How a Minimum time step Leads to the Construction of the Arrow of Time and the Formation of Initial Causal structure in space-time

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Abstract
Our view point is to assume that the cosmological constant is indeed invariant. And also done where we use an inflaton value due to use of a scale factor $a \sim a_{\min} t^\gamma$ if we furthermore use $\delta g_{tt} \sim a_{\min}^2 \cdot \phi_{initial}$ as the variation of the time component of the metric tensor $g_{tt}$ in Pre-Planckian space-time up to the Planckian space-time initial values. In doing so, we come up with a polynomial expression for a minimum time step, we can call $\Delta t$ which leads to a development of the arrow of time, and the preservation of information, of essential type, in cosmological early universe dynamics. In doing so we delineate where Causal structure as outlined by Dowker is relevant to space-time, which is integral to where we examine a non-singular beginning of space-time, albeit with a very small initial radii, of the order or smaller than Planck’s length in radii. We show an inter relationship between the formation of the Arrow of time, and Causal structure, assuming developments which are from the setting of $H = 0$ in the Friedman equation as a starting point and systematically allows delineation of where we can meaningfully discuss creation of Causal structure.

Key words Inflaton physics, causal structure, non Linear Electrodynamics.
1. Examination of the minimum time step, in Pre-Planckian Space-time as a Root of a Polynomial Equation.

We initiate our work, citing [1] to the effect that we have a polynomial equation for the formation of a root finding procedure for $\Delta t$, namely

$$
\Delta t \left( \frac{8\pi GV_0}{\sqrt{\gamma \cdot (3\gamma -1)} \cdot \Delta t - 1} \right)^2 + \left( \frac{8\pi GV_0}{\sqrt{\gamma \cdot (3\gamma -1)} \cdot \Delta t - 1} \right)^3 \approx \left( \frac{\gamma}{\pi G} \right)^{-1} \frac{48\pi h}{a_{\text{min}} \cdot \Lambda}
$$

From here, we then cited, in [1], using [2] a criteria as to formation of entropy, i.e. if $\Lambda$ is an invariant cosmological 'constant' and if Eq. (2) holds, we can use the existence of nonzero initial entropy as the formation point of an arrow of time.

$$
S_{\text{Arrow-of-time}} \equiv \pi \left( \frac{R_{\text{initial}} - c \cdot \Delta t}{l_{\text{Planck}}} \right)^2 \neq 0
$$

In short, our view, is that the formation of a minimum time step, if it satisfies Eq. (2) is a necessary and sufficient condition for the formation of an arrow of time, at the start of cosmological evolution we have a necessary and sufficient condition for the initiation of an arrow of time. In other words, Eq. (2) being non zero with a minimum time step, is necessary and sufficient for the formation of an arrow of time. The remainder of our article is focused upon the issues of a necessary and sufficient condition for causal structure being initiated, along the lines of Dowker, as in [3]

2. Considerations as to the start of causal structure of space-time

In [1] we make our treatment of the existence of causal structure, as given by writing its emergence as contingent upon having

$$
\left( \frac{R_{\text{initial}} - c \cdot \Delta t}{l_{\text{Planck}}} \right) \sim \mathcal{O}(1)
$$

We have assumed in writing this, that our initial starting point for which we can write a Friedman Equation with $H=0$ is a finite, very small ball of space-time and that within this structure that the Friedman Equation follows the following conventions, namely

$$
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3c^2} \cdot \rho_{\text{energy density}} - \frac{\kappa c^2}{r^2 \cdot G^2} + \frac{\Lambda}{3}\bigg|_{\text{relativistically-correct}}
$$

$$
\lim_{\kappa \to 0} \left( \frac{8\pi G}{3c^2} \cdot \rho_{\text{energy density}} + \frac{\Lambda}{3}\bigg|_{\text{relativistically-correct-flat-space}}
$$
The relativistically correct Friedman equation assumes, that within the confines of the regime for where \( H = 0 \) that we write \( \kappa \) equal to zero; i.e. there is no effective curvature within the confines of Pre-Planckian Space-time and that we make the following assumptions, namely that satisfying Eq. (3) above is contingent upon [4] where we are assuming that the volume is normalized to =1 , i.e. Planck length is set equal to zero.

