A new simple recursive algorithm for finding prime numbers using Rosser's theorem

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Abstract

In our previous work (The distribution of prime numbers: overview of n.ln(n), (1) and (2)) we defined a new method derived from Rosser's theorem (2) and we used it in order to approximate the nth prime number. In this paper we improve our method to try to determine the next prime number if the previous is known. We use our method with five intervals and two values for n (see Methods and results). Our preliminary results show a reduced difference between the real next prime number and the number given by our algorithm.

However long-term studies are required to better estimate the next prime number and to reduce the difference when n tends to infinity. Indeed an efficient algorithm is an algorithm that could be used in practical research to find new prime numbers for instance.

Keywords: Prime numbers, Algorithm prime numbers, Rosser's theorem.

Introduction

For more details, see the first study (The distribution of prime numbers: overview of n.ln(n))

We define the difference Δ as below:

$$\Delta = N - (n.ln(n)) \; n \in N* \; (1)$$

if N is the real nth prime number and $n.ln(n)$ the approximated nth prime number.

We define ζ as below:

$$\zeta = (n.ln(n)) - \Delta \; n \in N* \; (2)$$

We define ϵ as below:

$$\epsilon = \frac{\zeta}{\Delta} \; \Delta \neq 0 \; (3)$$

The aim is to know Δ to find the real nth prime number. In fact Δ is the difference between the real nth prime number and the number given by the empirical formula. According to (2) we have:

$$\zeta = (n.ln(n)) - \Delta \; n \in N*$$

$$\zeta = (n.ln(n)) - \frac{\zeta}{\epsilon} \; \epsilon \neq 0$$

$$\zeta = \frac{\epsilon(n.ln(n)) - \zeta}{\epsilon} \; \epsilon \neq 0$$

$$\epsilon\zeta + \zeta = \epsilon(n.ln(n))$$

$$\zeta(\epsilon + 1) = \epsilon(n.ln(n))$$

$$\zeta = \frac{\epsilon(n.ln(n))}{\epsilon + 1}$$

According to (3) we have:

$$\epsilon = \frac{\zeta}{\Delta} \; \Delta \neq 0$$

$$\Delta = \frac{\zeta}{\epsilon} \; \epsilon \neq 0$$

$$\Delta = \frac{\epsilon(n.ln(n))}{\epsilon^2+\epsilon} \; \epsilon \neq 0 \; (4)$$

Finally the real nth prime number is given by the following formula:

$$N = (n.ln(n)) + \Delta \; n \in N*$$

$$N = (n.ln(n))\frac{2+\epsilon}{1+\epsilon} \; \epsilon \neq -1$$

$$N = (n.ln(n))p \text{ with } 1 < p < 2, \; p = \frac{2+\epsilon}{1+\epsilon} \text{ and } n > 2 \; (5)$$

We must know p to find Δ and the real nth prime number. In the previous study we noticed that the value of ϵ (and consequently p) for a next prime number was very close to the value of p for a previous prime number (for the third interval of the previous study). In this work we confirm that two consecutive prime numbers seem to have values of p that are very close to each other. For this reason we try to use the value of p that is associated with the nth prime number to approximate the next prime number. However there are errors in the determination of the next prime number when we use this value to determine the next prime number. Because the value of p of the next prime number is unpredictable and is either greater than, smaller than or equal to the value of p of the previous prime number, we change the value of p that is associated with the nth prime number and we use three values of p to approximate the next prime number (see Methods and results).

Methods and results

Methods

In this work we use five intervals and two values for n as described below:

$n \in [2, 200]$ $n \in [1000, 1195]$ $n \in [1800, 1999]$ $n \in [2600, 2903]$ $n \in [999971, 10^6]$

$$n = 100000 \; n = (2 * 10^{17}) - 1$$

By using Microsoft Excel 2016 we calculate the value of p for each n of each interval and for each value of n (for more details see Introduction and the previous study). Then, in order to approximate the next prime number, we change the value of p as described below:

$$p \pm \frac{ln(ln(ln(n+1)))}{n+1} \text{ with } ln(ln(ln(n+1))) = ln^3(n+1)$$

We use these three values of p to approximate the next prime number, as below:

$$p_1 = p - \frac{ln(ln(ln(n+1)))}{n+1} \text{ with } ln(ln(ln(n+1))) = ln^3(n+1)$$

$$p_2 = p$$

$$p_3 = p + \frac{ln(ln(ln(n+1)))}{n+1} \text{ with } ln(ln(ln(n+1))) = ln^3(n+1)$$

Finally we have:

$$p_1 * ((n+1).ln(n+1))$$

$$\mathrm{p}_2 * ((n+1).ln(n+1))$$

$$\mathrm{p}_3 * ((n+1).ln(n+1))$$

Our results show that the next prime number defined as p(n+1) is close to one of these three values, with a margin of error and exceptions.

