Relativistic velocity stabilization of particle sets in gravity fields

Thierry De Mees
Independent researcher
thierrydemees @ telenet.be

Abstract. The analogy of electromagnetism for gravity was proposed by O. Heaviside in 1893 and applied by O. Jefimenko at the end of last millennium. In one intriguing example of two falling masses in a gravity field, he found that the two masses are mutually over-accelerating, more than the gravity acceleration field. I find here the result of his example in the form of a relativistic equation of velocity stabilization in that gravity field, related to the distance of the two masses. When I look for the conditions for the upper limit velocity, the speed of light, I deduce that the distance between the two masses at that relativistic speed equals the Planck length. Hence, this gives the first physical meaning of Planck length in a practical application, i.e. that very small particles such as gravitons and neutrinos with a rest mass can propagate in a gravity field at the speed of light without being just a wave that is propagated by the specific natural constants of a medium.

Keywords: Gravitomagnetism, Heaviside, Jefimenko, Planck length, neutrino, graviton.

Introduction

Oliver Heaviside (1850-1925), an electrical engineer, self-educated specialist in Maxwellian electromagnetism and mathematician, was able to greatly simplify Maxwell's 20 equations in 20 variables, replacing them by four equations in two variables. Today we call these 'Maxwell's equations' forgetting that they are in fact 'Heaviside's equations'.

Maxwell's theory predicts "electromagnetic waves" that travel with the speed of light. Heaviside reasoned that electromagnetic waves could travel on a telegraph cable too. Heaviside's equations, based on Maxwell's electromagnetic waves, worked for cables of all lengths.

Heaviside predicted that there was an conducting layer in the atmosphere which allowed radio waves to follow the Earth's curvature. Furthermore, Heaviside's operational calculus was rated as one of the three most important discoveries of the late 19th Century.

Heaviside was elected a Fellow of the Royal Society in 1891, perhaps the greatest honor he received. However it is doubtful if many people understand the greatness and significance of the achievements of this sad misunderstood genius.

One of Heaviside's ideas was that since Newton’s gravity law and Coulomb’s law are alike, it should be analyzed if there is a further analogy. This resulted in his brilliant 1893 paper: “A gravitational and electromagnetic analogy”.

In the end of last millennium, Oleg Jefimenko picked up Heaviside’s paper, and developed the theory further in several outstanding books, by observing that Maxwell’s equations are valid locally, but should be understood as provoked by electric charges at a distance. Hence, the retardation of the fields by the speed of light is a crucial element to describe electromagnetic events, and should be formulated mathematically.

1. Causal gravity with retarded time quantities

Since electromagnetism and gravity are causal, an equation describing a general event cannot express at the same time the cause and the effect. Jefimenko defines the “retarded” time in his equations that describes the effect of an event at a time \( t' = t - r/c \) in which \( r \) is the distance between cause and effect, and \( c \) the propagation velocity of the fields. Also the
present position vector \( \mathbf{r}_0 \) can be expressed in terms of the retarded position: \( \mathbf{r}_0 = \mathbf{r} - \mathbf{r} / c \).

Herein, the quantities \( \mathbf{r} \) and \( \mathbf{v} \) are retarded quantities.

Jefimenko found the gravity field created by a point mass in arbitrary motion as follows in the section 5-4:

\[
g = G \frac{m}{r^3} \left( \mathbf{r} \cdot \frac{\mathbf{v}}{c} \right) \left( \frac{\mathbf{v}}{c} \right) \left( 1 - \frac{\mathbf{v}^2}{c^2} \right) + \mathbf{r} \times \left( \frac{\mathbf{r} \times \frac{\mathbf{v}}{c}}{c^2} \right)
\]

This gravity field is measured for a point mass with instant velocity \( \mathbf{v} \) and instant acceleration \( \mathbf{v}' \) at a distance \( r \).

Heaviside’s gravity equation, found in 1893, is the same as above for non-accelerating masses. It can be written in terms of the present position, as defined in eq.(1):

\[
g = -G \frac{m_k}{r_0^3} \left( 1 - \frac{\mathbf{v}_0^2}{c^2} \right)^{3/2} \mathbf{r}_0
\]

2. Two masses falling in a gravity field

In the Example 13-2.8, Jefimenko takes two masses \( m \) and \( m' \) that are falling in a gravity field. When one mass is released, it falls with the acceleration of the gravity field. The second mass that is released however will also exert an acceleration upon the first mass, which is described by the eq.(2).

