Beal Conjecture Original Directly Proved

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Using a direct construction approach, the author proves the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. Two main types of equations are involved, namely, the equation $A^x + B^y = C^z$ and an equation which will be called a tester equation. A tester equation has similar properties as $A^x + B^y = C^z$, and will be used to determine the properties of $A^x + B^y = C^z$. Also, two types of tester equations, namely, a literal tester equation and a numerical tester equation will be applied. Each side of $A^x + B^y = C^z$ and a tester equation is reduced to unity by division. The non-unity sides are justifiably equated to each other to produce a new equation which will be called the master equation. The side of the master equation involving the terms of the tester equation will be called the tester side of the master equation. Three versions of the proof are presented. In Version 1 proof, the tester equation was the literal equation $G^m + H^n = I^p$, but in Versions 2 and 3 proofs, the tester equations were the numerical tester equations, $2^9 + 8^3 = 4^5$ and $3^3 + 6^3 = 3^5$, respectively. By inspection, using an approach in which the corresponding elements on the right and left sides of the master equation are equated to each other, it is determined that if $A^x + B^y = C^z$, A, B and C have a common prime factor. The proof is very simple, and occupies a single page, and high school students can learn it.

Beal Conjecture Original Directly Proved (Version 1)

(Proof using a literal tester equation)

In this version, using a literal tester equation, the author proves directly the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

Given: $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$.

Required: To prove that $A, B,$ and C, have a common prime factor.

Let G, H, I, m, n, p be positive integers such that $m, n, p > 2$, and $G^m + H^n = I^p$ is true. The equation $G^m + H^n = I^p$ will be called the tester equation (a true equation having similar properties as $A^x + B^y = C^z$ and which will be used to determine the properties of $A^x + B^y = C^z$).

Consider the true equations $A^x + B^y = C^z$ (1); and $G^m + H^n = I^p$ (2).

From (1), one obtains $\frac{A^x + B^y}{C^z} = 1$ (3); and similarly, from (2), one obtains $\frac{G^m + H^n}{I^p} = 1$ (4)

(Dividing both sides of each equation by the right side)

Let r, s, t be prime factors of A, B and C respectively such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E, F are positive integers. Also, let h, q, w be prime factors of G, H and I respectively such that $G = Jh$, $H = Kq$, and $I = Lw$, where J, K, L are positive integers, and $h = q = w$.

Then equations (3) and (4) become

$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = 1$ and $\frac{(Jh)^m + (Kq)^n}{(Lw)^p} = 1$, respectively.

Since $\frac{(Dr)^x + (Es)^y}{(Ft)^z} = 1$ and $\frac{(Jh)^m + (Kq)^n}{(Lw)^p} = 1$

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = \frac{\overbrace{(Jh)^m + (Kq)^n}^{\text{Tester side}}}{(Lw)^p} \quad (6) \text{ (Master equation)}$$

(By the transitive equality property)

One will next show that $r = s = t$ by inspection, using a correspondence approach in which the corresponding elements on the right and left sides of equation (6) are equated to each other.

Example on the principle applied below in tables **A** and **B**

If $\frac{U + V}{W} = \frac{3 + 5}{8} = 1$

Then either

$U = 3$, $V = 5$, and $W = 8$ **or**

$U = 5$, $V = 3$, and $W = 8$

In either case,

$$\frac{U + V}{W} = \frac{3 + 5}{8} = \frac{5 + 3}{8} = 1$$

$(Dr)^x = (Jh)^m$, $(Es)^y = (Kq)^n$, $(Ft)^z = (Lw)^p$, **or**

$(Dr)^x = (Kq)^n$, $(Es)^y = (Jh)^m$, $(Ft)^z = (Lw)^p$

A $Dr = Jh$; $x = m$; $D = J$; $r = h$;	$Es = Kq$, $y = n$; $E = K$; $s = q$;	$Ft = Lw$, $z = p$ $F = L$ $t = w$.	**Note:** $D, E, F, r, s, t, J, K, L$ h, q, w are all integers. $x, y, z > 2$; $m, n, p > 2$. $h = q = w$

From above, $r = h$, $s = q$, $t = w$. Since $h = q = w$ (given), $r = s = t$. Therefore, Dr, Es, and Ft have a common prime factor. Also, since $A = Dr$, $B = Es$, and $C = Ft$, $A, B,$ and C, have a common prime factor. .**OR**

B $Dr = Kq$; $x = n$; $D = K$; $r = q$;	$Es = Jh$, $y = m$; $E = J$; $s = h$;	$Ft = Lw$, $z = p$ $F = L$ $t = w$.	**Note:** $D, E, F, r, s, t, J, K, L$ h, q, w are all integers. $x, y, z > 2$; $m, n, p > 2$. $h = q = w$

From above, $r = q$, $s = h$, $t = w$. Since $h = q = w$ (given), $r = s = t$. Therefore, Dr, Es, and Ft have a common prime factor. Also, since $A = Dr$, $B = Es$, and $C = Ft$, $A, B,$ and C have a common prime factor. Observe above that in Table A or Table B, it is shown that $r = s = t$; and therefore, if $A^x + B^y = C^z$, then $A, B,$ and C have a common prime factor. The proof is complete.

