On the k-MACGA Mother Algebras of Conformal Geometric Algebras and the k-CGA Algebras

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Abstract

This note very briefly describes or sketches the general ideas of some applications of the $\mathcal{G}_{p,q}$ Geometric Algebra (GA) of a complex vector space $\mathbf{C}^{p,q}$ of signature (p,q), which is also known as the Clifford algebra $\mathcal{C}\ell(p,q)$. Complex number scalars are only used for the anisotropic dilation (directed scaling) operation and to represent infinite distances, but otherwise only real number scalars are used. The anisotropic dilation operation is implemented in Minkowski spacetime as hyperbolic rotation (boost) by an imaginary rapidity $\pm\varphi = \mathrm{atanh}\sqrt{1-d^2}$ for dilation factor $d>1$, using $+\varphi$ in the Minkowski spacetime of signature $(1,n)$ and $-\varphi$ in the signature $(n,1)$.

The $\mathcal{G}_{k(p+q+2),k(q+p+2)}$ Mother Algebra of CGA (k-MACGA) is a generalization of $\mathcal{G}_{p+1,q+1}$ Conformal Geometric Algebra (CGA) having k orthogonal $\mathcal{G}_{p+1,q+1}$: $p>q$ Euclidean CGA (ECGA) subalgebras and k orthogonal $\mathcal{G}_{q+1,p+1}$ anti-Euclidean CGA (ACGA) subalgebras with opposite signature. Any k-MACGA has an even $2k$ total count of orthogonal subalgebras and cannot have an odd $2k+1$ total count of orthogonal subalgebras.

The more generalized $\mathcal{G}_{l(p+1)+m(q+1),l\,(q+1)+m(p+1)}$: $p>q$ k-CGA algebra, for even or odd $k=l+m$, has any l orthogonal $\mathcal{G}_{p+1,q+1}$ ECGA subalgebras and any m orthogonal $\mathcal{G}_{q+1,p+1}$ ACGA subalgebras with opposite signature. Any $2k$-CGA with even $2k$ orthogonal subalgebras can be represented as a k-MACGA with different signature, requiring some sign changes.

All of the orthogonal CGA subalgebras are corresponding by representing the same vectors, geometric entities, and transformation versors in each CGA subalgebra, which may differ only by some sign changes.

A k-MACGA or a $2k$-CGA has even-grade $2k$-vector *geometric inner product null space* (GIPNS) entities representing general even-degree $2k$ polynomial implicit hypersurface functions F for even-degree $2k$ hypersurfaces, usually in a p-dimensional space or $(p+1)$-spacetime. Only a k-CGA with odd k has odd-grade k-vector GIPNS entities representing general odd-degree k polynomial implicit hypersurface functions F for odd-degree k hypersurfaces, usually in a p-dimensional space or $(p+1)$-spacetime. In any k-CGA, there are k-blade GIPNS entities representing the usual $\mathcal{G}_{p+1,q+1}$ CGA GIPNS 1-blade entities, but which are representing an implicit hypersurface function F^k with multiplicity k and the k-CGA null point entity is a k-point entity. In the conformal Minkowski spacetime algebras $\mathcal{G}_{p+1,2}$ and $\mathcal{G}_{2,p+1}$, the null 1-blade point embedding is a GOPNS null 1-blade point entity but is a GIPNS null 1-blade hypercone entity.

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1 Introduction

This note very briefly describes or sketches the general ideas of some applications of the $\mathcal{G}_{p,q}$ Geometric Algebra (GA) [9] of a complex vector space $\mathbf{C}^{p,q}$ of signature (p,q), which is also known as the Clifford algebra $\mathcal{C}\ell(p,q)$. For any Clifford geometric algebra $\mathcal{G}_{p,q}$, the conformal Clifford geometric algebra (CGA) is $\mathcal{G}_{p+1,q+1}$ [10]. Complex number scalars are only used for the anisotropic dilation (directed scaling) operation and to represent infinite distances, but otherwise only real number scalars are used.

