A Proof for infinitely many twin primes

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many “advanced mathematics tools” are used to solve them, but they are still unsolved. Based on the fundamental theorem of arithmetic and Euclid’s proof of endless prime numbers, we have proved there are infinitely many twin primes.

Introduction

Prime numbers[1] are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many “advanced mathematics tools” are used to solve them, but they are still unsolved.

I believe that prime numbers are “basic building blocks” of the natural numbers and they must follow some very simple basic rules and do not need “advanced mathematics tools” to solve them. One of the basic rules is the “fundamental theorem of arithmetic” and the “simplest tool” is Euclid’s proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic[2], which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.[1] Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof[3] that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just **n** primes, then the biggest prime will be labeled $\mathbf{P_n}$. Now we can form the number Q by multiplying together all these primes and adding 1, so

$$\mathbf{Q = (P_1 \times P_2 \times P_3 \times P_4... \times P_n) + 1}$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n.

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Twin Prime Conjecture: There are infinitely many twin primes.

A twin prime is a prime number that is either 2 less or 2 more than another prime number — for example, the twin prime pairs (11 and 13; 17 and 19; 41 and 43). In other words, a twin prime is a prime that has a prime gap of two.

Twin primes become increasingly rare as one examines larger ranges, in keeping with the general tendency of gaps between adjacent primes to become larger as the numbers themselves get larger. However, it is a longstanding conjecture that there are infinitely many twin primes. Work of Yitang Zhang[4] in 2013, as well as work by James Maynard, Terence Tao and others, has made substantial progress towards proving this conjecture, but at present it remains unsolved.

Except 5, any prime must have 1, 3, 7, or 9 as its last digit. Let $1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,…; $3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193….; $7 represents a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197…; and $9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,….

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,…; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,….; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77…; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,….

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

If a number N (N>19) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 11, it have 1/3 chance is divisible by 3 and 1/7 chance is divisible by 7, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, and 1/11 chance to be divisible by 11, so on, so we have terms: 1/3, 1/7x2/3, 1/11x2/3x6/7, 1/13x2/3x6/7x10/11, 1/17x2/3x6/7x10/11x12/13, 1/19x2/3x6/7x10/11x12/13x16/17….

Let N_o represent any odd number, the chance of N_o to be a non-prime is: [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x10/11x12/13x6/7x16/17x18/19) +(1/29x2/3x10/11x12/13x6/7x16/17x18/19x22/23) + (1/31x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29) + (1/37x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + …]------------Formula 1

Let $\sum$ represent the sum of the infinitely terms. According to Euclid's proof[3] that the set of prime numbers is endless, $\sum$ may be very close to 1 when N is growing to ∞, but always less than 1. Let $\Delta=1-\sum$, when N is growing to ∞, Δ may be very close to 0, but always more than 0. If Δ is 0, then there is no prime.

The chance of N_o to be a prime is: Δ=1-[(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x10/11x12/13x6/7x16/17x18/19) +(1/29x2/3x10/11x12/13x6/7x16/17x18/19x22/23) + (1/31x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29) + (1/37x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + …]-----------Formula 2

Every odd number (O1) with 1 as its last digit is a product of $1x$1, $3x$7 or $9x$9; the second $1 is decided by the first $1 and $7 is decided by $3, so we need to consider only $1, $3, and $9.

Every odd number (O3) with 3 as its last digit is a product of $3x$1 or $7x$9; $1 is decided by $3 and $9 is decided by $7, so we need to consider only $3 and $7.

Every odd number (O7) with 7 as its last digit is a product of $7x$1 or $3x$9; $1 is decided by $7 and $9 is decided by $3, so we need to consider only $3 and $7 (same as O3).

Every odd number (O9) with 9 as its last digit is a product of $1 x$9, $3x$3 or $7x$7; so we need to consider only $1, $3, and $7.

If a number (N>3) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, so on, so we have terms: 1/3, 1/7x2/3, 1/13x2/3x6/7…, For number N, there are N/10 odd number with 1 as its last digit, N/10 odd number with 3 as its last digit, N/10 odd number with 7 as its last digit, and N/10 odd number with 9 as its last digit.

