A Proof of Goldbach's Conjecture

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. We give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture , fundamental theorem of arithmetic, Euclid's proof of infinite primes

Introduction

Prime numbers[1] are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many “advanced mathematics tools” are used to solve them, but they are still unsolved.

I believe that prime numbers are “basic building blocks” of the natural numbers and they must follow some very simple basic rules and do not need “advanced mathematics tools” to solve them. One of the basic rules is the “fundamental theorem of arithmetic” and the “simplest tool” is Euclid’s proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic[2], which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.[1] Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof[3] that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just **n** primes, then the biggest prime will be labeled $\mathbf{P_n}$. Now we can form the number Q by multiplying together all these primes and adding 1, so

$$\mathbf{Q = (P_1 \times P_2 \times P_3 \times P_4 ... \times P_n) + 1}$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n.

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

If N is an even integer:

$N= N/2 + N/2 = (N/2+m) + (N/2-m)$; $m = 0, 1, 2, 3, \ldots\ldots M$. We need to prove $[(N/2+m]$ and $[N/2-m]$ can be primes at same time.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach's conjecture is about all numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, if we divide the numbers in groups, the Goldbach's conjecture will be much simpler.

Let N_o represent any odd number, the chance of N_o to be not a prime is: [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x10/11x12/13x6/7x16/17x18/19) +(1/29x2/3x10/11x12/13x6/7x16/17x18/19x22/23) + (1/31x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29) + (1/37x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + …]} -------Formula 1

Let ∑ represent the sum of the infinitely terms and Δ=1-∑, according to Euclid's proof[3] that the set of prime numbers is endless. Δ is the chance of any number to be a prime.

∑ may be very close to 1 when N is growing to ∞, but always less than 1. Let Δ=1-∑, when N is growing to ∞, Δ may be very close to 0, but always more than 0 according to Euclid's proof[3] that the set of prime numbers is endless. If Δ is 0, then there is no prime.

Let N_o represent any odd number, the chance of N_o to be a prime is: Δ=1-[[(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x10/11x12/13x6/7x16/17x18/19) +(1/29x2/3x10/11x12/13x6/7x16/17x18/19x22/23) + (1/31x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29) + (1/37x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x6/7x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + …]} -------Formula 2

Let us consider the following cases:

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 110, 120, 1120, 1130,…, then N/2 has only 0 or 5 as its last digit:

1a. Except 5, any prime must have 1, 3, 7, or 9 as its last digit. When both N and N/2 have 0 as their last digit, then N must be 20, 30, 40, 60, 80, 100, 120,…, N. For enough large number N, Let's consider $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$, O_1 and O_2 is an odd number. $O_1>O_2$, $O_1-O_2=2L+6$, L=0, 5, 10, 15, 20, 25, 30….L, $O_1-O_2=2L+6=6$, 26, 46, 66, 86, 106, 126,…(2L+6), then O_1 is an odd number with 3 as its last digit, O_2 is an odd number with 7 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$, O_1 and O_2 is an odd number. $O_1>O_2$, $O_1-O_2=2L+14$, L=0, 10, 20, 30….L, $O_1-O_2=2L+14=14$, 34, 54, 74, 94, 114, 134,…(2L+14), then O_1 is an odd number with 7 as its last digit, O_2 is an odd number with 3 as its last digit.

Then, we have odd number pairs as listed in table 1:

Table 1. The odd number pairs in $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$ and $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$

N-7	N-17	N-37	N-47	N-67	N-97	…	N/2+L+3	N/2-L-7	…	83	73	53	43	23	13	3
7	17	37	47	67	97	…	N/2-L-3	N/2+L+7	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Let $1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,…; $3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193….; $7 represents a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197…; and $9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,….

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,…; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,….; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77…; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,….

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number with 3 as its last digit is a product of $3x$1 or $7x$9; $1 is decided by $3 and $9 is decided by $7, so we need to consider only $3 and $7.

For number N, there are N/10 odd numbers. According to Euclid's proof, primes are endless and it is easy to prove that prime with 3 as its last digit is also endless.

If a number (N>3) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, so on, so we have terms: 1/3, 1/7x2/3, 1/13x2/3x6/7…, For number N, there are N/10 odd number with 1 as its last digit, N/10 odd number with 3 as its last digit, N/10 odd number with 7 as its last digit, and N/10 odd number with 9 as its last digit; The number (n) of primes in N/10 odd number with 3 as its last digit: n=N/10 -{N/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]} ----------(Formula 3)

For infinitely terms, the number will grow slowly and will be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid’s proof of endless prime numbers. Let $\sum_3$ represent the sum of the above infinitely terms. When N is growing to ∞, and $\Delta=1-\sum_3 >1-\sum$ may be close to 0, but never be 0.