The anisotropic dilation operation is implemented in Minkowski spacetime as hyperbolic rotation (boost) by an imaginary rapidity $\pm\varphi = \operatorname{atanh}\sqrt{1-d^2}$ for dilation factor $d>1$, using $+\varphi$ in the Minkowski spacetime of signature $(1,n)$ and $-\varphi$ in the signature $(n,1)$. See paper [5] on the Double Conformal Space-Time Algebra (DCSTA) for an example of the anisotropic dilation operation on general quadric surface entities.

In the conformal Clifford algebra $\mathcal{G}_{1,3+1}$ of the 3-D anti-Euclidean complex vector space $\mathbf{C}^{0,3}$, denoted $\mathcal{CS}$ [5], the distance function $d=\sqrt{2\mathbf{P}_{\mathcal{CS}}\cdot\mathbf{Q}_{\mathcal{CS}}}$ for distance between conformal points $\mathbf{P}_{\mathcal{CS}}$ and $\mathbf{Q}_{\mathcal{CS}}$ is imaginary when either *one* of the two points is the infinity point entity $\mathbf{e}_{\infty\mathcal{CS}}$ of $\mathcal{CS}$. In the conformal geometric algebra (CGA) $\mathcal{G}_{3+1,1}$ of the 3-D Euclidean real vector space $\mathbf{R}^{0,3}$, denoted $\mathcal{CE}$ [4], the distance function $d=\sqrt{-2\mathbf{P}_{\mathcal{CE}}\cdot\mathbf{Q}_{\mathcal{CE}}}$ for distance between conformal points $\mathbf{P}_{\mathcal{CE}}$ and $\mathbf{Q}_{\mathcal{CE}}$ is valid only for finite points, and then imaginary numbers are avoided entirely if the anisotropic dilation operation is not required.

The k-MACGA is the Mother Algebra [1] of Conformal Geometric Algebra [10], an algebra formed as the product of even $2k$ orthogonal CGAs, with k CGAs of signature $(p+1,\ q+1)$ and k CGAs of opposite signature $(q+1,\ p+1)$. All $2k$ CGAs are corresponding, having the same basis-blade coefficients and corresponding entities and operations, with sign changes as required. The k-CGA generalizes on k-MACGA to allow a product of any $k=l+m$ CGAs, with l corresponding CGAs of signature $(p+1,q+1)$ and m corresponding CGAs of opposite signature $(q+1,p+1)$.

A k-MACGA or a $2k$-CGA has even-grade $2k$-vector *geometric inner product null space* (GIPNS) entities representing general even-degree $2k$ polynomial implicit hypersurface functions F for even-degree $2k$ hypersurfaces, usually in a p-dimensional space or $(p+1)$-spacetime. Only a k-CGA with odd k has odd-grade k-vector GIPNS entities representing general odd-degree k polynomial implicit hypersurface functions F for odd-degree k hypersurfaces, usually in a p-dimensional space or $(p+1)$-spacetime. In any k-CGA, there are k-blade GIPNS entities representing the usual $\mathcal{G}_{p+1,q+1}$ CGA GIPNS 1-blade entities, but which are representing an implicit hypersurface function F^k with multiplicity k and the k-CGA null point entity is a k-point entity. In the conformal Minkowski spacetime algebras $\mathcal{G}_{p+1,2}$ and $\mathcal{G}_{2,p+1}$, the null 1-blade point embedding is a *geometric outer product null space* (GOPNS) null 1-blade point entity but is a GIPNS null 1-blade hypercone entity.

2 *k*-MACGA

The k-MACGA is the k Mother Algebra of Conformal Geometric Algebra (MACGA) for representing *general* even-degree $2k$ polynomial hypersurface entities, and also certain other *specific* degree $(2k+1)\ldots4k$ polynomial (cyclide or roulette) hypersurface entities.

