, P is not part of a twin prime pair. For example, 23 is an isolated prime, since 21 and 25 are both composite. We note that the repeated inevitable presence of Dimension YY grouping is nothing more than indicating the repeated occurrences of isolated prime. This constitutes yet another view on Dimension YY.

As general principles which are fully applicable except right at the beginning of prime & composite number integer sequences, prime gaps = 2, 4, 6,... are CIS and composite gaps = 1 & 2 are CFS. Composite numbers with composite gap = 1 are the "defacto" basic numbers needing to eternally recur simply because they are present in any two consecutive natural numbers [which themselves are also fundamentally and eternally present in the Prime-Composite mathematical landscape representing the 'composite gap = 1 signatures' to signify the actual prime gaps *per se* for non-twin prime numbers]. Composite numbers with composite gap = 2 can then be considered as the "default" basic numbers needing to eternally recur simply because they must be present as 'composite gap = 2 signatures' to signify the appearances of any prime numbers *per se*.

Thus another alternative and hugely advantageous view on prime numbers would stem from the perspective of this "manageable" CFS composite gaps [instead of the "unmanageable" CIS prime gaps] resulting in various observable clear-cut INTRINSIC patterns involving ALTERNATING PRESENCE and ABSENCE of composite numbers with composite gap = 2 in association with every CFS Dimensions representations of prime numbers with prime gaps ≥ 4. This all-important observation in the context of our Prime-Composite mathematical landscape can be classified as a mathematical law needing to be abided by all non-twin prime numbers. Twin primes with CFS Dimensions YY representations are always associated with a composite number with composite gap = 2, and are thus exempted from this law [now designated with the conveniently shortened name "Plus-Minus Composite Gap 2 Number Alternating Law"]. Two illustrative examples: a twin prime (with prime gap = 2) in its unique CFS Dimensions format is always followed by a composite number with composite gap = 2 [constant] pattern; and a cousin prime (with prime gap = 4) in its unique CFS Dimensions format is always followed by two composite numbers with composite gap = 1 & then one composite number with composite gap = 2 [combined] pattern ALTERNATING with three consecutive composite numbers with composite gap = 1 [non-combined] pattern. From this simple observation alone, one can rigorously observe that we can already / always generate an infinite magnitude of composite numbers from each of the composite gaps of 1 & 2 [automatically endowed with the same composite gaps of 1 & 2 respectively]. We can see that this composite gap = 2 ALTERNATING pattern behavior in cousin primes will NOT hold true unless twin primes & all other non-cousin primes are infinite in magnitude and integratedly supplying essential "driving mechanisms" to eternally sustain this composite gap = 2 ALTERNATING pattern behavior in cousin primes. Thus we have already discussed and established that (i) except for the very first CFS composite number 1 with associated composite gap = 3, composite numbers with composite gaps = 1 & 2 must both belong to CIS and (ii) twin primes and cousin primes in their CFS Dimensions formats are CIS closely intertwined together when depicted using composite numbers with composite gaps = 1 & 2 with each supplying their own peculiar (infinite) share of associated composite numbers with composite gap = 2 [thus contributing to the overall pool of composite numbers with composite gap = 2].

A inevitable mathematical statement in relation to "composite gap = 2 pool contribution" based on mathematical reasoning above is that, at the bare minimum, EITHER twin prime numbers OR at least one of the non-twin prime numbers must be infinite in magnitude. A beautiful natural question that follows is: Why then should all the generated sets of prime numbers from 'small gaps' [of 2 & 4] and 'large gaps' [of ≥ 6] alike not all belong to CIS thus allowing true uniformity in prime number distribution? Again we can see in Table 2 above depicting the Prime-Composite data for x = 64 that, for instance, prime numbers with prime gap = 6 must also persistently have this 'last-place' composite numbers endowed with composite gap = 2 intermittently appearing in certain rhythmic ALTERNATING patterns, thus complying with the Plus-Minus Composite Gap 2 Number Alternating Law. This CFS Dimensions representation for prime numbers with prime gaps = 6 will again generate their infinite share to the pool of associated composite numbers with composite gap = 2. The presence of this last-place composite numbers with composite gap = 2 in various alternating pattern in their appearances & non-appearances must SELF-GENERATINGLY be similarly extended in a mathematically consistent fashion *ad infinitum* to all remaining infinite number of prime gaps [which were not discussed in details above].