The chance of any odd number O1 to be a prime is: Δ_1=1-$\sum_1$=1-[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31)] + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]------------Formula 3

The sum of first 20 terms = [(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+ (1/53x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43) + (1/59x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53) + (1/61x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59) + (1/71x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61) + (1/73x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71)+ (1/79x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73) + (1/83x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79) + (1/89x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83) + (1/101x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89) + (1/103x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101) + (1/109x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101x102/103)] = 0.333333 +0.060606 + 0.046620 + 0.029444 +0.023043 + 0.017481+ 0.015789 + 0.011553+ 0.010747 + 0.008517 + 0.007506 + 0.007137+

0.006032 + 0.005783 + 0.005291 + 0.004953 + 0.004564 + 0.003976 + 0.003861 + 0.003613= 0.609849

For the first 20 term: $\sum_1$ =$\sum_1$= 0.609849, Δ_1=1-$\sum_1$ =0.390151, for any number N, Δ_1 >0

Or: Δ_1=1-$\sum_1$=1-[(1/3) + (1/7x2/3) + (1/11x2/3x6/7)] + (1/17x2/3x6/7x10/11) + (1/19x2/3x6/7x10/11x16/17) + (1/29x2/3x6/7x10/11x16/17x18/19) + (1/31x2/3x6/7x10/11x16/17x18/19x28/29) + (1/41x2/3x6/7x10/11x16/17x18/19x28/29x30/31) + (1/47x2/3x6/7x10/11x16/17x18/19x28/29x30/31x40/41)+…]------------Formula 4

The chance of any odd number O3($3 x $1， or $7 x $9) to be a prime is: Δ_3=1-$\sum_3$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x12/13x16/17x22/23x36/37)+ (1/47x2/3x6/7x12/13x16/17x22/23x36/37x42/43) +…]-----------Formula 5

The chance of any odd number O7 ($7 x $1, or $3 x $9) to be a prime is: Δ_7=1-$\sum_7$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]-----------Formula 5

The sum of first 20 terms = [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) +(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) +(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83) +(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) +(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) +(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) +(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + (1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) +

(1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137) + (1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157) =[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333+0.006468+0.005848+0.005073+0.004288+0.003997+0.003810+0.003574+0.003152+0.002899+0.002511+0.002403] =0.610677

For the first 20 term: $\sum_3 = \sum_7 = 0.610677$, $\Delta_3 = \Delta_7 = 1 - \sum_3 = 0.389323$

$\Delta_3 = \Delta_7$, or:

The chance of any odd number O3 ($1x $3, or $9 x $7) to be a prime is: $\Delta_3 = 1 - \sum_3 = 1-$[(1/3) + (1/11x2/3) + (1/19x2/3x10/11) + (1/29x2/3x10/11x18/19) + (1/31x2/3x10/11x18/19x28/29) + (1/41x2/3x10/11x18/19x28/29x30/31) +…]-----------Formula 6

The chance of any odd number O7 ($1 x $7, or $3 x $9) to be a prime is: $\Delta_7 = 1 - \sum_7 = 1-$[(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/23x2/3x10/11x12/13) + (1/31x2/3x10/11x12/13x22/23) + (1/43x2/3x10/11x12/13x22/23x30/31) +…]-----------Formula 7

For a large number N, Δ_3 should be very close to Δ_7. For or any number N, Δ_3 or $\Delta_7 > 0$

The chance of any odd number O9 to be a prime is: $\Delta_9 = 1 - \sum_9 = 1-$[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11)+ (1/17x2/3x6/7x10/11x12/13) +(1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) +(1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) +…] -------Formula 8

The sum of first 20 terms = [(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13)] +[1/2(1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) +(1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47) + (1/61x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53) + (1/67x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61) + (1/71x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67) +

(1/73x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71) + (1/83x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73) + (1/97x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83) + (1/101x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97) + 1/103x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101)] = 0.333333 + 0.095238 + 0.051948 + 0.039960+ 0.028207+ 0.019622 + 0.013926 + 0.011291 + 0.009914 + 0.009222 + 0.008241 + 0.007153 + 0.006097 + 0.005460 + 0.005076 + 0.004867 + 0.004222+ 0.003569 + 0.003393 + 0.003294 = 0.664033

For the first 20 term: $\sum_9$ = 0.664033, Δ_1=1-$\sum_9$ =0.335967, for any number N, Δ_9 >0

Euclid's proof that the set of prime numbers is endless, from these formulas, primes \$1, \$3, \$7, \$9 should be endless.