The $\mathcal{G}_{k(p+q+2),k(q+p+2)}$ Mother Algebra of CGA (k-MACGA) is a generalization of $\mathcal{G}_{p+1,q+1}$ Conformal Geometric Algebra (CGA) having k orthogonal $\mathcal{G}_{p+1,q+1} : p > q$ Euclidean CGA (ECGA) subalgebras and k orthogonal $\mathcal{G}_{q+1,p+1}$ anti-Euclidean CGA (ACGA) subalgebras with opposite signature. Each of the $2k$ orthogonal CGAs are corresponding copies, having the same scalar coefficients on all corresponding canonical basis-blades [11], except for some required sign changes due to opposite signatures such that all entities and operations are corresponding.

Any k-MACGA has an even $2k$ total count of orthogonal subalgebras and cannot have an odd $2k+1$ total count of orthogonal subalgebras.

3 *k*-CGA

More general than the k-MACGA is the k-CGA. The k-CGA is the k Conformal Geometric Algebra (CGA) for representing *general* even-degree or odd-degree k polynomial hypersurface entities, and also certain other *specific* degree $(k + 1)...2k$ polynomial (cyclide or roulette) hypersurface entities.

The more general $\mathcal{G}_{l(p+1)+m(q+1),l\,(q+1)+m(p+1)} : p > q$ k-CGA algebra, for even or odd $k = l + m$, has any l orthogonal $\mathcal{G}_{p+1,q+1}$ ECGA subalgebras and any m orthogonal $\mathcal{G}_{q+1,p+1}$ ACGA subalgebras with opposite signature. Each of the k orthogonal CGAs are corresponding copies, having the same scalar coefficients on all corresponding canonical basis-blades [11], except for some required sign changes due to opposite signatures such that all entities and operations are corresponding.

Any $2k$-CGA with even $2k$ orthogonal subalgebras can be represented as a k-MACGA with different signature, requiring some sign changes.

4 Monomial extraction operators

A monomial extraction operator T_s extracts the monomial s (acting also as symbolic index s) from a k-CGA k-vector entity $\mathbf{X}$ as $s = T_s \cdot \mathbf{X}$ by inner product. The T_s are derived from the k-CGA k-blade point embedding entity $\mathbf{P}_{\mathcal{K}} = \mathcal{K}(\mathbf{p}) = \prod \mathcal{C}^i(\mathbf{p}) = \prod \mathbf{P}_{\mathcal{C}^i} : i = 1...k$ as combinations of inverse basis k-blades that extract coefficient monomials from the point entity.

An entity $\mathbf{X}$ is formed as a linear combination of monomial extraction operators for different monomials s and represents a polynomial implicit hypersurface function F for an implicit hypersurface equation $F = 0$. $\mathbf{X}$ is called a geometric inner product null space (GIPNS) k-vector entity.

In any k-CGA, it should be possible to define, perhaps by an algorithm or formulas, the inner product null space k-vector monomial extraction operators T_s, similar to those in DCGA [4] and DCSTA [5], and also as in the forthcoming paper on TCGA (§10), which were defined manually by simple means.

5 Algebraic differential operators

The monomial extraction operator T_{x^k} for the monomial term x^k is a k-blade with an inverse. Since general degree k hypersurfaces are representable, the extraction operator for $T_{x^{k-1}}$ can also be defined as a k-vector, but it may not have an inverse. Using these two extraction operators, it is possible to form the product $D_x = kT_{x^{k-1}}T_{x^k}^{-1}$, which is clearly a differential operator on T_{x^k}. It can be shown that, using the commutator product $\times$, defined for any multivectors A and B as $A \times B = (AB - BA)/2 = -B \times A$, the differential operator D_x is a differential operator on all of the extraction operators T_s as $\partial_x T_s = D_x \times T_s$, where ∂T_s is another combination of extraction operators that correctly represents the directional derivative in the x direction. There are similar differential operators D_y, D_z etc. for each independent variable of the space that is embedded in the k-CGA.