The sum of first 20 terms=[(1/3 + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) +(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) +(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83)

+(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) +(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) +(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) +(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + (1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + (1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137) + (1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157) =N/10[1/3+1/10.5+1/22.75+1/32.23+1/46.33+1/77.92+1/93.07+1/104.15+1/120+1/164.61+1/171.01+1/197.14+1/233.2+1/250.2+1/262.47+1/279.80+1/317.27+1/344.97+1/398.24+1/416.11+=[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333+0.006075+0.005848+0.0050773+0.004288+0.003997+0.003810+0.003574+0.003152+0.002899+0.002511+0.002403+0.002331] =0.6102883

For N=600, the smallest prime $1 is 11, it decided the possible largest prime $3 is 53, the smallest prime $9 is 19, but 3 x 3 is 9, so the possible largest prime $7 is 47, 47 x 3 x3=423 (the next will 67 x 3 x3=603>600), so we have: Prime number with 3 as its last digit=600/10 –{600/10[(1/3)+ (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47)] = 600/10-600/10[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333=60-60x0.566654=60-34=26, it is 3 less than 29 primes (in total 60 odd number) with 3 as their last digit from 1 to 600 due to the first three odd numbers 3, 13, and 23 is too small and cannot be divisible by primes 7, 11, 13, 17, or 19. When N is big enough, the calculated number will be very close to the real number. . For N=600, we have Δ=1-∑=1-0.566654=0.433346, every odd number with 3 as its last digit has almost 43% chance to be a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime; for a number bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime.

Every odd number with 7 as its last digit is a product of $3x$9 or $7x$1; $1 is decided by $7 and $9 is decided by $3, so we need to consider only $3 and $7.

The number (n) of primes in N/10 odd number with 7 as its last digit is: n=N/10 -{N/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + …]} --------Formula 3.

Let $\sum_7$ represent the sum of the above infinitely terms. $\sum_7 = \sum_3$, when N is growing to ∞, and $\Delta=1-\sum_7 > 1-\sum$ may be close to 0, but never be 0.

That mean we have almost same number of primes with 3 as their last digit as the number of primes with 7 as their last digit. From above formula, we can know smaller number N has high percentage to be primes than bigger number N.

For formula 3, we also can know: for any odd number (O7) with 7 as its last digit, the chance of N-O7 to be an odd number (but not prime) with 3 as its last digit is: [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) …] -------Formula 4

If this O7 is a prime $7, the chance of (N-$7) to be an odd number (but not prime) with 3 as its last digit is smaller than (N-O7). Why?

Let see $7=7 first (left side of table 1), When $7 is 7, $N = O_1+O_2$, O_1-O_2 = (N-7)-7=2L+6. Only 56, 126, 196,…, is divisible by 7, however, L=0, 10, 20, 30, 40…. and (2L+6) =6, 26, 46, 66, 86, 106, 126, 146, 166, 186, 206…, only 1 in 3 is shown on (2L+6) that is divisible by 7, so the term (1/7x2/3), not (1/7) should be taken off from Formula 3.

The chance of (N-7) to be an odd number (not prime) with 3 as its last digit is: [(1/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…] -------------Formula 5.

For the next prime $7=17, the chance of (N-17) to be an odd number (not prime) with 3 as its last digit is: [(1/3) +(1/7x2/3) (1/13x2/3x6/7) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + …] ----------- Formula 6

Term (1/17x2/3x6/7x12/13) should be taken off from Formula 3, so on.... Finally, we have:

[(1/3) + (1/13x2/3x6/7) + (1/23x2/3x12/13x6/7x16/17) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ …]------------Formula 7

For the right side of table 1, starting from \$3=3, $N = O_1+O_2$, $O_1-O_2 = (N-3)-3=2L+14$, The chance of (N-3) to be an odd number (not prime) with 7 as its last digit is: [(1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43)…]-------------Formula 8.

Term (1/3) should be taken off from Formula 3.