The preceding few paragraphs above then provide the rigorous proofs for Polignac's and Twin prime conjectures in that we have mathematically shown in a self-consistent manner that prime gaps are [necessarily] infinite (arbitrarily large) in magnitude with each individual prime gap [necessarily] generating prime numbers which are again infinite in magnitude. The Plus-Minus Composite Gap 2 Number Alternating Law, only clearly seen when prime & composite numbers are depicted in their CFS Dimensions formats and respective prime & composite number gaps, is crucial in regards to achieving those rigorous proofs. To comply with this Information-Complexity conservation, which is literally the appropriate recurrences of Dimensions $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, ..., $(2x - \infty)^1$ [all endowed with the same exponent] on an eternal basis, is what we dubbed "Dimensional analysis homogeneity" for prime [and composite] numbers whereby this 'same exponent' has to be consistently 1. Incorrect / incomplete recurrences in any of those mentioned Dimensions or in their exponents [e.g. using exponents $\frac{2}{5}$ or $\frac{3}{5}$ instead of exponent 1] would have the dire consequence of "Dimensional analysis non-homogeneity" resulting in drastically incorrect or incomplete representation of all known prime numbers. The fixed mathematical landscape "pages" for prime numbers will have to permanently display Chaos [sensitivity to initial conditions viz. positions of subsequent prime numbers are "sensitive" to positions of initial prime numbers] and Fractals [manifesting fractal dimensions with self-similarity viz. those aforementioned Dimensions for prime numbers must always be present, albeit in a non-identical manner, for all ranges of x]. Advocated in another manner, the Chaos and Fractals phenomena of those Dimensions for prime numbers above must always be correctly present signifying the accurate composition of prime and composite numbers for different (predetermined) mathematical landscape "pages" for prime numbers that are self-similar but never identical.

In this paper, we regard a 'conjecture' to become a 'hypothesis' when that particular conjecture has been rigorously proven to be true. Abiding to this notational use for those terms, we should now call Polignac's and Twin prime conjectures as Polignac's and Twin prime hypotheses.

5. Polignac's and Twin prime hypotheses

The lemmas and propositions from the preceding section above should now provide all necessary evidences to support the following Theorem I to IV (Virtual container) which will be seen to further contribute towards fully strengthening the rigorous proofs for Polignac's and Twin prime conjectures in a succinct manner. Only after successfully procuring those rigorous proofs are we finally permitted to term Polignac's and Twin prime conjectures more appropriately as Polignac's and Twin prime hypotheses.

The complete set of prime gaps are traditionally & conveniently dubbed as belonging to 'small gaps' and 'large gaps'. In this paper, we arbitrarily denote prime numbers with 'small gaps' as having prime gaps = 2 & 4 and prime numbers with 'large gaps' as having prime gaps ≥ 6. We have already established in our previous section above that (i) composite numbers with composite gap = 1 simply represent the infinite magnitude of all possible prime gaps except that for twin primes, viz. prime gaps = 4, 6, 8,... [twin primes whose prime gap = 2 will always require composite numbers with composite gap = 2 representations] and (ii) composite numbers with composite gap = 2 simply represent the appearances of any prime numbers which must compulsorily be present on an indefinite basis. Furthermore, we ingeniously establish through the Plus-Minus Composite Gap 2 Number Alternating Law that composite numbers with composite gaps = 2 present in each of the prime numbers with prime gaps ≥ 4 situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence for relevant composite numbers with composite gap = 2. This is the dominant underlying driving mechanism for the infinite magnitude of prime numbers generated by each of the prime gaps ≥ 4 scenario in a mathematically consistent manner. The case for twin primes with prime gap = 2 scenario with its own underlying driving mechanism can best be understood as the special situation of "rhythmic patterns with CONTINUAL presence" for relevant composite numbers with composite gap = 2. All these prime number INTERLINKED driving mechanisms must be perpetually present (viz. must be "self-generating") in every single prime gap in order to contribute towards generating the [complete] infinite size pool of composite numbers with composite gap = 2.

Alphonse de Polignac (1826 - 1863) was a French mathematician. In 1849, the year he was admitted to Polytechnique, he made what is known as Polignac's conjecture which relates the complete set of prime numbers to all prime gaps = 2, 4, 6, ..., ∞ [viz. all the even numbers]. We reiterate here again that Twin prime conjecture, which relates twin prime numbers to prime gap = 2, is nothing more than a subset of Polignac's conjecture.

Theorem I. The set of prime numbers P_n = 2, 3, 5, 7, 11, ..., ∞ is infinite in magnitude with each and every conceivable prime number [but not its actual identity] irrefutably, accurately and completely represented by the following formula involving prime gaps G_{Pi} viz.

$$P_{n+1} = 2 + \sum_{i=1}^{n} G_{Pi} \text{ <——- Equation (1)}$$

whereby prime number is represented here with the aid of 'n' notation instead of the usual 'i' notation used in this research paper; and i & n = 1, 2, 3, 4, 5, ..., ∞. The number '2' in this formula represent P_1, the very first (and only even) prime number.

Proof. We treat and closely analyze the function in Eq. 1 as a unique mathematical object looking for key intrinsic properties and behaviors. By definition, each prime number is assigned with a unique prime gap. Prime numbers and (thus) prime gaps are known to be infinite in magnitude. As the original true equation containing all possible prime numbers by itself (viz. without computationally supplying prime gaps {or its *proxy* composite gaps} as "input information" to generate the necessary prime numbers {or its *proxy* composite numbers} as "output complexity"), this equation will intrinsically incorporate the actual presence [but not the actual locations] of the complete set of prime numbers. See Proposition 5.1 below, which is based on the language using cardinality and pigeonhole principle, for further supporting details. The proof is now complete for Theorem I.

