On the Logical Inconsistency of Einstein’s Time Dilation

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27 March, 2017

ABSTRACT

Time dilation is a principal feature of the Special Theory of Relativity. It is purported to be independent of position, being a function only of uniform relative velocity, via the Lorentz Transformation. However, it is not possible for a ‘clock-synchronised stationary system’ of observers $K$ to assign a definite time to any ‘event’ relative to a ‘moving system’ $k$ using the Lorentz Transformation. Consequently, the Theory of Relativity is false due to an insurmountable intrinsic logical contradiction.

1 Introduction

In a previous paper [1] I proved that a system of clock-synchronised stationary observers is inconsistent with the Lorentz Transformation. Assuming both leads to a contradiction. Herein I synchronise clocks in Einstein’s ‘stationary system’ $K$ by mathematical construction and prove that his ‘stationary system’ $K$ cannot then assign any definite time $\tau$ anywhere in his ‘moving system’ $k$ for any given position $x$ and time $t$ in his ‘stationary system’ $K$. From this it follows immediately that Einstein’s ‘time dilation’ is false because there is no common determinable time dilation for all observers in Einstein’s ‘stationary system’ $K$.

2 Stationary and moving clocks

In §4 of his 1905 paper, Einstein [2] compared one clock ‘at rest’ relative to the ‘moving system’ $k$, with all the synchronised identical clocks ‘at rest’ relative to his ‘stationary system’ $K$:

“…we imagine one of the clocks which are qualified to mark the time $t$ when at rest relatively to the stationary system, and the time $\tau$ when at rest relatively to the moving system, to be located at the origin of the co-ordinates of $k$, and so adjusted that it marks the time $\tau$. What is the rate of this clock, when viewed from the stationary system?”

“Between the quantities $x$, $t$, and $\tau$, which refer to the position of the clock, we have, evidently, $x = vt$ and

$$\tau = \frac{1}{\sqrt{1 - v^2/c^2}} \left( t - \frac{vx}{c^2} \right).$$

“Therefore,

$$\tau = t \sqrt{1 - v^2/c^2} = t - \left( 1 - \sqrt{1 - v^2/c^2} \right) t$$

“whence it follows that the time marked by the clock (viewed in the stationary system) is slow by $1 - \sqrt{1 - v^2/c^2}$ seconds per second, …” [2, §4]

In Einstein’s notation the coordinates of his assumed system of clock-synchronised stationary observers $K$ are $x, y, z, t$, those corresponding to the ‘moving system’ $k$ are $\xi, \eta, \zeta, \tau$, illustrated in figure 1, for his initial conditions.

The Lorentz Transformation is,

$$\tau = \beta \left( t - \frac{vx}{c^2} \right), \quad \xi = \beta (x - vt), \quad \eta = y, \quad \zeta = z,$$

$\beta = 1/\sqrt{1 - v^2/c^2}$. (1)

Note that according to the Lorentz Transformation the time $\tau$ depends upon both $t$ and $x$. Einstein specifically set $x = 0 = \xi$ for $\tau = t = 0$, shown in figure 1.
According to Einstein’s time dilation, \( \tau > t \) at the time \( t \) of Einstein’s ‘stationary system’ \( K \) the clock at Einstein’s ‘stationary system’ \( K \) reads time \( \tau \), reads the same time \( \tau \) at all positions \( x \) in the \( K \) system. The clock at the origin of the ‘moving system’ \( k \), where \( \xi = 0 \), reads \( \tau > 0 \). The origin \( \xi = 0 \) has advanced a distance \( x = vt \).

After a time \( t > 0 \) the origin of Einstein’s ‘moving system’ \( k \) has advanced a distance \( x = vt \), illustrated in figure 2. At this time \( t \) all the observers in Einstein’s ‘stationary system’ \( K \) read the same time \( t \) on their clocks no matter where they are located, because their clocks are synchronised. The clock at Einstein’s \( \xi = 0 \) of the ‘moving system’ \( k \) reads time \( \tau > 0 \). According to Einstein’s time dilation, \( \tau = t/\beta \).

All of Einstein’s ‘stationary observers’ in \( K \) are entitled to look at the same moving clock. An observer located at any \( x^* \neq x \) in Einstein’s ‘stationary system’ \( K \) can observe the clock in the ‘moving system’ \( k \) at any synchronised time \( t \) of Einstein’s ‘stationary system’ \( K \), to see what the clock reads. However, Einstein’s assumption that a system of clock-synchronised stationary observers is consistent with the Lorentz Transformation is demonstrably false. An observer \( x^* \) does not find the same \( \tau \) or the same \( \xi \) as observer \( x \) does.

### 3 Systems of stationary observers

The Lorentz Transformation between systems of observers stationary with respect to their own systems is [1, §2],

\[
\begin{align*}
\tau &= \beta \left( t - vx/c^2 \right), \\
x_k &= \kappa x, \\
\xi_k &= \beta \left( x_k - vt_k \right) = \beta \left( \left( \kappa/\beta^2 + v^2/c^2 \right)x + vt \right), \\
t_k &= t + (\kappa - 1)vx/c^2, \\
\beta &= 1/\sqrt{1-v^2/c^2}, \\
\kappa &= \in \mathbb{R}.
\end{align*}
\]

(2)