For the next prime 13, the chance of (N-13) to be an odd number (not prime) with 3 as its last digit is: [(1/3)+(1/7x2/3) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]---------Formula 9

The term (1/13x2/3x6/7) should be removed from Formula 3, so on... Finally, the remainder term in Formula 7 are cancelled

For number N, let n_7 be the number of primes (N-\$7) with 3 as its last digit which matches a prime (\$7) with 7 as its last digit and n_3 be the number of primes (N-\$3) with 7 as its last digit which matches a prime (\$3) with 3 as its last digit $n_7 = n_3$, $n= n_7 +n_3= 2n_3$. n is the total number of primes (N-\$3) and (N-\$7) that have 3 or 7 as their last digital and match \$7 or \$3 :

$n= n_3 +n_7 = n_3$-{(n_3-1)[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + …]} + n_7-{(n_7-1)[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + …]} $=2n_3 = n_3-[(n_3-1)(\sum_3)] + n_7-[(n_7-1)(\sum_7)] =n-n\sum_3+\sum_3=[n(1-\sum_3)+\sum_3]=n\Delta+\sum=n\Delta_3+(1-\Delta_3)=1+n\Delta_3-\Delta_3=1+(n-1)\Delta_3>1$ because $n>1$ and $\Delta_3>0$, The results show: When both N and N/2 have 0 as their last digit, there is at least one pair primes in which one prime has 7 as its last digit and another has 3 as its last digit and their sum is N (see table 1). This is an extreme situation, normally, more than 1 pair primes can be found. For N=600, n=(600/2)x(1/10)=30, $\sum_3=0.57$ and $\Delta_3=0.43$, $n\Delta_3+\sum_3=30\text{x}0.57=13.5$, in fact, 600 can be expressed as the sum of 15 pairs of primes which has 7 as one prime last digit and 3 as another prime last digit.

For enough large number N, Let's consider $N=O_1+O_2=(N/2+L+1)+(N/2-L-1)$, O_1 and O_2 are odd numbers. $O_1>O_2$, $O_1-O_2=2L+2$, L=0, 10, 20, 30….L, $O_1-O_2=2L+2=2, 22, 42, 62, 82, 102, 122,…(2L+2)$, then O_1 is an odd number with 1 as its last digit, O_2 is an odd number with 9 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+9)+(N/2-L-9)$, O_1 and O_2 are odd numbers. $O_1>O_2$, $O_1-O_2=2L+18$, L=0, 10, 20, 30….L, $O_1-O_2=2L+18=18, 38, 58, 78, 98, 118, 138,…(2L+18)$, then O_1 is an odd number with 9 as its last digit, O_2 is an odd number with 1 as its last digit.

These odd number pairs are listed in table 2:

Table 2. The odd number pairs in $N=O_1+O_2=(N/2+L+1)+(N/2-L-1)$ and $N=O_1+O_2=(N/2+L+9)+(N/2-L-9)$

N-9	N-19	N-29	N-39	N-59	N-79	N-89	…	N/2+L+1	N/2-L-9	…	101	71	61	41	31	11
9(3x3)	19	29	39	59	79	89	…	N/2-L-1	N/2+L+9	…	N-101	N-71	N-61	N-41	N-31	N-11

Every odd number (O1) with 1 as its last digit is a product of $1x$1, $3x$7, and $9x$9. For $1x$1 and $9x$9, every odd number with 1 as its last digit will cost 2 $1 or $9. The first $9 is 19, but the odd number 9 = 3x3, so 3 is the smallest prime for $9 and the number (n) of primes in N/10 odd number with 1 as its last digit: n=N/10 -{N/10[(1/3) + ½[(1/11x2/3)] + (1/13x2/3x10/11)+ ½[(1/19x2/3x10/11x18/19)] +(1/23x2/3x10/11x18/19x18/19) + ½[(1/29x2/3x10/11x18/19x18/19x22/23)] + ½[(1/31x2/3x10/11x18/19x18/19x22/23x28/29)] + ½[(1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31)] + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41)+…]}------------Formula 10

For infinitely terms, the number will grow slowly and can be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid's proof of endless prime numbers. $\sum_1$ represent the sum of the infinitely terms and $\Delta_1=1-\sum_1$. The sum of Formula 10 is smaller to the sum of Formula 1, that is $\sum_1<\sum$, so $\Delta_1>\Delta$.