Theorem II. The set of prime gaps G_{Pi} = 2, 4, 6, 8, 10, ..., ∞ is infinite (arbitrarily large) in magnitude with each and every conceivable prime gap irrefutably, accurately and completely represented by Dimensions $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, ..., $(2x - \infty)^1$ which must satisfy Information-Complexity conservation in a self-consistent manner. Furthermore, this nominated method of prime gap representation using these Dimensions is purportedly the only (solitary) way to achieve the necessary conservation.

Proof. The relevant part of the proof from Proposition 4.2 stated that all prime numbers can be represented by the Dimension $(2x - N)^1$ with N ≥ 4 for any given x value (except for the x = 1 value). If each prime number is endowed with a specific prime gap value, then each such prime gap must [via logical mathematical deduction] be able to be represented by the mentioned Dimension $(2x - N)^1$. The preceding mathematical statement is absolutely correct as there is, by definition, a unique prime gap value associated with each prime number. Proposition 5.1 below predominantly based on cardinality language then provides supporting details that prime gaps are infinite (arbitrarily large) in magnitude. The proof is now complete for Theorem II.

Theorem III. To maintain Dimensional analysis (DA) homogeneity, those aforementioned Dimensions $(2x - N)^1$ [compulsorily endowed with exponent 1] from Theorem II above must repeat themselves on an indefinite basis in the following specific combinations – (i) Dimension $(2x - 4)^1$ only appearing as twin [two-times-in-a-row] and quadruplet [four-times-in-a-row] sequences, and (ii) Dimensions $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$, $(2x - 8)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of [even numbers] 2, 4, 6, 8, 10,..., ∞. To accommodate the (only) even prime number '2', the exceptions to this DA homogeneity compliance will expectedly occur right at the beginning of the prime number sequence – (i) the one-off appearance of Dimension $(2x - 2)^1$, (ii) the one-off appearances of Dimension $(2x - 4)^1$ and Dimension $(2x - 5)^1$ in a solitary [isolated] manner, and (iii) the one-off appearance of Dimension $(2x - 4)^1$ as a quintuplet [five-times-in-a-row] sequence. Theorem III can be more succinctly stated as the eternal repetitions of well-ordered sets constituted by Dimensions $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$, $(2x - 8)^1$, ..., $(2x - \infty)^1$. These sequentially arranged sets consist of countable finite sets (CFS) whereby from x = 11 onwards, each set will always commence initially as 'baseline' Dimension $(2x - 4)^1$ at x = odd number values and will always end with its last Dimension at x = even number values. Each set will also have varying cardinality with the value derived from 2, 4, 6, 8, 10, 12,..., ∞; and the correctly combined sets will always intrinsically generate the two infinite sets of prime and, by default, composite numbers in an integrated manner whereby at ever larger x values, Prime-$\pi(x)$ will overall become larger but with a DECELERATING trend and Composite-$\pi(x)$ will overall become larger but with an ACCELERATING trend.

Proof. Theorem III simply represent a mathematical summary of all the expressed characteristics of Dimension $(2x - N)^1$ when used to represent prime numbers with intrinsic display of Dimensional analysis homogeneity. This summary has mathematically been rigorously derived in Section 3 & 4 above. See Proposition 5.2 below for further supporting details on the Dimensional analysis aspect. The proof is now complete for Theorem III.

Theorem IV. Condition 1. The presence of any Dimension(s) that do not repeat itself (themselves) on an indefinite basis or with exponent other than 1 will give rise to the incomplete set of prime numbers or incorrect set of non-prime numbers ["the DA-wise mathematical impossibility argument" associated with inevitable *de novo* DA non-homogeneity], together with Condition 2. The presence of all Dimensions that do repeat themselves on an indefinite basis or with exponent of 1 will give rise to the complete set of prime numbers ["the DA-wise one and only one mathematical possibility argument" associated with inevitable *de novo* DA homogeneity] from Theorem III above, fully support the rather mute but whole point of study in that the CFS Dimensions format Virtual container representations of prime (and composite) numbers [and their respective gaps] are proven to be completely accurate when these two (mutually inclusive) conditions are met.

Proof. Theorem IV simply reflect the proof from Theorem III on all prime numbers which will be associated with Dimensional analysis homogeneity. In addition, Theorem IV also include the corollary on the inevitable appearance of incomplete prime numbers or non-prime numbers [which will always be associated with Dimensional analysis non-homogeneity] being tightly incorporated into this mathematical framework. See Propositions 5.1 & 5.2, and Corollary 5.3 below for further supporting details. The proof is now complete for Theorem IV.

Ignoring the glitch caused by the (only) even prime number '2' at the commencement of prime number sequence, we can further analyze the two components prime numbers and composite numbers in terms of (i) measurements based on cardinality of countable infinite set (CIS) and (ii) the pigeonhole principle which states that if n items are put into m containers, with n>m, then at least one container must contain more than one item. The Composite gaps can only be "finitely" constituted by the numerical values 1 & 2 (except the first Composite gap = 3) and the Prime gaps can only be "infinitely" [in an arbitrarily large magnitude] constituted by the numerical values 2, 4, 6, ..., ∞ (except the first Prime gap = 1). We note that the ordinality of all infinite prime (and infinite composite) numbers is "fixed" implying that each one of the infinite well-ordered Dimension sets conforming to the countable finite set (CFS) type as constituted by Dimensions $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$, $(2x - 8)^1$, ..., $(2x - \infty)^1$ on the respective gaps for prime (and composite) numbers, must also be "fixed". Only by this method alone can we then accommodate each and every one of the infinite prime (and composite) numbers.

