THE REAL-ZEROS OF JONES POLYNOMIAL OF TORUS

CHANG LI

Dedicated to My Parents.

Abstract. This article proved two theorems and presented one conjecture about the real-zeros of Jones Polynomial of Torus. Topological quantum computer is related to knots/braids theory where Jones polynomials are characters of the quantum computing. Since the real-zeros of Jones polynomials of torus are observable physical quantities, except the real-zero at 1.0 there exists another distinguished real-zero in $1 < r < 2$ for every Jones polynomial of Torus, these unique real zeros can be IDs of torus knots in topological quantum computing.

1. The Real-Zeros of Jones Polynomial of Torus

The real-zeros of Jones polynomial of torus $V_t(p, q)$ are distributed on x axis. In physics, since real-zeros of an equation usually represent observable values, it is interested to investigate them in advanced.

$V_t(p, q) = 1 - t^{p+1} - t^{q+1} + t^{p+q}, t \in \mathbb{C}$

Figure 1. Real Zeros of Jones Polynomial Torus

Theorem 1. For all Jones polynomials of Torus $V(p, q)$ there are two positive real-zeros with one is $+1$ and another is inside $1 < r < 2$.

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Proof. Suppose a positive real-zero of \( V_t(p, q) \) is \( r \) where \( 0 < r < +\infty \), then there is a \( r \) so that \( V_r(p, q) = 0 \). By Cauchy theorem, \( \|r\| < 1 + max(\|a_i\|) \leq 2 \). We can represent \( V_r(p, q) = 0 \) as \( (1 - r^p)(1 - r^q) = (r - 1)(r^p + r^q) \). If we suppose \( 0 < r < 1 \), then \( 0 < r^p < 1 \) and \( 0 < r^q < 1 \), the left side of equation is positive, but since \( r - 1 < 0 \), the right side is negative. So there is \( 1 < r < 2 \).

By Descartes’ rule of signs, there are two signs change in the coefficients of equation (6), so there are two positive zeros. Obviously, +1 is a zero, another positive real-zero is in \( 1 < r < 2 \).

**Theorem 2.** For all Jones polynomials of Torus \( V(p, q) \), for both \( p \) and \( q \) are positive even integers, there are no negative zeros; for one of \( p \) or \( q \) is odd and another is even, there is one negative zero; for both \( p \) and \( q \) are odd, there are two negative zeros.

Proof. Replace \(-t\) in Jones polynomial \( V_t(p, q) \) as \( V_{-t}(p, q) \) Descartes’s rule can be applied to decide all the negative real-zeros.

* Suppose \( p = 2m, q = 2n \), then \( V_{-t}(p, q) = 1 + t^{p+1} + t^{q+1} + t^{p+q} \). Because no signs change, there are no negative real-zeros.

* Suppose \( p = 2m + 1, q = 2n, V_{-t}(p, q) = 1 - t^{p+1} + t^{q+1} - t^{p+q} \). Because \( V_{-t}(p, q) = V_{-t}(q, p) \), it is also true for \( p = 2m, q = 2n+1 \).

* Suppose \( p = 2m + 1, q = 2n + 1 \), then \( V_{-t}(p, q) = 1 - t^{p+1} - t^{q+1} + t^{p+q} \). It is the case of positive real-zeros, so there are two negative real-zeros \(-1\) and \( r \) with \( -1 < r < 0 \).

**Conjecture 1.** For \( p \geq 2, q \geq 3 \), \( Ze(p, q) \) is the positive real-zero of \( V_t(p, q) \) which is not equal to \(+1\), then there is approximation formula below

\[
\frac{\ln(Ze(p, q) - 1)}{\ln(Ze(p, q + 1) - 1)} \approx \frac{\ln(p + q - 3)}{\ln(p + q - 2)}
\]

This conjecture shows the relationship of the real-zeros distribution of Jones Polynomial of Torus. Computer software has partially verified this conjecture.

**References**


Neatware, 2900 Warden Ave, P.O.Box 92012, Toronto, Ontario M1W3Y8, Canada
Email address: changli@neatware.com