Beal Conjecture Original Directly Proved (Version 2)
(Proof using a numerical tester equation)

In this version, using a numerical tester equation, the author proves the original Beal conjecture that if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor.

Given: $A^x + B^y = C^z$, where A, B, C, x, y, z be positive integers such that $x, y, z > 2$.

Required: To prove that $A, B,$ and C, have a common prime factor.

Consider the true equations $A^x + B^y = C^z$ (1); and $2^9 + 8^3 = 4^5$ (2).

Equation (2) will be called a tester equation (a true equation having similar properties as $A^x + B^y = C^z$ and which will be used to determine the properties of $A^x + B^y = C^z$).

From (1), one obtains $\frac{A^x + B^y}{C^z} = 1$ (3); and similarly, from (2), one obtains $\frac{2^9 + 8^3}{4^5} = 1$ (4.)

(Dividing both sides of each equation by the right side)

Let r, s, t be prime factors of A, B and C respectively such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E, F are positive integers. Then equation (3) becomes

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = 1 \quad (5).$$

Since $\frac{(Dr)^x + (Es)^y}{(Ft)^z} = 1$, and $\frac{2^9 + 8^3}{4^5} = 1$

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = \frac{\overbrace{2^9 + 8^3}^{\text{Tester side}}}{4^5}$$

(By the transitive equality property)

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = \frac{\overbrace{(2 \bullet 1)^9 + (2 \bullet 4)^3}^{\text{Tester side}}}{(2 \bullet 2)^5} \quad (6) \quad \text{(Master equation)}$$

(Note: D, E, F, r, s, t, are all integers; $x, y, z > 2$)

One will next show that $r = s = t$ by inspection, using a correspondence approach in which the corresponding elements on the right and left sides of equation (6) are equated to each other.

Note:

Example on the principle applied below in tables **A** and **B**

If $\frac{U + V}{W} = \frac{3 + 5}{8} = 1$

Then either

$U = 3$, $V = 5$, and $W = 8$ **or**

$U = 5$, $V = 3$, and $W = 8$

In either case,

$$\frac{U + V}{W} = \frac{3 + 5}{8} = \frac{5 + 3}{8} = 1$$

$(Dr)^x = 2^9$, $(Es)^y = 8^3$, $(Ft)^z = 4^5$, or

$(Dr)^x = 8^3$, $(Es)^y = 2^9$, $(Ft)^z = 4^5$

A $A = Dr = 2 \bullet 1$, $x = 9$ $D = 1$ $r = 2$	$B = Es = 8 = 2 \bullet 4$, $y = 3$ $E = 4$ $s = 2$	$C = Ft = 4 = 2 \bullet 2$, $z = 5$ $F = 2$ $t = 2$

From above, $r = 2, s = 2, t = 2$, and therefore, $r = s = t$, and Dr, Es, and Ft have a common prime factor, 2. Since $A = Dr, B = Es$, and $C = Ft$, A, B, and C have a common prime factor; **OR**

B $A = Dr = 2 \bullet 4$, $x = 3$ $D = 4$ $r = 2$	$B = Es = 2 = 2 \bullet 1$, $y = 9$ $E = 1$ $s = 2$	$C = Ft = 4 = 2 \bullet 2$, $z = 5$ $F = 2$ $t = 2$

From above, $r = 2, s = 2, t = 2$, and therefore, $r = s = t$, and Dr, Es, and Ft have a common prime factor, 2. Since $A = Dr$, $B = Es$, and $C = Ft$, A, B, and C have a common prime factor. Observe above that in either Table A or Table B, it is shown that $r = s = t$; and therefore, if $A^x + B^y = C^z$, then A, B, and C, have a common prime factor. The proof is complete.

Beal Conjecture Original Directly Proved (Version 3)
(Proof using a numerical tester equation)

In this version, using a numerical tester equation, the author proves the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor.

Given: $A^x + B^y = C^z$, where A, B, C, x, y, z be positive integers such that $x, y, z > 2$.

Required: To prove that A, B, and C, have a common prime factor.

Consider the true equations $A^x + B^y = C^z$ (1); and $3^3 + 6^3 = 3^5$ (2).