Proposition 5.1. Prime gaps are infinite (arbitrarily large) in magnitude with each individual prime gap generating prime numbers which are again infinite in magnitude.

Proof. Let the cardinality of (i) all known prime numbers (CIS) derived from all prime gaps 2, 4, 6,..., ∞ sets (CIS) = T, (ii) all prime numbers (CIS) derived from prime gap 2 set (CIS) = T_2, all prime numbers (CIS) derived from prime gap 4 set (CIS) = T_4, all prime numbers (CIS) derived from prime gap 6 set (CIS) = T_6, etc. Paradoxically $T = T_2 + T_4 + T_6 +... + T_\infty$ is mathematically valid despite $T = T_2 = T_4 = T_6 =... = T_\infty$ (when defined in terms of the 'well-ordering principle' applied to the cardinality of each set). But if prime numbers derived from one or more prime gap(s) are finite in magnitude of the CFS variety, this will breach the CIS 'uniformity' property resulting in (i) DA non-homogeneity and (ii) the inequality $T > T_2 + T_4 + T_6 +... + T_\infty$. In the language of pigeonhole principle, residual prime numbers (still CIS in magnitude) not accounted for by the CFS-type prime gap(s) will have to be [incorrectly] contained in one (or more) of the other prime gap(s). Ditto for composite numbers with a similar argument able to be conjured up for the case if composite numbers from one of the composite gaps are finite in magnitude and of the CFS variety. Furthermore, the Plus-Minus Composite Gap 2 Number Alternating Law has an underlying built-in intrinsic mechanism to automatically apply to [and further generate] all prime gaps in a mathematically consistent *ad infinitum* manner. The above arguments thus constitute the rigorous proof for Proposition 5.1 as it has been shown that (i) prime gaps and (ii) prime numbers generated from each of the prime gaps, must all consist of CIS. The proof is now complete for Proposition 5.1.

Proposition 5.1 fully encompass Polignac's and Twin prime conjectures. When Proposition 5.1 is rigorously proven to be correct, it will be the overall mathematical ["quantitative"] statement to fully describe the complete set of prime numbers as generated by the Sieve of Erastosthenes algorithm. This complete set of prime numbers derived from all prime gaps can be fully represented by the Dimensions $(2x - N)^1$ concept as rigorously stated in Theorem I - IV above. As Theorem I - IV is not falsifiable, our respectful opinion is that they must act as valid Virtual container for all prime & composite numbers, and their respective gaps. Table 2 and Figure 1 on Prime-Composite mathematical landscape clearly depict perpetual ["qualitative"] features / patterns supporting (i) the Plus-Minus Composite Gap 2 Number Alternating Law (which literally can be stated as composite numbers with composite gaps = 2 present in each of the prime numbers with prime gaps $\geq$ 4 situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence for relevant composite numbers with composite gap = 2), and (ii) the composite numbers with composite gaps = 2 continual appearances in each of the (twin) prime numbers with prime gap = 2 situation.

From all the above tedious mathematical reasoning, Polignac's and Twin prime conjectures have now been proven to be true thus becoming Polignac's and Twin prime hypotheses with the overall implication that all prime numbers generated from each of the infinite (arbitrarily large) in magnitude prime gaps 2, 4, 6, ..., ∞ are again infinite in magnitude. The four steps ('mathematical foot-prints') in specific sequence required to prove Theorem I - IV can be outlined next as:

Step 1: Use the 2-variable formula with 'prime number' variable & 'prime gap' variable to "contain" all prime numbers without knowing their true identities (in a virtual manner). *Step 2:* Use Dimensions $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, ..., $(2x - \infty)^1$ to "contain" all prime gaps without knowing their true identities (in a virtual manner). *Step 3:* Define DA homogeneity as the perpetual recurrences of specific groupings of those Dimensions with exponent 1 for all ranges of x. *Step 4:* Supporting mathematical arguments that the one (single) DA homogeneity possibility will represent all prime numbers in a complete manner whereas the more than one (multiple) DA non-homogeneity possibilities will represent prime numbers in an incomplete manner or will (incorrectly) represent non-prime numbers.

Proposition 5.2. Only the defined Dimensional analysis homogeneity will always result in the correct & complete set of prime numbers.

Proof. The DA definition is completely dependent on these Dimensions. As all prime (and composite) numbers are "fixed", we can deduce from Figure 1 and Table 2 above that there is one (and only one) way to represent Information-Complexity conservation using our defined Dimensions. Thus, there is one (and only one) way to depict all prime numbers using these Dimensions in a self-consistent manner and this can only be achieved with the one (and only one) DA homogeneity possibility. The proof is now complete for Proposition 5.2.

Corollary 5.3. The defined Dimensional analysis non-homogeneity will always result in the incorrect & / or incomplete set of prime numbers.