Every odd number (O9) with 9 as its last digit is a product terms of $1 x$9, $3x$3 or $7x$7, for $3x$3 and $7x$7, every odd number with 1 as its last digit will cost 2 $3 or $7. We need to consider only $1, $3, and $7 and the number (n) of primes in N/10 odd number with 9 as its last digit: n=N/10 -{N/10[[1/2(1/3)]+ [1/2(1/7x2/3)]+ (1/11x2/3x6/7) + [1/2(1/13x2/3x6/7x10/11)]+ [1/2(1/17x2/3x6/7x10/11x12/13)] +[1/2(1/23x2/3x6/7x10/11x12/13x16/17)] + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) +[1/2(1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31)] + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + [1/2(1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41)] + [1/2(1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)]…]}-----------Formula 11

[1/2(1/3)] and [1/2(1/7x2/3)] are the of the largest terms and total is 0.21, so sum of Formula 11 ($\sum_9$) is less than the sum of formula 10 ($\sum_1$) or formula 1($\sum$),that is $\sum_9<\sum$, so $\Delta_9>\Delta$.

That means there is some higher chance to have at least one pair primes in which one prime has 9 as its last digit and another has 1 as its last digit and their sum is N (see table 2).

For N = 600 (see table 3), 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit and 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime with 9 as its last digit.

7 Prime	17 Prime	27 3x9	37 Prime	47 prime	57 3x19	67 Prime	77 7x11	87 3x29	97 Prime	107 prime	117 3x3x13	127 Prime	137 prime	147 3x7x7	157 prime	167 prime	177 3x59	187 11x17
593 Prime	583 11x53	573 3x191	563 Prime	553 7x79	543 3x181	533 13x41	523 prime	513 3x3x3x19	503 Prime	493 17x29	483 3x7x23	473 11x43	463 prime	453 3x151	443 Prime	433 prime	423 3x3x47	413 7x59
223 prime	233 prime	243 3x3x3x3x3	253 11x23	263 prime	273 3x7x13	283 prime	293 prime	303 3x101	313 Prime	323 17x19	333 3x111	343 7x7x7	353 Prime	363 3x11x11	373 Prime	383 Prime	393 3x131	403 13x31

377 13x29	367 prime	357 3x7x17	347 prime	337 prime	327 3x109	317 Prime	307 prime	297 3x3x33	287 7x41	277 Prime	267 3x89	257 Prime	247 13x19	237 3x79	227 Prime	217 7x31	207 3x3x23	197 Prime
387 3x3x43	397 prime	407 11x37	417 3x139	427 7x61	437 23x19	447 3x149	457 prime	467 prime	477 3x3x53	487 prime	497 7x71	507 3x13x13	517 11x47	527 17x31	537 3x179	547 prime	557 prime	567 3x3x3x3x7
213 3x71	203 7x29	193 prime	183 3x61	173 Prime	163 prime	153 3x3x17	143 11x13	133 7x19	123 3x41	113 3x3x13	103 prime	93 3 x31	83 prime	73 7x11	63 3x3x7	53 prime	43 prime	33 3x11
																3 Prime	13 Prime	23 prime
																597 3x199	587 prime	577 prime
589 19x31	579 3x193	569 Prime	559 13x43	549 3x3x61	539 7x7x11	529 23x23	519 3x178	509 Prime	499 Prime	489 3x163	479 Prime	469 7x67	459 3x3x3x17	449 Prime	439 Prime	429 3x11x13	419 Prime	409 Prime
11 Prime	21 3x7	31 Prime	41 Prime	51 3x17	61 Prime	71 Prime	81 3x3x3	91 7x13	101 Prime	111 3x37	121 11x11	131 Prime	141 3x47	151 Prime	161 7x23	171 3x3x19	181 Prime	191 Prime

381 3x127	371 7x53	361 19x19	351 3x3x3 x13	341 11x31	331 Prime	321 3x107	311 Prime	301 7x43	291 3x97	281 Prime	271 Prime	261 3x3x2 9	251 Prime	241 Prime	231 3x7x1 1	221 13x17	211 Prime	201 3x67
219 3x73	229 Prime	239 Prime	249 3x83	259 7x37	269 Prime	279 3x3x3 1	289 17x17	299 13x23	309 3x103	319 11x29	329 7x47	339 3x113	349 Prime	359 Prime	369 3x3x4 1	379 prime	389 Prime	399 3x7x19
209 11x19	199 Prime	189 3x3x7	179 Prime	169 13x13	159 3x53	149 Prime	139 Prime	129 3x43	119 7x17	109 Prime	99 3x3x11	89 Prime	79 Prime	69 3x23	59 Prime	49 7x7	39 3x13	29 Prime
391 17x23	401 Prime	411 3x137	421 Prime	431 Prime	441 3x3x7 x7	451 11x41	461 Prime	471 3x157	481 13x37	491 Prime	501 3x167	511 7x73	521 Prime	531 3x3x5 9	541 Prime	551 19x29	561 3x11x17	571 Prime
																	591 3x197	581 7x81
																	9 3x3	19 Prime

1b. When both N has 0 as its last digit, and N/2 has 5 as its last digit.