The differential operators provide a standard method for entity analysis of the k-vector geometric entities of the k-CGA. Entity analysis includes extracting the center position and other parameters of a k-vector geometric entity.

6 GIPNS *k*-blade entities

In any k-CGA (more generally than k-MACGA), there are k-blade GIPNS entities representing the usual $\mathcal{G}_{p+1,q+1}$ CGA GIPNS 1-blade entities, but which are presenting an implicit hypersurface function F^k with multiplicity k. The k-CGA null point entity is a k-point entity. In the conformal Minkowski spacetime algebras $\mathcal{G}_{p+1,2}$ and $\mathcal{G}_{2,p+1}$, the null 1-blade point embedding is a GOPNS null 1-blade point entity but is a GIPNS null 1-blade hypercone entity. For examples, see the papers [4] and [5].

The k-blade entities $\mathbf{X}_{\mathcal{K}} = \prod \mathbf{X}_{\mathcal{C}^i} : i = 1...k$ are simply the product (geometric or outer) of the corresponding CGA entities $\mathbf{X}_{\mathcal{C}^i}$ in each of the k CGAs of the k-CGA. The k-CGA null point entity is the k-blade point $\mathbf{P}_{\mathcal{K}} = \prod \mathbf{P}_{\mathcal{C}^i}$ which embeds a point $\mathbf{p}$ of the embedded vector space. For an inner product null space k-blade entity $\mathbf{X}_{\mathcal{K}}$, $F^k(\mathbf{p}) \simeq \mathbf{P}_{\mathcal{K}} \cdot \mathbf{X}_{\mathcal{K}}$, where $\simeq$ means "up to a homogeneous scalar factor". Since the implicit hypersurface functions F of algebraic geometry are homogeneous, representing an implicit hypersurface equation $F = 0$, scalar factors do not affect hypersurface representation. Metrical results are affected by scalar factors, and for metrical results there must be a chosen homogeneous normalized form for each entity from which to obtain metrical results of the expected scale. Otherwise, the k-blade entities are just the analogs of the 1-blade CGA entities.

7 GIPNS *k*-vector entities

A k-MACGA or a $2k$-CGA has even-grade $2k$-vector *geometric inner product null space* (GIPNS) entities [11] representing general even-degree $2k$ polynomial implicit hypersurface functions F for even-degree $2k$ hypersurfaces, usually in a p-dimensional space or $(p+1)$-spacetime. For examples, see the papers [4] and [5].

Only a k-CGA with odd k has odd-grade k-vector GIPNS entities representing general odd-degree k polynomial implicit hypersurface functions F for odd-degree k hypersurfaces, usually in a p-dimensional space or $(p+1)$-spacetime. For example an of odd-grade k-CGA, see TCGA (§10).

In a k-CGA, the geometric inner product null space (GIPNS) k-vector entities [11] are linear combinations of the k-vector extraction operators T_s, as described in §4 and as exemplified in [4] and [5]. The extraction operators include terms for *general* degree k hypersurface entities and certain other *specific* degree $(k+1)...2k$ (cyclide or roulette) hypersurface entities.

A k-CGA GIPNS k-vector hypersurface entity $\mathbf{\Omega}$ represents an implicit hypersurface function $F(\mathbf{p}) \simeq \mathbf{P}_{\mathcal{K}} \cdot \mathbf{\Omega}$, up to scale. The entity $\mathbf{\Omega}$ must have a certain normalized form to represent F at a desired or standard scale. Note that, for the k-blade entities we have F^k (k power), not F. Using a differential operator D_x on the k-vector entity $\mathbf{\Omega}$ gives $\partial_x F(\mathbf{p}) = \mathbf{P}_{\mathcal{K}} \cdot (D_x \times \mathbf{\Omega})$. Using a differential operator D_x on the GIPNS k-blade entity $\mathbf{X}_{\mathcal{K}}$ gives $kF^{k-1}(\mathbf{p})\partial_x F(\mathbf{p}) \simeq \mathbf{P}_{\mathcal{K}} \cdot (D_x \times \mathbf{X}_{\mathcal{K}})$. This is consistent with the chain rule of differentiation. An entity can be differentiated in succession in any sequence (e.g., as $\partial_{xy}\mathbf{\Omega} = \frac{\mathrm{d}^2\mathbf{\Omega}}{\mathrm{d}x\mathrm{d}y} = D_y \times (D_x \times \mathbf{\Omega})$ etc.) for mixed partial derivatives.