If the masses are sufficiently close to each other, the retardation can be neglected and the eq.(2) can be written as:

\[
g = -G \frac{m}{r^3} \left( \mathbf{r} \left( 1 - \frac{\mathbf{v}^2}{c^2} \right) \mathbf{r} + \mathbf{r} \times \left( \frac{\mathbf{a}}{c^2} \right) \right)
\]

Herein \( r \) is the distance between the masses. We use a less approximate equation as Jefimenko to analyze the example further than he did. Eq.(3) can be written as:

\[
g = -G \frac{m}{r^3} \left( 1 - \frac{\mathbf{v}^2}{c^2} \right) - G \frac{m \mathbf{r} \cdot (\mathbf{r} \mathbf{a})}{r^2 c^2} + G \frac{m a}{r c^2}
\]

The two first terms represent the attraction of the masses \( m \) and \( m' \), which can be regarded outside the scope of investigation, since the analysis concerns here the falling in the external gravity field, represented by \( a \).

It is however clear that the first term will in many cases be larger than the third term. The second term is zero if the masses fall from the same height, but the last two terms vanish if the masses are both in line with the fall direction. In that case, both masses will fall at the same acceleration except that they will attract each other by the first term. After an infinite time, the velocity becomes the speed of light.

In the case that the two masses are falling at the same height, one can investigate the last term of eq.(5) and disregard the two first terms for the time being.

The acceleration by the last term must be added to the original acceleration \( a \), and in the starting position, this results in:

\[
a_t = \left( 1 - \frac{\mathbf{v}_0^2}{c^2} \right) + G \frac{m}{r c^2} a_0
\]

Since we can also write:

\[
a_{n+1} = \left( 1 - \frac{\mathbf{v}_n^2}{c^2} \right) + G \frac{m}{r c^2} a_n
\]

or with eq.(5) and when considering that:

\[v_0 < v_1 < ... < v_n < c\] for a large \( n \), and the distance \( r \) is considered a constant, it results that, under certain conditions:

\[
a_{n+1} = a_0 \prod_{i=0}^{n} \left( 1 - \frac{\mathbf{v}_i^2}{c^2} \right) + G \frac{m}{r c^2}
\]

The velocity for a large \( n \) depends from the field retardations between the masses \( m \) and \( m' \), and so, from the distance \( r \) between the masses and the actual speeds. If the speeds are very high, the communication speed of the fields between the masses decreases.

It follows that the acceleration in eq.(7) tends to increase as long as the velocity remains below the velocity \( v_n < \sqrt{Gm/r} \). If \( v_n > \sqrt{Gm/r} \), the acceleration tends to reduce.

Hence, I have proven that the equation \( v = \sqrt{Gm/r} \) is an intrinsic gravity equation and that in gravity fields, the falling velocity stabilizes to a limited value, in principle after an infinite time. Remark also that this velocity has nothing to do at all with the orbital velocity in a circular orbit, because the value of \( r \) id the distance between the masses \( m \) and \( m' \), and not the height in a gravity field.

It is thus predictable that when objects are falling, the value that is found is dependendent from the communication between gravity masses, but the result of eq.(10) is surprising.

3. Planck length found by gravitomagnetism

If the masses \( m \) and \( m' \) are elementary de Broglie half-particles that can be represented by a wave with wavelength \( \lambda \), the distance between them is then assumed to be \( r = \lambda/2 \).

The well-known equation of energy equivalence

\[
mc^2 = \hbar \nu
\]

can then be used in the eq.(10), of which the velocity is then \( c \), and then becomes, after some elementary manipulations:

\[
\lambda = \sqrt{2Gm/c^3}
\]

It can be concluded that the Planck length can be deduced from the physical situation of falling elementary masses in a gravity field, stabilizing by self-acceleration at the speed of light.

Remark that an important difference here with the transmission of light is that we have masses and not waves. Hence, the propagation speed is not that of a wave in a medium, but that of rest masses at the speed of light.
In the case of half-particles or even particles falling at the same height at the speed of light, the first term eq.(4) has also become zero, which means that eq.(12) becomes an exact solution.

4. Conclusion

In this paper, it has been found that two particles, freely falling at the same height in a gravity field, will get self-accelerated beyond the acceleration value of the gravity field in which the masses fall. The velocity is however limited by a stabilization velocity given by eq.(10).

When the finding is applied to small particles for a velocity at the speed of light, we find that the distance between the masses in order to obtain that speed must be given as a Planck length.

Hence, some very small particles such as maybe gravitons or neutrinos could well be described by the Planck dimensions and represent a reality, as found in this paper. They could propagate as fast as the speed of light without being propagated waves, but real particles with a rest mass.

References