Proof. Proposition 5.2 equates DA homogeneity with the correct & complete set of prime numbers with full mathematical consistency. There are "more than one" DA non-homogeneity possibilities. For instance, if a particular $(2x - 4)^1$ Dimension derived from $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$,..., $(2x - \infty)^1$ terminates prematurely and does not perpetually repeat [resulting in loss of continuity and thus depicting one DA non-homogeneity possibility]; then there are intuitively two 'broad' DA possibilities here; namely, (one) DA homogeneity possibility and "all others" endowed with DA non-homogeneity possibilities. This meant that the mathematical consistency of Dimensions $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$, $(2x - 8)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of [even numbers] 2, 4, 6, 8, 10,..., ∞ will be halted without justification. For optimal clarity, we have treated all those Dimensions above using exponents and depict them as $(2x - 4)^1$, $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$, $(2x - 8)^1$,..., $(2x - \infty)^1$. Then a particular Dimension, using the $(2x - 4)^1$ example (endowed with exponent 1), that stop recurring at some point in the prime number sequence would have DA non-homogeneity and be depicted against-all-trends as $(2x - 4)^0$ when endowed with a totally different exponent – which is arbitrarily set as 0 in this case. Thus a Dimension that stop recurring will result in the well-ordered CFS sets from the progressive groupings of [even numbers] 2, 4, 6, 8, 10,..., ∞ for Dimensions $(2x - 5)^1$, $(2x - 6)^1$, $(2x - 7)^1$, $(2x - 8)^1$,..., $(2x - \infty)^1$ to stop existing (and ultimately for sequential prime numbers to stop appearing) at that point using this grouping method – with the likely ensuing outcome that prime and composite numbers are overall [incorrectly] finite in magnitude. Finally, a Dimension with fractional exponent values (other than 1) will always result in non-prime and non-composite (fractional) numbers. The proof is now complete for Corollary 5.3.

Thus the seemingly small but utterly essential sequential mathematical steps in (i) representing all prime numbers using a '2-variable function' (made up of prime number variable and prime gap variable) and (ii) then further representing all prime gaps with the defined Dimensions, will crucially allow proper DA process to happen in the absolute correct way. The 'strong' principle argument mathematical end-result is that DA homogeneity will equate to the complete set of prime numbers whereas DA non-homogeneity will not equate to the complete set of prime numbers. One could additionally advocate for a 'weak' principle argument supporting DA homogeneity for prime numbers in that nature should not "favor" any particular Dimension(s) to terminate and therefore DA non-homogeneity does not, and cannot, exist for prime numbers.

By an L-function, we generally mean a Dirichlet series with a functional equation and an Euler product. Contextually, the simplest example of an L-function is Riemann zeta function on which the 1859 Riemann conjecture / hypothesis is based upon. L-functions are ubiquitous in number theory and have applications to mathematical physics and cryptography. They arise from and encode information about a number of mathematical objects and it is necessary to exhibit these objects along with the L-functions themselves since typically we need these objects to compute L-functions. For examples, L-functions can come from modular forms, elliptic curves, number fields, and Dirichlet characters, as well as more generally from automorphic forms, algebraic varieties, and Artin representations. The resulting mammoth 'L-functions and Modular Forms Database' (LMFDB) creation by an international group of mathematicians is based on these examples. The LMFDB website location address is http://www.lmfdb.org/ with its launching celebrated on May 10, 2016. Thus LMFDB can be considered an uncharted mathematical terrain providing a detailed atlas of mathematical objects that highlights deep relationships and serves as a guide to latest research happening in physics, computer science and mathematics. [The LMFDB was first conceived at an American Institute of Mathematics workshop in 2007. Elliptic curves arise naturally in many parts of mathematics and can be described by a simple cubic equation. They also form the basis of cryptographic protocols used by most of the major internet companies including Google, Facebook and Amazon. Modular forms are more mysterious objects constituted by complex functions with an almost unbelievable degree of symmetry. The two mathematical worlds of elliptic curves and modular forms are remarkably connected via their L-functions. It is this deep connection that was in essence required in the late 20th century by famous British number theorist Andrew John Wiles to successfully achieve his proof of Fermat's Last Theorem.]

In the grand scheme of things, this paper manifests the classically encountered phenomenon that pure and applied mathematics during, and resulting from, the derivation of many mathematical proofs are largely inseparable. Some of the less conventional aspects of the resulting applied mathematics in regards to 'Living Things' and 'Nonliving Things' are outlined next towards the end of the Conclusion section below. Being Incompletely Predictable entities, there are notable near-identical occurrences of exception-to-the-rule events in prime numbers in that the very first & only even prime number is '2', and in Riemann zeta function in that the very first & only negative Gram [y=0] intercept at the designated $\sigma = \frac{1}{2}$ critical line occur only when parameter t = 0 with the resulting $\zeta(\frac{1}{2})$ = -1.4603545 (rounded off to seven decimal places) value. We speculatively hope and selfishly dream that this applied mathematics pathway resulting from solving Polignac's and Twin prime conjectures (in this research paper), and Riemann conjecture / hypothesis (in our other research paper) could one day even lead to providing scientific breakthrough answers on whether 'Living Things' arise via either the Evolution process [as per atheist belief] or the Creation process [as per religious belief]!