\[
\frac{\Delta E\Delta t}{Volume} \sim \left[ \frac{\hbar}{Volume} \cdot (\delta g_{tt} \sim a_{\text{min}}^2 \cdot \phi_{\text{initial}}) \right]_{\text{Pre-Planckian}}
\]

\[
i.e. \text{ the regime of where we have the initiation of causal structure, if allowed would be contingent upon the behavior of [5,6,7]}
\]

\[
g_{tt} - \delta g_{tt} \approx a_{\text{min}}^2 \phi
\]

\[
i.e. \text{ the right hand side of Eq. (6) is the square of the scale factor, which we assume is } \sim 10^{-11}, \text{ due to '}[5,6], \text{ and an inflaton given by [8]}
\]

\[
a \approx a_{\text{min}} t'
\]

\[
\Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \frac{8\pi GV_o}{\gamma (3\gamma - 1) \cdot t} \right\}
\]

\[
\Leftrightarrow V \approx V_o \cdot \exp \left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\}
\]

These are the items which were enfolded into the derivation of Eq. (1) of reference [1]. I.e. our following claim is that Causal structure commences if we can say the following,

\[
g_{tt} \sim \delta g_{tt} \approx a_{\text{min}}^2 \phi_{\text{initial}} \ll 1 \quad \Rightarrow \quad \left( \frac{R_{\text{initial}}}{l_{\text{Planck}}} \sim c \cdot \Delta t \right)_{\text{Planck}} \sim \mathcal{O}(1)
\]

3. Conclusion, so what is the root of our approximation for a time step?

Here for the satisfying of Eq. (8) is contingent upon \( R_{\text{initial}} \sim c \cdot \Delta t \) as an initial event horizon, of our bubble of space-time being of the order of magnitude of Planck Length, for the satisfaction of forming a regime of space time which may have causal structure as given by Dowker [3], i.e. at the boundary of a space–time initial bubble [5,6] which may contravene the Penrose conjecture [9] as to initial singularities.

Furthermore, this is not incommensurate with what Penrose wrote himself in [10], namely reviewing the Weyl Curvature hypothesis, as given in [10], i.e. singularities as presumed in initial space-time are very different from singularities of black holes, and that modification of the Weyl curvature hypothesis, may be allowing for what Penrose referred to as gravitational clumping initially to boost the initial entropy, above a presumed initial value. I.e. this we believe is commensurate with Eq. (2) above, and is crucially important.

We close this inquiry by noting that what we have done is also conditional upon [11, 12] to the effect that we can write the genesis of our time step formula, as given by Eq. (1) above as crucially dependent upon, the following
\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} \cdot \rho_0 + \frac{2\kappa}{r_0^2 a^2} \left( \text{Newtonian} \right) \]
\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3c^2} \cdot \rho_{\text{Energy-Density}} - \frac{r_{e,c}^2}{r_{c,e}^2 a^2} + \frac{\Lambda}{3} \left( \text{relativistically-correct flat-space} \right) \]
\[ \kappa \rightarrow 0 \quad \frac{8\pi G}{3c^2} \cdot \rho_{\text{Energy-Density}} + \frac{\Lambda}{3} \left( \text{relativistically-correct flat-space} \right) \]
\[ \frac{2\kappa}{r_0^2 a^2} \left( \text{Newtonian} \right) \rightarrow - \frac{\kappa c^2}{r_{c,e}^2 a^2} \]

In the third line of Eq. (9) the essential substitution is to go from a \( \kappa \) in the Newtonian case where we have the universe as bounded if \( \kappa < 0 \) in a gravitational sense, or unbounded in a gravitational sense, if \( \kappa > 0 \) to the question of, in relativity of negative curvature, \( \kappa < 0 \), or positive curvature, with \( \kappa > 0 \). Here what we do, in our own adaptation of Eq. (9) is to realize that the Newtonian case involves the conservation of energy for an expanding universe, while we conflate the 2nd and 3rd relativistic case involves changing from a mass density, \( \rho_0 \), by an energy Density, \( \rho_{\text{Energy-Density}} \) which is a generalized version of what we are attempting to analyze.

So, in the Pre Planckian regime of Space-time, our initial assumption is twofold, i.e. we assume that we cannot reference either \( \kappa < 0 \) or \( \kappa > 0 \). The default choice we will pick is, in the Pre Planckian space-time to simplify our analysis, is to then set \( \kappa = 0 \). And then we will re image the energy density which will then be in conjunction with a revitalized version of a modified early universe version of the Heisenberg Uncertainty principle. With this we conclude the basis of our inquiry and answering of the FQXI question as given in this contest.

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References


