THE REAL-ZEROS OF JONES POLYNOMIAL OF TORUS

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Dedicated to My Parents.

Abstract. This article proved two theorems and presented one conjecture about the real-zeros of Jones Polynomial of Torus. Topological quantum computer is related to knots/braids theory where Jones polynomials are characters of the quantum computing. Since the real-zeros of Jones polynomials of torus are observable physical quantities, except the real-zero at 1.0 there exists another distinguished real-zero in 1 ¡ r ¡ 2 for every Jones polynomial of Torus, these unique real zeros can be IDs of torus knots in topological quantum computing.

1. The Real-Zeros of Jones Polynomial of Torus

The real-zeros of Jones polynomial of torus $V_t(p, q)$ are distributed on x axis. In physics, since real-zeros of an equation usually represent observable values, it is interested to investigate them in advanced.

$$V_t(p,q) = 1 - t^{p+1} - t^{q+1} + t^{p+q}, t \in \mathbb{C} \tag{1.1}$$

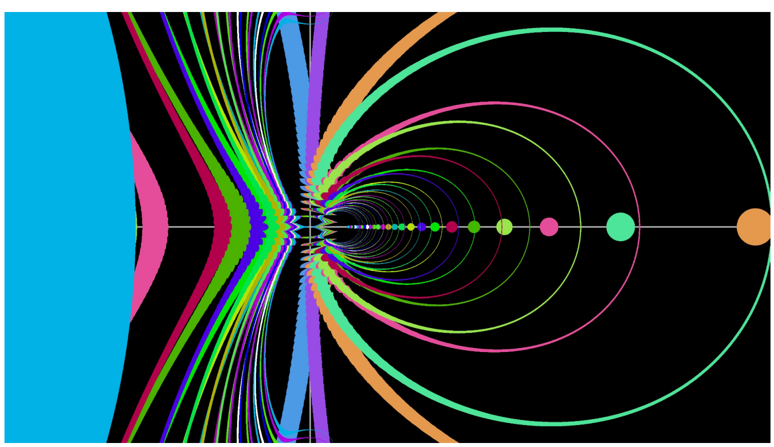

Figure 1. Real Zeros of Jones Polynomial Torus

Theorem 1. *For all Jones polynomials of Torus $V(p, q)$ there are two positive real-zeros with one is $+1$ and another is inside $1 < r < 2$.*

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Proof. Suppose a positive real-zero of $V_t(p,q)$ is r where $0 < r < +\infty$, then there is a r so that $V_r(p,q) = 0$. By Cauchy theorem, $\|r\| < 1 + max(\|a_i\|) \leq 2$. We can represent $V_r(p,q) = 0$ as $(1-r^p)(1-r^q) = (r-1)(r^p + r^q)$. If we suppose $0 < r < 1$, then $0 < r^p < 1$ and $0 < r^q < 1$, the left side of equation is positive, but since $r - 1 < 0$, the right side is negative. So there is $1 \leq r < 2$.

By Descartes' rule of signs, there are two signs change in the coefficients of equation (6), so there are two positive zeros. Obviously, +1 is a zero, another positive real-zero is in $1 < r < 2$. □

Theorem 2. *For all Jones polynomials of Torus $V(p,q)$, for both p and q are positive even integers, there are no negative zeros; for one of p or q is odd and another is even, there is one negative zero; for both p and q are odd, there are two negative zeros.*

Proof. Replace $-t$ in Jones polynomial $V_t(p,q)$ as $V_{-t}(p,q)$ Descartes's rule can be applied to decide all the negative real-zeros.

* Suppose $p = 2m, q = 2n$, then $V_{-t}(p,q) = 1 + t^{p+1} + t^{q+1} + t^{p+q}$. Because no signs change, there are no negative real-zeros.

* Suppose $p = 2m+1, q = 2n$, $V_{-t}(p,q) = 1 - t^{p+1} + t^{q+1} - t^{p+q}$. Because $V_{-t}(p,q) = V_{-t}(q,p)$, it is also true for p = 2m, q = 2n+1.

* Suppose $p = 2m+1, q = 2n+1$, then $V_{-t}(p,q) = 1 - t^{p+1} - t^{q+1} + t^{p+q}$. It is the case of positive real-zeros, so there are two negative real-zeros -1 and r with $-1 < r < 0$. □

Conjecture 1. *For $p \geq 2, q \geq 3$, $Ze(p,q)$ is the positive real-zero of $V_t(p,q)$ which is not equal to +1, then there is approximation formula below*

$$\frac{\ln(Ze(p,q)-1)}{\ln(Ze(p,q+1)-1)} \approx \frac{\ln(p+q-3)}{\ln(p+q-2)} \tag{1.2}$$

This conjecture shows the relationship of the real-zeros distribution of Jones Polynomial of Torus. Computer software has partially verified this conjecture.

References

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