8 k-versors and $2k$-versors

In a $\mathcal{G}_{p+1,q+1}$ CGA, the GIPNS 1-blade hypersphere or hyperpseudosphere (in a spacetime) entity $\mathbf{S}_{\mathcal{C}}$ is the 1-versor for inversion (*inversor*) in the hypersphere. The *inversion* of a CGA entity $\mathbf{X}_{\mathcal{C}}$ in $\mathbf{S}_{\mathcal{C}}$ is $\mathbf{X}'_{\mathcal{C}} = \mathbf{S}_{\mathcal{C}}\mathbf{X}_{\mathcal{C}}\mathbf{S}_{\mathcal{C}}$. The GIPNS 1-blade hyperplane entity $\mathbf{\Pi}_{\mathcal{C}}$ is the 1-versor for reflection (*reflector*) in the hyperplane. The reflection of a CGA entity $\mathbf{X}_{\mathcal{C}}$ in $\mathbf{\Pi}_{\mathcal{C}}$ is $\mathbf{X}'_{\mathcal{C}} = \mathbf{\Pi}_{\mathcal{C}}\mathbf{X}_{\mathcal{C}}\mathbf{\Pi}_{\mathcal{C}}$.

The CGA 2-versors are the following. The translation operator (*translator*) $T = \mathbf{\Pi}_{\mathcal{C}}\mathbf{\Pi}_{\mathcal{C}}$ is just successive reflections in two parallel hyperplanes that are separated by half the translation vector displacement $\mathbf{d}$. The rotation operator (*rotor*) $R = \mathbf{\Pi}_{\mathcal{C}}\mathbf{\Pi}_{\mathcal{C}}$ is just successive reflections in two non-parallel spatial hyperplanes subtending half the rotation angle. The hyperbolic rotation operator (*boost*) $B = \mathbf{\Pi}_{\mathcal{C}}\mathbf{\Pi}_{\mathcal{C}}$ is just successive reflections in a time-like hyperplane followed by reflection in a space-like hyperplane subtending half the hyperbolic angle (this also makes an anisotropic dilation operator for imaginary hyperbolic angle φ). The isotropic dilation operator (*dilator*) $D = \mathbf{S}_{\mathcal{C}}\mathbf{S}_{\mathcal{C}}$ is just successive inversions of two hyperspheres, first of radius r_1 then of radius r_2, for dilation by factor $d = r_2^2/r_1^2$, often choosing $r_1 = 1$ and $r_2 = \sqrt{d}$.

In k-CGA, the corresponding CGA 1-versors for an operation are multiplied together by geometric or outer product as the k-CGA k-versor for the same effective operation on k-CGA entities.

In k-CGA, the corresponding CGA 2-versors for an operation are multiplied together by geometric or outer product as the k-CGA $2k$-versor for the same effective operation on k-CGA entities.

For efficient operation of the k-CGA k-versors or $2k$-versors, each of the k individual CGA 1-versors or 2-versors $V_{\mathcal{C}^{i+1}} : i = 0...(k-1)$ is used in succession on a k-CGA entity $\mathbf{\Omega} = \mathbf{\Omega}_0$ as $\mathbf{\Omega}_{i+1} = V_{\mathcal{C}^{i+1}}\mathbf{\Omega}_i V_{\mathcal{C}^{i+1}}^{-1}$ to obtain the transformed entity $\mathbf{\Omega}' = \mathbf{\Omega}_k$. This operation is known as *outermorphism*. The direct operation of high-grade k-CGA k-versors or $2k$-versors on high-grade k-vector entities, without successive CGA versor operations, can be very inefficient for computations.