Or: Δ_9=1-$\sum_9$=1-[(1/3) + (1/7x2/3)+ (1/13x2/3x6/7)+ (1/17x2/3x6/7x12/13) + (1/19x2/3x6/7x12/13x16/17) + (1/23x2/3x6/7x12/13x16/17x18/19) + (1/29x2/3x6/7x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x12/13x16/17x18/19x22/23x28/29) + (1/43x2/3x6/7x12/13x16/17x18/19x22/23x28/29x36/37) + (1/47x2/3x6/7x12/13x16/17x18/19x22/23x28/29x42/43) +…] -------Formula 9

The chance for O3 following a non-prime, O1, to be a prime is: Δ_3=1-$\sum_3$=1-{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]

The chance for O3 following a prime, \$1, to be a prime is: Δ_3=1-$\sum_3$=1-{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…] - 1/\$1x2/3x6/7x….}

Let assume the pair, $\$1_{x}$,$\$3_{x}$ is the largest twin pair, however we know both $\$1_{x}$ and $\$3_{x}$ are endless, there are $\$1_{x+1}$, $\$1_{x+2}$, $\$1_{x+3}$,… $\$1_{x+n}$, and $\$3_{x+1}$, $\$3_{x+2}$, $\$3_{x+3}$,… $\$3_{x+n}$. We need only find if there is at least 1 of $\$3_{x+n}$ follow 1 of $\$1_{x+n}$. any \$1 can be divisible by only 1 and itself, so $\$1_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 14, 16, or 18) cannot be divisible by $\$1_{x+n}$ because the smallest \$1 is 11 and the next \$1 is 31.

The chance for O3 following a non-prime, O1, to be a prime is: Δ_3=1-$\sum_3$=1-{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) +

(1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]

The chance for O3 following a prime, \$1, to be a prime is: Δ_3=1-$\sum_3$=1-{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…] - 1/$\$1_{x+1}$}

The O3 next to $\$1_{x+1}$ is only 2 (also is true for 4, 6, 8, 10, 12, 14, 16, or 18 difference) different from $\$1_{x+1}$, so the term is a single term 1/$\$1_{x+1}$. For limited n primes, $\$1_{x+n}$, we will have – [1/$\$1_{x+1}$+ 1/$\$1_{x+2}$+ 1/$\$1_{x+3}$,…+ 1/$\$1_{x+n}$]. If the number n of primes \$1 ≥ $\$1_{x+1}$, then: n(1/$1_{x+1}$+ 1/$\$1_{x+2}$+ 1/$\$1_{x+3}$,…+ 1/$\$1_{x+n}$)= ($\$1_{x+1}$/$\$1_{x+1}$ + $\$1_{x+1}$/$\$1_{x+2}$ + $\$1_{x+1}$/$\$1_{x+3}$,…. $\$1_{x+1}$/$\$1_{x+n}$)= (1+ $\$1_{x+1}$/$\$1_{x+2}$ + $\$1_{x+1}$/$\$1_{x+3}$,…. $\$1_{x+1}$/$\$1_{x+n}$)>1, so n$\Delta_3$=n-n$\sum_3$ + (1+ $\$1_{x+1}$/$\$1_{x+2}$ + $\$1_{x+1}$/$\$1_{x+3}$,…. $\$1_{x+1}$/$\$1_{x+n}$), even if $\sum_3$ =1, nΔ_3>1, so there is at least 1 more $\$3_{x+n}$ following one of $\$1_{x+n}$ for every n primes of $\$1_{x+n}$, for n=100, \$1 is endless, then twin prime pairs (\$1, \$3) is endless. It is easy to prove (\$7,\$9), (\$9, \$1), (\$3, \$7, p+4 prime), (\$7, \$1, p+4 prime), (\$1, \$7, sexy prime), (\$7, \$3), (\$3, \$9), (p, p+8), (p, p+10), (p, p+12), (p, p+14), (p, p+16), and (p, p+18) are endless.

References:

1. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
2. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., Section 2, Lemma 5.
3. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., p. 10, section 2.
4. Zhang, Yitang (2014). "Bounded gaps between primes".Annals of Mathematics. 179 (3): 1121–1174