The Inverse Stationary Lorentz Transformation is [1, §2]

\[
\begin{align*}
t &= \beta \left( t + vx/c^2 \right), \\
x_k &= \kappa x, \\
\xi_k &= \beta \left( x_k + vt \right) = \beta \left( \left( \kappa/\beta^2 + v^2/c^2 \right)x - vt \right), \\
t_k &= t - (\kappa - 1)vx/c^2, \\
\beta &= 1/\sqrt{1-v^2/c^2}, \\
\kappa &= \in \mathbb{R}.
\end{align*}
\]

(3)

By means of the Inverse Stationary Lorentz Transformation (3),

\[
\Delta t_k = \Delta t = \Delta t - \sqrt{1-v^2/c^2}.
\]

This is Einstein’s time-dilation equation. However, if \( l_0 \) is the length of a ‘rigid’ rod in the moving system \( k \), according to the system \( K \) its length is [1, §4],

\[
\Delta x = \beta l_0 = \frac{l_0}{\sqrt{1-v^2/c^2}}.
\]

which is length expansion, not length contraction. Thus, although no observer in the stationary system \( K \) is clock-synchronised, every observer \( x_k \) of the stationary system \( K \) observes the same time-interval \( \Delta t \) in \( K \) and the same time-dilated interval \( \Delta t \) in \( k \), but at the expense of length contraction and clock-synchronisation [1, §4]. This is irreconcilable with Einstein’s theory.

### 4 Clock-synchronised observers

The Clock-Synchronised Lorentz Transformation is [1, §5],

\[
\begin{align*}
\tau_k &= \beta \left( \tau - vx/c^2 \right), \\
x_k &= \kappa x, \\
\xi_k &= \beta \left( x_k - vt \right), \\
\beta &= 1/\sqrt{1-v^2/c^2}, \\
\kappa &= \in \mathbb{R}.
\end{align*}
\]

(4)

Although all observers in \( K \) are clock-synchronised to a common time \( t \), only \( x_1 \) is not a function of the time \( t \). Thus, only \( x_1 \) is stationary. All other observers in \( K \) cannot be stationary.

The Inverse Clock-Synchronised Lorentz Transformation is [1, §5],

\[
\begin{align*}
t_k &= \beta \left( t + vx/c^2 \right), \\
x_k &= \kappa x, \\
\xi_k &= \beta \left( \xi_k + vt \right), \\
\beta &= 1/\sqrt{1-v^2/c^2}, \\
\kappa &= \in \mathbb{R}.
\end{align*}
\]

(5)

Although all observers in \( k \) are clock-synchronised to a common time \( \tau \), only \( \xi_1 \) is not a function of the time \( \tau \). Thus, only \( \xi_1 \) is stationary. All other observers in \( k \) cannot be stationary.

From this it follows that, from their vantage points, no two observers in the ‘stationary system’ \( K \) read either the same time or same time-interval on the moving clocks in system \( k \);
Since all observers are clock-synchronised with respect to their own systems, all observers in the \( K \) system observe the common clock time-interval \( \Delta t \). Observer \( x_\kappa \) of system \( K \) watches the clock of \( \xi_\kappa \) in the ‘moving system’ \( k \) and observes the clock time-interval \( \Delta t_\kappa \) of \( \xi_\kappa \) in system \( k \). Then from (4),

\[
\Delta t_\kappa = \kappa \beta \Delta t.
\]

Thus, after a time-interval \( \Delta t \) in \( K \) any observer \( x_\kappa \) in the clock-synchronised system \( K \) reads the time-interval \( \Delta t_\kappa \) at \( \xi_\kappa \) in the \( k \) system. Each observer \( x_\kappa \) observes a different time and a different time-interval on the corresponding clock held by observer \( \xi_\kappa \) in system \( k \). For example, the observer \( \kappa = 1 \) located at \( x_1 \) in system \( K \) observes not time dilation at \( \xi_1 \) but time expansion at \( \xi_1 \): \( \Delta t_1 = \beta \Delta t \). The observer \( \kappa = 1/\beta \) located at \( x_{1/\beta} \) in system \( K \) observes no change in the time-interval of the clock at \( \xi_{1/\beta} \) in system \( k \): \( \Delta t_{1/\beta} = \Delta t \). The observer \( \kappa = 1/\beta^2 \) located at \( x_{1/\beta^2} \) observes the time-interval \( \Delta t_{1/\beta^2} \) at \( \xi_{1/\beta^2} \) in system \( k \):

\[
\Delta t_{1/\beta^2} = \frac{\Delta t}{\beta} = \Delta t \sqrt{1 - \frac{v^2}{c^2}},
\]

which is Einstein’s time dilation equation. In all cases, \( \Delta t_\kappa = \kappa \tau_\kappa \), in accordance with (4), since \( \Delta t_1 = \beta \Delta t \). Conversely, all observers \( \xi_\kappa \) in the \( k \) system read a common time \( \tau \) and common time-interval \( \Delta \tau \), finding that the clock at \( x_\kappa \) in \( K \) reads, from (5), the time-interval,

\[
\Delta t_\kappa = \kappa \beta \Delta \tau.
\]

Neither system \( K \) nor system \( k \) can assign any particular time-interval to one another because no observer in the one observes the same time-interval at all locations in the other.

5 Conclusions

Einstein’s ‘clock-synchronised stationary system’ \( K \) cannot assign any common time dilation to the observers in his ‘moving system’ \( k \). A system of clock-synchronised stationary observers is not consistent with the Lorentz Transformation. Einstein’s time dilation is inconsistent with the Lorentz Transformation. It is therefore false. Hence, the Theory of Relativity is false.