9 Exemplary 1-MACGAs

The Mother Algebras of Conformal Geometric Algebras (k-MACGAs) have an even $2k$ count of k orthogonal ECGA subalgebras and k corresponding orthogonal ACGA subalgebras of opposite signature, with $2k$ corresponding entities and operations. Prior work on Double Conformal Geometric Algebra (DCGA) [7][3][6][4][2] and Double Conformal Space-Time Algebra [5][8] are both 2-CGAs and each can be represented as a 1-MACGA, requiring only some sign changes.

9.1 MACST

The $\mathcal{G}_{4,8}$ Double Conformal Space-Time Algebra (DCSTA) [5] uses two orthogonal $\mathcal{G}_{2,4}$ Conformal Space-Time Algebra (CSTA) subalgebras. The $\mathcal{G}_{4,2}$ Conformal Time-Space Algebra (CTSA) is an alternative form of $\mathcal{G}_{2,4}$ CSTA with opposite signature, requiring some sign changes. The $\mathcal{G}_{8,4}$ Double Conformal Time-Space Algebra (DCTSA) is an alternative form of $\mathcal{G}_{4,8}$ DCSTA with opposite signature that uses two orthogonal $\mathcal{G}_{4,2}$ CTSA subalgebras.

The $\mathcal{G}_{6,6}$ *Mother Algebra of Conformal Spacetime* (MACST) uses one $\mathcal{G}_{4,2}$ CTSA subalgebra and one corresponding orthogonal $\mathcal{G}_{2,4}$ CSTA subalgebra. MACST has all of the same entities and operations as DCTSA or DCSTA with different signature, requiring some sign changes.

9.2 MACS

The $\mathcal{G}_{8,2}$ Double Conformal Geometric Algebra (DCGA) [4] uses two orthogonal $\mathcal{G}_{4,1}$ Conformal Geometric Algebra (CGA) subalgebras. The $\mathcal{G}_{1,4}$ Conformal Space Algebra (CSA) is an alternative form of $\mathcal{G}_{4,1}$ CGA with opposite signature, requiring some sign changes. The $\mathcal{G}_{2,8}$ Double Conformal Space Algebra is an alternative form of $\mathcal{G}_{8,2}$ DCGA with opposite signature and uses two orthogonal $\mathcal{G}_{1,4}$ CSA subalgebras.

The $\mathcal{G}_{5,5}$ *Mother Algebra of Conformal Space* (MACS) uses one $\mathcal{G}_{4,1}$ CGA subalgebra and one corresponding orthogonal $\mathcal{G}_{1,4}$ CSA subalgebra. MACS has all of the same entities and operations as DCGA or DCSA with different signature, requiring some sign changes.

$\mathcal{G}_{5,5}$ MACS is the spatial subalgebra of the $\mathcal{G}_{6,6}$ MACST spacetime algebra. Any MACST entity or operation can be projected onto MACS as a MACS entity or operation. Examples of this projection are found in [5] and [8] in the discussion of the anisotropic dilation operation on general quadric spatial surface entities. The projection is a subalgebra unit pseudoscalar projection $\mathcal{P}_{\mathbf{I}}$.

10 An exemplary 3-CGA

A paper on the $\mathcal{G}_{3(2+1),3}$ *Triple Conformal Geometric Algebra for Cubic Plane Curves* (TCGA) is in preparation for publication. TCGA is an exemplary k-CGA, using three corresponding orthogonal $\mathcal{G}_{2+1,1}$ CGAs, with odd-grade ($k = 3$)-vector entities representing *general* cubic polynomial implicit hypersurface (plane curve) functions F for general cubic hypersurfaces (plane curves) in a 2-dimensional space. TCGA also has 3-vector entities representing certain other *specific* degree 4, 5, 6 polynomial (cyclide or roulette) hypersurfaces (plane curves).