Proposed algorithm

We propose the following recursive algorithm as a way to approximate the next prime number if we know a nth prime number:

1. We define p(n) as the nth prime number. We want to approximate the next prime number p(n+1) with the value of p that is associated with n.

2. We calculate n.ln(n)

3. We calculate Δ

4. We calculate n.ln(n) - Δ

5. We calculate ϵ

6. We calculate p_1, p_2 and p_3 as described above

7. We approximate p(n+1) using the three values of p :

$p(n+1) \approx p_1 * ((n+1).ln(n+1))$

Or $p(n+1) \approx p_2 * ((n+1).ln(n+1))$

Or $p(n+1) \approx p_3 * ((n+1).ln(n+1))$

The real next prime number p(n+1) is close to one of these three values, with a margin of error and several exceptions.

Results

n	p(n)	p1	p2	p3	p(n+1) (p1)	p(n+1) (p2)	p(n+1) (p3)	Real next prime number
175	1039	1.14672182	1.14954286	1.15236391	1043.5228	1046.08996	**1048.65713**	**1049**
176	1049	1.14993182	1.15274069	1.15554956	**1053.54281**	1056.11624	1058.68966	**1051**
1158	9349	1.14386326	1.14444114	1.14501902	9353.49293	9358.21834	**9362.94375**	**9371**
1180	9533	1.14159283	1.14216111	1.14272938	**9537.47382**	9542.22146	9546.9691	**9539**
1900	16363	1.14036498	1.14073525	1.14110551	**16367.4388**	16372.7531	16378.0675	**16369**
1901	16369	1.14010367	1.14047375	1.14084384	16373.4365	**16378.7515**	16384.0665	**16381**
2642	23719	1.13912624	1.13940047	1.1396747	23723.4062	23729.1173	**23734.8283**	**23741**
2643	23741	1.13969691	1.13997104	1.14024518	**23745.4113**	23751.1228	23756.8343	**23743**
100000	1299709	1.128903959	1.128912894	1.128921828	1299712.84	**1299723.126**	1299733.412	**1299721**
999971	15485473	1.12090981	1.12091077	1.12091174	15485476.3	15485489.6	**15485502.9**	**15485497**
999972	15485497	1.12091034	1.12091131	1.12091227	15485500.3	15485513.6	**15485526.9**	**15485537**
999973	15485537	1.12091203	1.120913	1.12091397	**15485540.3**	15485553.6	15485566.9	**15485539**
999974	15485539	1.12091098	1.12091194	1.12091291	**15485542.3**	15485555.6	15485568.9	**15485543**
$(2.10^{17}) - 1$	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

[1] 8512677386048191019

[2] 1.068436045689152299107750 60

[3] 1.0684360456891523056288297 9

[4] 1.0684360456891523121499089 8

[5] 8512677386048191010.6

[6] 8512677386048191062.6

[7] 8512677386048191114.5

[8] 8512677386048191063

p(n+1) (p) refers to the number given by our method with the value of p_1, p_2 or p_3.

Discussion

We notice that the real next prime number is close to one of the three values of p(n+1). However there is often a margin of error and there are several exceptions. Moreover it is impossible to predict the exact value of p that is associated with the next prime number. Because the value of p of the next prime number is unpredictable and is either greater than, smaller than or equal to the value of p of the previous prime number, we must use the three values of p. Consequently the speed of the algorithm is slow and this is a problem if we want to find new prime numbers.

We fail to approximate the next prime number with p_2 or p_3 when there are twin primes (n and n+2 are both prime) but not with p_1.

Finally our method seems to be particularly effective even if the value of n is increased ($n = (2.10^{17}) - 1)$) because we found the exact value (-0.4) of the next prime number using p_2, suggesting that our algorithm may be used to find new prime numbers. However we fail to determine the exact value of other next prime numbers (with $n = (2.10^{17})$, $n = (2.10^{17}) + 1$ and $n = (2.10^{17}) + 2$, data not shown here).

Conclusion

In this work we approximated the next prime number using the value of p of the previous prime number. It is clear that the value of the next prime number is close to one of the three values of p(n+1) but there is a margin of error and several exceptions. Further investigations are needed to improve this algorithm. Is it possible to predict the exact value of p of the next prime number using the value of p of the previous prime number? Is it possible to decrease the margin of error (when n tends to infinity)?

Tools

Statistics. Statistics were performed using Microsoft Excel 2016.

The list of prime numbers used in this study. http://compoasso.free.fr/primelistweb/page/prime/liste_ online.php

The next prime numbers. The next prime numbers were found using http://www.numberempire.com/primenumbers.php

References

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