Equation (2) will be called a tester equation (a true equation having similar properties as $A^x + B^y = C^z$ and which will be used to determine the properties of $A^x + B^y = C^z$).

From (1), one obtains $\dfrac{A^x + B^y}{C^z} = 1$ (3); and similarly, from (2), one obtains $\dfrac{3^3 + 6^3}{3^5} = 1$ (4.)

(Dividing both sides of each equation by the right side)

Let r, s, t be prime factors of A, B and C respectively such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E, F are positive integers. Then equation (3) becomes

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = 1 \quad (5).$$

Since $\dfrac{(Dr)^x + (Es)^y}{(Ft)^z} = 1$, and $\dfrac{3^3 + 6^3}{3^5} = 1$

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = \frac{\overbrace{3^3 + 6^3}^{\text{Tester side}}}{3^5}$$

(By the transitive equality property)

$$\frac{(Dr)^x + (Es)^y}{(Ft)^z} = \frac{\overbrace{(3 \bullet 1)^3 + (3 \bullet 2)^3}^{\text{Tester side}}}{(3 \bullet 1)^5} \quad (6) \text{ (Master equation)}$$

(Note: D, E, F, r, s, t, are all integers; $x, y, z > 2$)

One will next show that $r = s = t$ by inspection, using a correspondence approach in which the corresponding elements on the right and left sides of equation (6) are equated to each other.

Note:
Example on the principle applied below in tables **A** and **B**

If $\dfrac{U+V}{W} = \dfrac{3+5}{8} = 1$

Then either

$U = 3$, $V = 5$, and $W = 8$ **or**

$U = 5$, $V = 3$, and $W = 8$

In either case,

$$\frac{U+V}{W} = \frac{3+5}{8} = \frac{5+3}{8} = 1$$

$(Dr)^x = 3^3$, $(Es)^y = 6^3$, $(Ft)^z = 3^5$, or
$(Dr)^x = 6^3$, $(Es)^y = 3^3$, $(Ft)^z = 3^5$

A $A = Dr = 3 \bullet 1$, $x = 3$ $D = 1$ $r = 3$	$B = Es = 6 = 3 \bullet 2$, $y = 3$ $E = 2$ $s = 3$	$C = Ft = 3 = 3 \bullet 1$, $z = 5$ $F = 1$ $t = 3$

From above, $r = 3$, $s = 3$, $t = 3$, and therefore, $r = s = t$, and Dr, Es, and Ft have a common prime factor, 3. Since $A = Dr, B = Es$, and $C = Ft$, A, B, and C have a common prime factor; **OR**

B $A = Dr = 6 = 3 \bullet 2$, $x = 3$ $D = 2$ $r = 3$	$B = Es = 3 = 3 \bullet 1$, $y = 3$ $E = 1$ $s = 3$	$C = Ft = 3 = 3 \bullet 1$, $z = 5$ $F = 1$ $t = 3$

From above, $r = 3$, $s = 3$, $t = 3$, and therefore, $r = s = t$, and Dr, Es, and Ft have a common prime factor, 3. Since $A = Dr$, $B = Es$, and $C = Ft$, A, B, and C have a common prime factor.

Observe above that in either Table A or Table B, it is shown that $r = s = t$; and therefore, if $A^x + B^y = C^z$, then A, B, and C, have a common prime factor. The proof is complete.

Conclusion

Using a direct construction approach, the author proved the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. Two main types of equations were involved, namely, the equation $A^x + B^y = C^z$ and an equation which was called a tester equation. A tester equation has similar properties as $A^x + B^y = C^z$, and was used to determine the properties of $A^x + B^y = C^z$. Two types of tester equations, namely, a literal tester equation and a numerical tester equation were applied. Each side of $A^x + B^y = C^z$ and a tester equation was reduced to unity by division. The non-unity sides were justifiably equated to each other to produce a new equation which was called the master equation. The side of the master equation involving the terms of the tester equation was called the tester side of the master equation. Three versions of the proof were presented. In Version 1 proof, tester equation was the literal equation $G^m + H^n = I^p$, but in Versions 2 and 3 proofs, the tester equations were the numerical tester equations, $2^9 + 8^3 = 4^5$ and $3^3 + 6^3 = 3^5$, respectively.. By inspection, using an approach in which the corresponding elements on the right and left sides of the master equation are equated to each other. An interesting observation was that it did not matter, so far as the determination of the common factors were concerned, whether the elements in the first term of the numerator on the left side of the master equation were equated to the elements in either the first or second term of the numerator on the right side of the master equation. The common factor results were always the same.

It was determined that if $A^x + B^y = C^z$, then A, B and C have a common prime factor. The proof is very simple, and occupies a single page, and even high school students can learn it.

PS
Previously, the author proved the equivalent Beal conjecture: viXra:1609.0383
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