The TCGA paper, which is in preparation for publishing, provides the full details of the TCGA GIPNS 3-vector monomial *extraction operators* T_s that form the basis for all of the TCGA GIPNS 3-vector entities. Algebraic differential operators D_x and D_y are also given, which are valid on all of the extraction operators (e.g., as $D_x \times T_s$ etc.) using the commutator product $\times$. The TCGA paper also includes the full details of the anisotropic dilation operation on the TCGA GIPNS 3-vector general cubic plane curve entities.

11 Conclusion

This note has described a sketch of the general ideas of some applications of the $\mathcal{G}_{p,q}$ Geometric Algebra (GA) [9] of a complex vector space $\mathbf{C}^{p,q}$ of signature (p,q), which is also known as the Clifford algebra $\mathcal{C}\ell(p,q)$. For any Clifford geometric algebra $\mathcal{G}_{p,q}$, the conformal Clifford geometric algebra (CGA) is $\mathcal{G}_{p+1,q+1}$ [10].

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The k-MACGA is the k Mother Algebra of Conformal Geometric Algebra (MACGA) for representing *general* even-degree $2k$ polynomial hypersurface entities, and also certain other *specific* degree $(2k+1)...4k$ polynomial (cyclide or roulette) hypersurface entities.

More general than the k-MACGA is the k-CGA. The k-CGA is the k Conformal Geometric Algebra (CGA) for representing *general* even-degree or odd-degree k polynomial hypersurface entities, and also certain other *specific* degree $(k+1)...2k$ polynomial (cyclide or roulette) hypersurface entities.

As examples, $\mathcal{G}_{4,8}$ DCSTA [5] is represented by $\mathcal{G}_{6,6}$ MACST with different signature, requiring some sign changes. $\mathcal{G}_{8,2}$ DCGA [4] is represented by $\mathcal{G}_{5,5}$ MACS with different signature, requiring some sign changes.

$\mathcal{G}_{n,n}$ Mother Algebras (MA) [1] for representing general even-degree polynomial hypersurfaces with conformal versor operations for translation, rotation, and isotropic dilation are achieved. Furthermore, in the superalgebra $\mathcal{G}_{n+1,n+1}$ MA of a spacetime it is possible to form the hyperbolic rotor (boost versor) for the anisotropic dilation (directed scaling) operation on the general degree k polynomial hypersurfaces represented in the subalgebra $\mathcal{G}_{n,n}$ MA of a space. After the directed scaling operation, the resulting entity in $\mathcal{G}_{n+1,n+1}$ MA can be projected back onto the $\mathcal{G}_{n,n}$ MA as an entity in $\mathcal{G}_{n,n}$ MA.

In the k-CGA of k corresponding orthogonal CGAs for *space* with k-vector entities representing general degree k spatial hypersurfaces, the anisotropic dilation operation on the general degree k spatial hypersurface entities is available as the spacetime boost (hyperbolic rotation) operation in the larger k-CGA of k corresponding orthogonal CGAs for *spacetime*. This is demonstrated in the forthcoming paper on TCGA (§10).

In conclusion, this note[1] contains a sketch of ideas that answer or solve many research goals. These goals have included Mother Algebra representations of general degree k polynomial hypersurfaces having all of the usual CGA conformal operations and also an anisotropic dilation operation. However, one goal of some researchers, to find k-*blades* (not k-vectors) that represent general degree k hypersurfaces, is not met here in this note, and may not be a possible goal. Other goals, of other researches, that are not met here include operations for shearing and other operations for projective geometry in computer graphics. However, some operations for the projections of plane curves have been demonstrated in [6][2], and these projection operations may generalize into other k-CGAs for the projections of general degree k plane curves in a $(n > k)$-space where the plane curves are represented as GIPNS intersections (wedges) of GIPNS k-vector general degree k surface entities with GIPNS k-blade plane entities.

Future papers may elaborate further on the ideas presented in this note.

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