6. Conclusions

The following overall property has been harnessed in this paper to obtain the successful proofs on Polignac's and Twin prime conjectures: The countable infinite set (CIS) of [Completely Predictable] natural numbers 1, 2, 3, 4, 5,... with its countable finite set (CFS) natural number gap = 1 are completely constituted by two complementary sets of numbers, namely (i) the CIS of [Incompletely Predictable] prime numbers 2, 3, 5, 7, 11,... with CIS prime number gaps = 2, 4, 6, 8, 10,... and (ii) the CIS of [Incompletely Predictable] composite numbers 1, 4, 6, 8, 9,... with CFS composite gaps = 1 & 2. This overall property is applicable to all prime and composite numbers alike except right at the beginning of their number sequences due to the one-off exception that the very first (and only) prime number '2' is an even number.

It is commonly advocated that the rigorous proof for Riemann conjecture / hypothesis would be instrumental in proving the efficacy of techniques that estimate prime counting function (traditionally denoted by $\pi(x)$) efficiently and reasonably well. This point to a deep-seated connection between Riemann conjecture / hypothesis and prime numbers. In this research paper, we have provided aesthetical if not rigorous proofs for Polignac's and Twin prime conjectures by using a Virtual container [literally equivalent to the relevant Theorem I to IV above] and lessons arising out of this method should also be applicable in regards to rigorous proof for Riemann conjecture / hypothesis, and *vice versa*. Initially by essentially using the unique CFS Dimensions representation for all prime & composite number gaps, we outlined the full self-consistent mathematical contents of our rigorous proofs mainly using Information-Complexity conservation together with its naturally occurring Plus-Minus Composite Gap 2 Number Alternating Law. The rigorous arguments for the inevitable presence of this Alternating Law have been mathematically provided for in this paper. This Law can best be perceived as a "descriptive" law on certain observed self-generating rhythmic behaviors [alternating presence and absence] of composite numbers with composite gap = 2 required to accurately "describe" the actual appearances of all prime numbers except twin prime numbers (which are endowed with the smallest possible prime gap = 2). Twin primes will still manifest rhythmic behaviors that are exempted from obeying this Alternating Law in the sense that composite numbers with composite gap = 2 occurrences have to recur on a continual [instead of alternating] basis for twin prime numbers. We subsequently employed Dimensional analysis on those Dimensions to show that the complete & correct sets of prime (& composite) numbers will always have to comply with Dimensional analysis homogeneity, and that the incomplete or incorrect set(s) of prime (& composite) numbers will always have features of Dimensional analysis non-homogeneity. The CFS gaps = 1 & 2 [of finite magnitude] for composite numbers can be seen to confer great advantages since they are relatively easy to work with. This is in sharp contrast to the CIS gaps = 2, 4, 6, 8, 10,... [of infinite magnitude] for prime numbers which must be mathematically more difficult to work with.

Proving Riemann, Polignac's and Twin prime conjectures / hypotheses can simplistically be seen as theoretically needing to analyze relevant "elements" of infinite magnitude whereby each and every one of these "elements" needs to [impossibly] be proven (totaling an infinite number of times) to fulfill certain "criteria". This three open problems would purportedly belong to 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' in the sense that we postulate the rigorous proofs for those problems should arbitrarily be conducted in the following sequence: (i) initially having to invoke the 'solitary-style' Virtual container research method to mathematically contain those CIS "elements" and (ii) subsequently / concurrently having to invoke the correct analysis on finite number of "special properties" arising from the relevant Virtual container. We point out here that due to the beastly nature of these types of open problems, the footprints of our rigorous proofs for Polignac's and Twin prime conjectures / hypotheses have not been easily or fully depicted sequentially as (i), and then followed by (ii). One could envision our new Virtual container research method to anticipatedly be accepted as [futuristic] applied mathematics for solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' containing Incompletely Predictable "elements" of infinite magnitude. In regards to proving Riemann conjecture / hypothesis, we have similarly utilized our Virtual container research method to achieve this goal in our other research paper [6].

"Prime numbers can be described as atoms. What mathematicians have been missing is a kind of mathematical number spectrometer. Chemists have an atomic spectrometer machine that, if we give it a molecule, will tell us the atoms that it is built from. Mathematicians have never invented a mathematical version of this. The proof of Riemann conjecture / hypothesis would have given us perfect understanding on how prime numbers work, and translating this into essential knowledge allowing construction of this prime number spectrometer. Suddenly all cryptic codes are breakable. No internet transaction would be safe as the whole of e-commerce depends on the integrity of humongous [non-prime] numbers (molecules) to be anonymously or secretly constituted from its basic prime numbers (atoms). In other words, breaching this integrity by identifying the prime numbers constituents of relevant humongous numbers using prime number spectrometer would have massive implication in that it has now brought the whole of e-commerce to its knees overnight."

The truthfulness of the preceding narrative paragraph can now be beautifully refuted by us here as follows. Having solved Riemann or Polignac's or Twin prime conjecture / hypothesis is simply irrelevant because the CIS of prime numbers must be treated as Incompletely Predictable / Pseudorandom numbers abiding by ”Complex Elementary Fundamental Laws” that usually involve Incompletely Predictable Laws [and not ”Simple Elementary Fundamental Laws” that usually involve Completely Predictable Laws]. Thus the construction of prime number spectrometer is intuitively and literally a mathematical impossibility. We have thus dispelled the doom-and-gloom prophecy that financial disaster might follow when successful proof of Riemann conjecture / hypothesis occur.

