Conjecture on the Fibonacci numbers with an index equal to 2p where p is prime

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Abstract. In this paper I make the following conjecture: If F(2*p) is a Fibonacci number with an index equal to 2*p, where p is prime, p ≥ 5, then there exist a prime or a product of primes q1 and a prime or a product of primes q2 such that F(2*p) = q1*q2 having the property that q2 – 2*q1 is also a Fibonacci number with an index equal to 2^n*r, where r is prime or the unit and n natural. Also I observe that the ratio q2/q1 seems to be a constant k with values between 2.2 and 2.237; in fact, for p ≥ 17, the value of k seems to be 2.236067(...).

Conjecture:

If F(2*p) is a Fibonacci number with an index equal to 2*p, where p is prime, p ≥ 5, then there exist a prime or a product of primes q1 and a prime or a product of primes q2 such that F(2*p) = q1*q2 having the property that q2 – 2*q1 is also a Fibonacci number with an index equal to 2^n*r, where r is prime or the unit and n natural.

Note:

I observe that the ratio q2/q1 seems to be a constant k with values between 2.2 and 2.237; in fact, for p ≥ 17, the value of k seems to be 2.236067(...).

Verifying the conjecture:
(for the first thirteen such Fibonacci numbers)

: for p = 5, we have F(10) = 55 = 5*11 and 11 – 2*5 = 1 = F(1), where 1 = 2^0*1;

[note the fact that 11/5 = 2.2]

: for p = 7, we have F(14) = 377 = 13*29 and 29 – 2*13 = 3 = F(4), where 4 = 2^2;

[note the fact that 199/89 = 2.230769...]

: for p = 11, we have F(22) = 17711 = 89*199 and 199 – 2*89 = 21 = F(8), where 8 = 2^3;

[note the fact that 199/89 = 2.235955...]

: for p = 13, we have F(26) = 121393 = 233*521 and 521 - 2*233 = 55 = F(10), where 10 = 2*5 and 5 is prime;

[note the fact that 521/233 = 2.236051...]

: for p = 17, we have F(34) = 5702887 = 1597*3571 and 3571 - 2*1597 = 377 = F(14), where 14 = 2*7 and 7 is prime;

[note the fact that 3571/1597 = 2.236067...]

: for p = 19, we have F(38) = 39088169 = 37*113*9349 and 9349 - 2*37*113 = 987 = F(16), where 16 = 2^4;

[note the fact that 9349/(37*113) = 2.236067...]

: for p = 23, we have F(46) = 1836311903 = 139*461*28657 and 139*461 - 2*28657 = 6765 = F(20), where 20 = 2^2*5 and 5 is prime;

[note the fact that (139*461)/28657 = 2.236067...]

: for p = 29, we have F(58) = 591286729879 = 59*19489*514229 and 59*19489 - 2*514229 = 121393 = F(26), where 26 = 2*13 and 13 is prime;

[note the fact that (59*19489)/514229 = 2.236067...]

: for p = 31, we have F(62) = 4052739537881 = 557*2417*3010349 and 3010349 - 2*557*2417 = 317811 = F(28), where 28 = 2^2*7 and 7 is prime;

[note the fact that 3010349/(557*2417) = 2.236067...]

: for p = 37, we have F(74) = 1304969544928657 = 73*149*2221*54018521 and 54018521 - 2*73*149*2221 = 5702887 = F(34), where 34 = 2*17 and 17 is prime;

[note the fact that 54018521/(557*73*149*2221) = 2.236067...]

: for p = 41, we have F(82) = 61305790721611591 = 2789*59369*370248451 and 370248451 - 2*2789*59369 = 39088169 = F(38), where 38 = 2*19 and 19 is prime;

[note the fact that 370248451/(2789*59369) = 2.236067...]

: for p = 43, we have F(86) = 420196140727489673 = 6709*144481*433494437 and 6709*144481 - 2*433494437 = 102334155 = F(40), where 40 = 2^3*5 and 5 is prime;

[note the fact that 370248451/(2789*59369) = 2.236067...]

: for p = 47, we have F(94) = 19740274219868223167 = 2971215073*6643838879 and 6643838879 - 2*2971215073 = 701408733 = F(44), where 44 = 2^2*11 and 11 is prime;

[note the fact that 6643838879/2971215073 = 2.236067...]