Table 3. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L-2)$

N-7	N-17	N-37	N-47	N-67	N-97	…	N/2+L-2	N/2-L-2	…	83	73	53	43	23	13	3
7	17	37	47	67	97	…	N/2-L+2	N/2+L+2	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

In same way as 1a, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 7 as its last digit and their sum is N (see table 3).

Table 4. The odd number pairs in $N=O_1+O_2=(N/2+L-4)+(N/2-L+4)$ and $N=O_1+O_2=(N/2+L-6)+(N/2-L+6)$

N-9	N-19	N-29	N-59	N-79	N-89	…	N/2+1-4	N/2-L+6	…	101	71	61	41	31	11
9(3x3)	19	29	59	79	89	…	N/2-L+4	N/2+L-6	…	N-101	N-71	N-61	N-41	N-31	N-11

There is some higher chance to have at least one pair primes in which one prime has 9 as its last digit and another has 1 as its last digit than in which one prime has 3 as its last digit and another has 7 as its last digit and their sum is N (see table 4).

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 112, 122, 1122, 1132,…, then N/2 has only 6 or 1 as its last digit.

   2a. When any even integer (N) has 2 as its last digit, such as 12, 32, 52, 112, 1132,…, then N/2 has 6 as its last digit:

Table 5. The odd number pairs in $N=O_1+O_2=(N/2+L-3)+(N/2-L+3)$ and $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$

N-9	N-19	N-29	N-59	N-79	N-89	…	N/2+L-3	N/2-L-3	…	83	73	53	43	23	13	3
9(3x3)	19	29	59	79	89	…	N/2-L+3	N/2+L+3	…	N-83	N-73	N-53	N-43	N-23	N-13	N-3

Due to $\Delta_9 > \Delta_7$ or Δ_3 , there is some higher chance to have at least one pair primes in which one prime has 9 as its last digit and another has 3 as its last digit than in which one prime has 3 as its last digit and another has 7 as its last digit and their sum is N.

Table 6. The odd number pairs in $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$ and $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$

N-11	N-31	N-41	N-61	N-71	N-101	…	N/2+L-5	N/2-L+5	…	101	71	61	41	31	11
11	31	41	61	71	101	…	N/2-L+5	N/2+L-5	…	N-101	N-71	N-61	N-41	N-31	N-11

Due to $\Delta_1 > \Delta_7$ or Δ_3 , there is some higher chance to have at least one pair primes in which both prime has 1 as its last digit than in which one prime has 3 as its last digit and another has 7 as its last digit and their sum is N.

2b. When any even integer (N) has 2 as its last digit, such as 22, 42, 62, 82, 102, 1122,…, then N/2 has 1 as its last digit:

Table 7. The odd number pairs in $N=O_1+O_2=(N/2+L+2)+(N/2-L+8)$ and $N=O_1+O_2=(N/2-L+2)+(N/2+L+8)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L+2	N/2-L+2	…	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	…	N/2-L+8	N/2+L+8	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 7 is same to table 5, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 8. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L-0)$ and $N=O_1+O_2=(N/2+L-0)+(N/2-L+0)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L+0	N/2-L+0	…	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	…	N/2-L-0	N/2+L-0	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 8 is same to table 6, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

3a. When any even integer (N) has 4 as its last digit, such as 24, 44, 64, 84, 104, 1124,…, then N/2 has 2 as its last digit:

Table 9. The odd number pairs in $N=O_1+O_2=(N/2+L+1)+(N/2-L+7)$ and $N=O_1+O_2=(N/2-L+1)+(N/2+L+7)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L+1	N/2-L+1	…	83	73	63	53	43	33	23	13	3

9	19	29	39	49	59	69	79	…	N/2-L+7	N/2+L+7	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 9 is same to table 7, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 10. The odd number pairs in $N=O_1+O_2$=(N/2+L-1)+(N/2-L-1) and $N=O_1+O_2$=(N/2+L-1)+(N/2-L-1)

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L-1	N/2-L-1	…	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	…	N/2-L-1	N/2+L-1	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 10 is same to table 8, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

3b. When any even integer (N) has 4 as its last digit, such as 14, 34, 54, 74, 94, 1114,…, then N/2 has 7 as its last digit:

Table 11. The odd number pairs in $N=O_1+O_2$=(N/2+L+6)+(N/2-L+2) and $N=O_1+O_2$=(N/2-L+6)+(N/2+L+2)

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L+6	N/2-L+6	…	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	…	N/2-L+2	N/2+L+2	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 11 is same to table 9, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 12. The odd number pairs in $N=O_1+O_2=(N/2+L+4)+(N/2-L+4)$ and $N=O_1+O_2=(N/2+L+4)+(N/2-L+4)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L+4	N/2-L+4	…	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	…	N/2-L+4	N/2+L+4	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 12 is same to table 10, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

4a. When any even integer (N) has 6 as its last digit, such as 26, 46, 66, 86, 106, 1126,…, then N/2 has 3 as its last digit:

Table 13. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L+6)$ and $N=O_1+O_2=(N/2-L+0)+(N/2+L+6)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L+0	N/2-L+0	…	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	…	N/2-L+6	N/2+L+6	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 13 is same to table 11, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 14. The odd number pairs in $N=O_1+O_2$=(N/2+L-2)+(N/2-L+8) and $N=O_1+O_2$=(N/2+L+8)+(N/2-L-2)

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L-2	N/2-L-2	…	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	…	N/2-L+8	N/2+L+8	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 14 is same to table 12, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

4b. When any even integer (N) has 6 as its last digit, such as 16, 36, 56, 76, 96, 1116,…, then N/2 has 8 as its last digit:

Table 15. The odd number pairs in $N=O_1+O_2$=(N/2+L-5)+(N/2-L+1) and $N=O_1+O_2$=(N/2-L+5)+(N/2+L+1)

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L-5	N/2-L+5	…	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	…	N/2-L+1	N/2+L+1	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 15 is same to table 13, there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 16. The odd number pairs in $N=O_1+O_2$=(N/2+L+3)+(N/2-L+3) and $N=O_1+O_2$=(N/2+L+3)+(N/2-L+3)

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L+3	N/2-L+3	…	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	…	N/2-L+3	N/2+L+3	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 16 is same to table 14, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

5a. When any even integer (N) has 8 as its last digit, such as 28, 48, 68, 88, 108, 1128,…, then N/2 has 4 as its last digit:

Table 17. The odd number pairs in $N=O_1+O_2$=(N/2+L-1)+(N/2-L+5) and $N=O_1+O_2$=(N/2-L-1)+(N/2+L+5)

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L-1	N/2-L-1	…	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	…	N/2-L+5	N/2+L+5	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 17 is same to table 15, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 18. The odd number pairs in $N=O_1+O_2$=(N/2+L-3)+(N/2-L+7) and $N=O_1+O_2$=(N/2+L+7)+(N/2-L-3)

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L-3	N/2-L-3	…	91	81	71	61	51	41	31	21	11

11	21	31	41	51	61	71	81	…	N/2-L+7	N/2+L+7	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 18 is same to table 16, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

5b. When any even integer (N) has 8 as its last digit, such as 18, 38, 58, 78, 98, 1118,…, then N/2 has 9 as its last digit:

Table 19. The odd number pairs in $N=O_1+O_2=(N/2+L-6)+(N/2-L+0)$ and $N=O_1+O_2=(N/2-L-6)+(N/2+L+0)$

N-9	N-19	N-29	N-39	N-49	N-59	N-69	N-79	…	N/2+L-6	N/2-L-6	…	83	73	63	53	43	33	23	13	3
9	19	29	39	49	59	69	79	…	N/2-L+0	N/2+L+0	…	N-83	N-73	N-63	N-53	N-43	N-33	N-23	N-13	N-3

Table 19 is same to table 17, there is at least one pair primes in which one prime has 3 as its last digit and another has 9 as its last digit and their sum is N.

Table 20. The odd number pairs in $N=O_1+O_2=(N/2+L+2)+(N/2-L+2)$ and $N=O_1+O_2=(N/2+L+2)+(N/2-L+2)$

N-11	N-21	N-31	N-41	N-51	N-61	N-71	N-81	…	N/2+L+2	N/2-L+2	…	91	81	71	61	51	41	31	21	11
11	21	31	41	51	61	71	81	…	N/2-L+2	N/2+L+2	…	N-91	N-81	N-71	N-61	N-51	N-41	N-31	N-21	N-11

Table 20 is same to table 18, there is at least one pair primes in which both primes have 1 as its last digit and their sum is N.

For any even number, Goldbach's conjecture is true.

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1. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
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