Without going into finer details using number theory, the irrationality measure (or irrationality exponent or approximation exponent or Liouville-Roth constant) of any real number is a measure of how ”closely” it can be approximated by rationals. For a rational number, the irrationality measure is 1. The Thue-Siegel-Roth theorem states that for an algebraic irrational number, viz. real but not rational number, then the irrationality measure is 2. Transcendental irrational numbers have irrationality measure 2 or greater; for instance, the transcendental Euler's number e (= 2.718281828459...) has irrationality measure equal to 2. The [seemingly] simplistic-looking Liouville numbers is typified by Liouville's constant, sometimes also called Liouville's number, a real number defined by $L = \sum_{n=1}^{\infty} 10^{-n!} = 0.110001000000000000000001...$ with '!' denoting factorial. These numbers are irrational numbers of [the relatively more ”complex”] transcendental types instead of [the relatively less ”complex”] algebraic types; and their numerical make-up consist of just '0' and '1' digits. Despite this apparently simple-looking numerical make-up of Liouville numbers (as opposed to more complicated-looking numerical make-up of e), they are precisely those numbers [paradoxically] having infinite irrationality measure. For the above, we would assign all [Completely Predictable] rational numbers to obeying Simple Elementary Fundamental Laws, and all [Incompletely Predictable] irrational numbers to obeying Complex Elementary Fundamental Laws.

We now endeavor to compare, contrast and reconcile the two entities 'Living Things' and 'Nonliving Things'. Rigorous mathematical proofs must obviously be associated with 100% certainty. This can only apply to Simple and Complex Elementary Fundamental Laws on 'Nonliving Things'. Diverging onto proofs for Simple and Complex Emergent Fundamental Laws on 'Living Things', one observe that they can never be associated with perfect 100% certainty simply because we are dealing with ”ALIVE” Living Things with dynamic spatial and temporal properties that could not be totally predictable. In this setting, the proofs for the Simple cases [e.g. physiologically modeling Cardiac Output (CO) equals to Heart Rate (HR) multiplied by Stroke Volume (SV) in the Cardiovascular System (CVS)] will comparatively be less challenging to derive than the Complex cases [e.g. physiologically modeling complex Human Brain functions using Neural Networks in the Central Nervous System (CNS)].

Note that the terms 'Elementary' and 'Emergent' are used here in the preceding and subsequent paragraphs to, respectively, denote 'Nonliving Things' and 'Living Things'. In real life situation for 'Living Things', there will always be the perpetual presence of infinitesimally tiny [and unpredictable] ”Chaos and Fractals physiological variability”, for instance, in the Simple Emergent Fundamental Law CO = HR X SR. This variability phenomenon will inevitably occur even in the most relaxed state of a person in deep sleep whereby dynamic processes such as intrinsic neuro-endocrine continuous signal input to the heart must occur on a permanent basis thus giving rise to this variability.

For the medically oriented readers, we finish off this paper by touching on Evidence based Medicine (EBM) and Evidence based Practice (EBP). Both could comply with either Simple or Complex Emergent Fundamental Laws on Living Things (namely, Human Beings in this scenario). EBM is typically depicted pictorially as a 'Pyramidal hierarchy of Literature Review' classifying available medical research materials into [the most powerful] Systematic Reviews down to [the least powerful] Expert Opinion.

Then EBP = Clinician Experience + Patient Expectation + Best Practice; with Best Practice being roughly equated with EBM. For doctors and medical researchers confronted daily with responsibly following and improving up-to-date EBP and EBM, they must be familiar with most statistical tools employed in medical research with the classic example being research hypothesis expressed as a null hypothesis [the ”devil's advocate” position] and alternative hypothesis. The level of statistical significance for hypothesis testing is often expressed as the so-called *p*-value. Whilst there is relatively little justification why a [cut-off] significance level of 0.05 is widely used in academic research [rather than 0.01 or 0.10]; we could be particularly more confident in our results by setting a more stringent level of (say) 0.01 [a 1% chance or less; 1 in 100 chance or less]. Despite this experimental / research tactic, we could strive to, but never, achieve perfect or 100% confidence in our results by setting ever more stringent levels.

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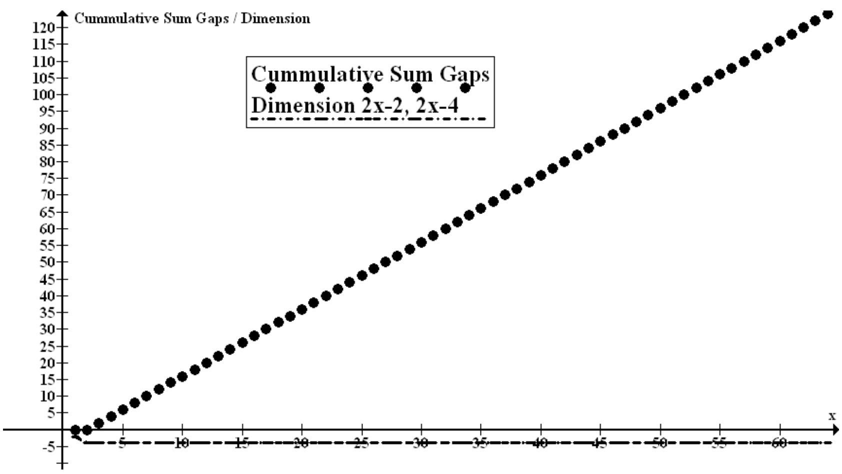
Appendix 1. Tabulated and graphical depictions on Even-Odd mathematical landscape for x = 64



Even Odd.png

Figure 2: Even-Odd mathematical (graphed) landscape using data obtained for x = 64.

