The Dangers of Adopting Results from One Discipline in Another.

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Abstract.

Quite often in the sciences, results from one area are used in other quite different scenarios. However, it is crucial in these cases to ensure beyond all doubt that any assumptions made in deriving the initial result are fulfilled in all subsequent applications. It is the purpose of this note to draw attention to this highly important, but often neglected, point.
Introduction.

It is quite common practice to adopt and use a well-known result from one area of the physical sciences in another. However, when doing so, great care must be exercised to ensure that any basic assumptions made in deriving the original result are satisfied exactly in the new circumstances. This sounds almost trivially obvious but, unfortunately, the care is not always exercised in practice. A typical example is provided by the manipulation involved in writing the expression for the mean-square relative fluctuation in number of particles in terms of the isothermal compressibility and then using the resulting expression to discuss behaviour below the condensation temperature. The manipulation involved crucially assumes that the pressure is dependent on both the volume and temperature but, below the condensation temperature, the pressure is independent of the volume and so the quoted manipulation is invalid and the value of the said fluctuations below the condensation temperature may not be evaluated using the expression involving the isothermal compressibility. However, as may be noted from the mentioned references, this has not always been the case. The example does serve to illustrate that, even at fairly elementary levels, great care must be exercised when using established results in areas other than that in which they first occurred; here the original result being essentially from pure mathematics and then used subsequently in statistical physics.

Open, Closed and Isolated Systems.

Early in any introduction to thermodynamics, the ideas of open, closed and isolated systems are met. In truth, the notions are quite straightforward and initially cause no difficulty: a closed system is one which may exchange energy with its surroundings but not matter, otherwise it is deemed open. An isolated system is one which may exchange neither matter nor energy with its surroundings.

Obviously these definitions of both closed and isolated systems represent idealisations; neither can exist in practice although, in the context of present understanding, the Universe as a whole might be termed an isolated system. With the modern understanding of mass/energy equivalence, the exchange of energy between a system and its surroundings might be claimed to constitute an exchange of mass. However, it might be noted that, according to the usual mathematical formulation of thermodynamics, the First and Second Laws may be combined so that the equation

$$TdS = dU + pdV$$

holds for closed systems but, if the system is open, this equation is replaced by

$$TdS = dU + pdV - \mu dN$$

where all the symbols have their usual meanings. It is immediately apparent that the openness of the system finds expression in the inclusion of the term $\mu dN$. Hence, the openness is linked with a change in number of particles and so “no exchange of matter” is effectively replaced by “no exchange of particles”. To the founding fathers of the subject ‘thermodynamics’, this change would have been of little consequence but, today, following advances in physics, this seemingly small point is one which can no longer be ignored.
The Second Law of Thermodynamics.

Recently\(^4\), questions were raised over the correctness of identifying the statement “the entropy never decreases” as a statement of the Second Law of thermodynamics. It was shown that such an identification is of dubious validity. In fact, since the entropy function of classical thermodynamics is defined by

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dS = d'Q/T,
\]

it is immediately clear that this entropy function does decrease whenever heat is removed from a system at positive absolute temperatures. Hence, at the very least the range of validity of the quoted statement as a wording of the Second Law of Thermodynamics must come into question. Nevertheless, this statement is taken in areas outside physics as a statement of the Second Law of Thermodynamics. For example, in a well-known book on the molecular biology of the cell\(^5\), one reads

*Cells must obey the laws of physics and chemistry. The rules of mechanics and of the conversion of one form of energy to another apply just as much to a cell as to a steam engine. There are, however, puzzling features of a cell that, at first sight, seem to place it in a special category. It is common experience that things left to themselves eventually become disordered: buildings crumble, dead organisms become oxidised, and so on. This general tendency is expressed in the Second Law of Thermodynamics, which states that the degree of disorder in the universe (or in any isolated system in the universe) can only increase.*

There are several points contained in this wording which deserve examination. Firstly, simply linking the ‘degree of order’ with entropy requires explanation. It is an identification which does occur in physics but it is usually encountered when considering either the entropy function of statistical mechanics or that of information theory. Both these entropy functions are statistical in origin whereas that of classical thermodynamics is not; that entropy function’s change is irrevocably linked with heat flow either into, for a positive entropy change, or out of, for a negative entropy change, the system in question. This point does, of course, raise once again the validity of the assumption that all three entropy functions mentioned are equivalent\(^6\). Secondly, of course, since it is implied that ‘degree of disorder’ is linked with the notion of the entropy function of classical thermodynamics, the range of validity of the statement must be queried as discussed earlier. Thirdly, given the earlier definition of an ‘isolated’ system, a discussion of what processes may occur in such a system to allow any change at all in the entropy is required. This quoted piece seems a good example of where a result from one discipline (in this case classical thermodynamics) has been quoted in another totally different discipline (in this case molecular biology).

Hence, in the field of molecular biology, is the quoted rule correct? That is for others to say but it is certainly true that the statement is not a completely general statement of the Second Law of Classical Thermodynamics; the valid statements of that law are still embodied only in the familiar statements due to Kelvin and Clausius and also that more abstract one due to Carathéodory. While the relations linking these three forms of the Second Law are well-known\(^7\), the link with the entropy increase statement, while well-known, is based on much less solid foundations.
References.


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