In Figure 2, Dimensions 2x - 2 & 2x - 4 are symbolically represented by -2 & -4 with 2x - 4 displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must reset itself onto this (Grand-Total Gaps) 'baseline' Dimension after the initial Dimension 2x - 2 on a permanent basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by the two negative integers are Completely Predictable with both Even-$\pi(x)$ and Odd-$\pi(x)$ becoming larger at a constant rate. We note that there is a complete absence of Chaos & Fractals phenomena being manifested in our graph.

x	E_i or O_i, Gaps	ΣEO_x-Gaps	Dimension	x	E_i or O_i, Gaps	ΣEO_x-Gaps	Dimension
1	O1, 2	0	2x-2	33	O17, 2	62	Y
2	E1, 2	0	Y	34	E17, 2	64	Y
3	O2, 2	2	Y	35	O18, 2	66	Y
4	E2, 2	4	Y	36	E18, 2	68	Y
5	O3, 2	6	Y	37	O19, 2	70	Y
6	E3, 2	8	Y	38	E19, 2	72	Y
7	O4, 2	10	Y	39	O20, 2	74	Y
8	E4, 2	12	Y	40	E20, 2	76	Y
9	O5, 2	14	Y	41	O21, 2	78	Y
10	E5, 2	16	Y	42	E21, 2	80	Y
11	O6, 2	18	Y	43	O22, 2	82	Y
12	E6, 2	20	Y	44	E22, 2	84	Y
13	O7, 2	22	Y	45	O23, 2	86	Y
14	E7, 2	24	Y	46	E23, 2	88	Y
15	O8, 2	26	Y	47	O24, 2	90	Y
16	E8, 2	28	Y	48	E24, 2	92	Y
17	O9, 2	30	Y	49	O25, 2	94	Y
18	E9, 2	32	Y	50	E25, 2	96	Y
19	O10, 2	34	Y	51	O26, 2	98	Y
20	E10, 2	36	Y	52	E26, 2	100	Y
21	O11, 2	38	Y	53	O27, 2	102	Y
22	E11, 2	40	Y	54	E27, 2	104	Y
23	O12, 2	42	Y	55	O28, 2	106	Y
24	E12, 2	44	Y	56	E28, 2	108	Y
25	O13, 2	46	Y	57	O29, 2	110	Y
26	E13, 2	48	Y	58	E29, 2	112	Y
27	O14, 2	50	Y	59	O30, 2	114	Y
28	E14, 2	52	Y	60	E30, 2	116	Y
29	O15, 2	54	Y	61	O31, 2	118	Y
30	E15, 2	56	Y	62	E31, 2	120	Y
31	O16, 2	58	Y	63	O32, 2	122	Y
32	E16, 2	60	Y	64	E32, 2	124	Y

Table 3: Even-Odd mathematical (tabulated) landscape using data obtained for x = 64.

The definitive derivation of the data in Table 3 is given next and this is clearly illustrated by two examples given for position x = 31 & 32. For i & x ∈ 1, 2, 3, ..., ∞; ΣEO_x-Gap = ΣEO_{x-1}-Gap + Gap value at E_{i-1} or Gap value at O_{i-1} whereby (i) E_i or O_i at position x is determined by whether the relevant x value belongs to an even (E) or odd (O) number, and (ii) both ΣEO_1-Gap and ΣEO_2-Gap = 0. Example for position x = 31: 31 is an odd number (O16). Our desired Gap value at O15 = 2. Thus ΣEO_{31}-Gap (58) = ΣEO_{30}-Gap (56) + Gap value at O15 (2). Example 2 for position x = 32: 32 is an even number (E16). Our desired Gap value at E15 = 2. Thus ΣEO_{32}-Gap (60) = ΣEO_{31}-Gap (58) + Gap value at E15 (2).

Using the relevant data above, we have now painstakingly tabulate (in Table 3) and graphically map (in Figure 2) the [Completely Predictable] Even-Odd mathematical landscape for x = 64. Legend: E = even, O = odd. Involved Dimensions are 2x - 2 & 2x - 4 with Y denoting Dimension 2x - 4 for visual clarity. This Even-Odd mathematical landscape, made up of Dimension 2x - 4 (except for the very first and only Dimension 2x - 2), will intrinsically incorporate even and odd numbers in an integrated manner. Except for the very first odd number, we note that all Completely Predictable even and odd numbers, and all their identical gaps, can be represented by the countable finite set of [single] Dimension 2x - 4.